An algorithm for converting an electrical circuit to a bond graph : a formalization of the inspection method

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An Algorithm for Converting an Electrical Circuit to a Bond Graph: a Formalization of the Inspection Method

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Abstract

In this article, we show that the algorithm proposed by Karnopp and Rosenberg for converting a circuit into a bond graph is not fully adapted to the bond graph formalism. This formalism implicitly deletes information which is useless for describing the circuits' functioning. But this information is not systematically deleted in the bond graphs obtained with their algorithm. We propose an algorithm based on another of their method, called the inspection method. It was not yet formalized and allows to obtain bond graphs where the useless information is entirely deleted. This algorithm uses the Tutte decomposition of graphs and associates a unique bond graph with a given circuit.

1   INTRODUCTION

Since the introduction of the bond graph representation [1], algorithms have been proposed to convert a physical system to a bond graph. This step is central for simulating the system because it determines the efficiency of the bond graph algorithms. Some physical systems can easily be represented by linear graphs. Electrical circuits are examples of such systems. In this special case, Karnopp and Rosenberg recommended an algorithm in [2] which transforms a linear graph into a bond graph. It is now widely used and it will be called from now on the straightforward algorithm. They also vaguely described another conversion procedure called the inspection method. This paper describes the advantages of this last method. An algorithm which formalizes it is proposed.

An interesting property of the bond graph formalism is first pointed out: it does not encode information which is contained in the classical component-connection representation but which is useless for analyzing the circuits' functioning. In a second step, the bond graphs obtained with the straightforward algorithm and with the inspection method are compared. This study shows that the useless information is not entirely deleted in the bond graphs obtained with the first method but is systematically suppressed with the second one. In other words, the bond graphs obtained with the straightforward method are not fully adapted to their formalism. An algorithm which formalizes the inspection method is proposed in the third part.

2   BOND GRAPH FORMALISM

2.1   Hypothesis

Physical electrical circuits are modeled by discrete elements, connected to each other with ideal wires. The circuits are supposed to be composed entirely of dipoles, selected among ideal current sources (noted S_f), voltage sources (S_e), resistors (R), inductances (L) or capacitors (C). In this article, the dipoles are physically non-oriented, that is, capacitors are not polarized and sources are reversible. Given these hypothesis, a component-connection representation of an electrical circuit is an edge-labeled graph. Each dipole in the circuit is associated with an edge in the component-connection representation and each equipotential connection is associated with a vertex. Consequently, there is one and only one component-connection representation for a given circuit. From now on they will not be distinguished.

Besides, the circuit's functioning is supposed to be entirely described by Kirchhoff's and Ohm's laws. Consequently, two circuits are said *equivalent* or "functioning in the same way" if there exists a bijection between the edges of the component-connection representation such that corresponding edges are associated with the same

dipole and such that the same current flows through those corresponding dipoles. The two circuits in Figure 1 have equivalent systems of equations obtained by applying Kirchhoff's and Ohm's laws. Therefore, the same current will flow through corresponding dipoles, which means that these two circuits are equivalent. This example shows that two circuits functioning in the same way may have distinct component-connection representations. In other words, this representation encodes a piece of information which is useless for describing the circuit's functioning. This piece of information is specified in the next paragraph.
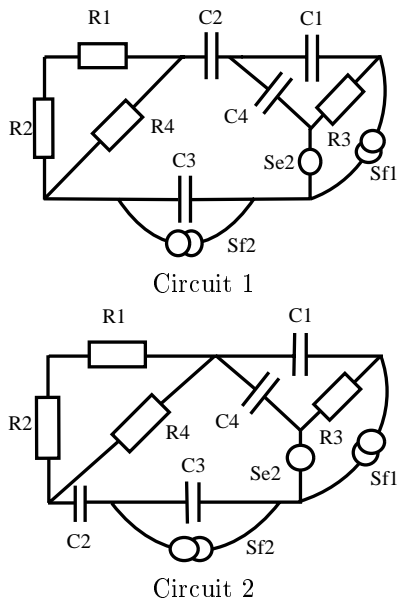

Figure 1: Circuits 1 and 2 are equivalent.

2.2 Useless Information

By defining the 2-isomorphism and by comparing the electrical properties of circuits which are 2-isomorphic, Whitney has formalized in [3] a well known property of electrical circuits. This property is that the permutation of elements or blocks of elements in series does not modify the circuit's functioning. For example, the two circuits of Figure 1 are equivalent because they only differ from the order of their blocks of elements in series. In other words, the information about the order of elements or blocks of elements in series is useless for studying the circuit's functioning. In the next paragraph, we show that the bond graph representation uses a formalism which implicitly deletes this useless information.

2.3 An Interesting Property of the Bond Graph Formalism

Bond graphs are vertex-labeled graphs used to represent physical systems and especially electrical networks [4]. With the bond graph formalism, dipoles are connected to each other with edges called bonds and with *parallel* or *serial* vertices called junctions. By definition, a parallel junction connects two elements having identical terminal voltages and the same current flows through elements which are connected by a serial junction. The parallel and serial junctions are symbolized respectively with "0" or "1". Figure 2 shows a circuit and a bond graph associated with this circuit. The five dipoles are connected using two junctions and six bonds. The resistor R1, the capacitor and the voltage source are connected using a serial junction because the same current flows through them. This set of three dipoles, the resistors R2 and R3 are connected using a parallel junction because the same voltage appears between their terminals.
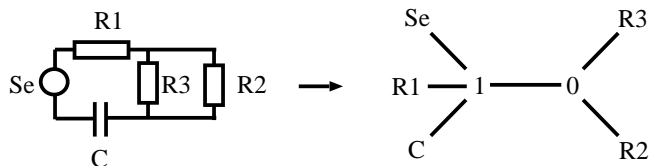

Figure 2: A circuit and an associated bond graph.

With this formalism, the order of elements in series may be deleted. For example, if the capacitor and the voltage source are permuted in the circuit of Figure 2, the component-connection representation is modified but not the bond graph. In other words, a property of Kirchhoff's laws is integrated in the bond graph formalism. But having a suitable formalism is not sufficient: the bond graph associated with a circuit has to be computed.

3 FROM A CIRCUIT TO A BOND GRAPH

3.1 The Inspection Method

The inspection method consists in identifying the circuit's topology in terms of serial and parallel connections. Then, elements in series are connected by using a serial junction and elements in parallel are connected by using a parallel junction. The bond graph of Figure 2 is for example obtained by using this method. Karnopp and Rosenberg did not propose an algorithm formalizing this method but they have instead proposed the algorithm

briefly described in the next paragraph.

3.2 The Straightforward Algorithm

The algorithm proposed by Karnopp and Rosenberg can be decomposed into two main steps. In the first step, each equipotential connection of the circuit is associated with a parallel junction in the bond graph and each dipole of the circuit is inserted between these parallel junctions by using a serial junction. In the second step, an arbitrary parallel junction is deleted and simplification rules are applied. The deleted parallel junction may correspond to the ground of the circuit. Figure 3 shows a circuit and the two main steps of the algorithm proposed by Karnopp. In this example, the point A is chosen as the ground.
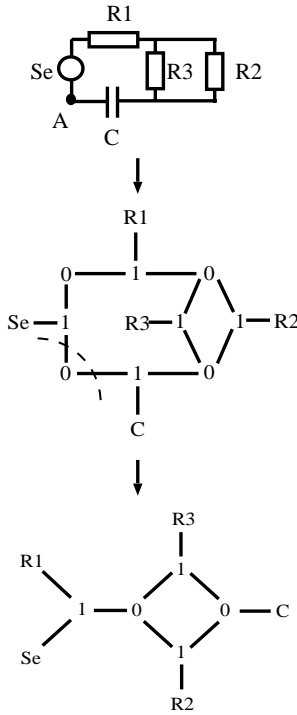

Figure 3: The two main steps of Karnopp's transformation algorithm.

3.3 Comparison of the Two Methods

The two methods are obviously not equivalent because a choice of a particular equipotential connection is necessary in the straightforward algorithm but not in the inspection method.

The information about the order of elements in series is not entirely deleted in bond graphs obtained with the

straightforward algorithm. For example, by permuting the voltage source and the capacitor in the circuit of Figure 3, the bond graph is modified. This is an important drawback because it contradicts the interesting property of the bond graph formalism described in paragraph 2.3. The algorithm has such a drawback because every serial (resp. parallel) connection is not expressed, that is, two dipoles which are in series (resp. in parallel) in the circuit are not systematically connected in the bond graph using a serial (resp. parallel) junction. For example, the capacitor and the voltage source are in series in the circuit of Figure 3 but they are not connected using a serial junction in the bond graph. Birkett and Roe have proposed another algorithm in [5]: the bond graph depends on a particular planar representation of the initial circuit. Unfortunately it has the same drawback.

The bond graphs obtained with the inspection method do not have this drawback because, by definition, elements which are in series in the circuit are systematically connected by using a serial junction in the bond graph. The next section describes an algorithm which formalizes this method.

4 FORMALISATION OF THE INSPECTION METHOD

The proposed transformation algorithm can be decomposed into three main steps. The first step consists in identifying the serial and parallel connections in the circuit: a particular decomposition of graphs proposed by Tutte is used. Next, each element of this decomposition in transformed into a bond graph compound. In the last step, these compounds are connected to each other to obtain the final bond graph.

The important notion of m-connexity is presented briefly here but Berge describes the graph theory terminology more precisely in [6]. Let G be a connected graph. An n-separation of G is a pair (H, K) of edge-disjoint subgraphs of G satisfying the following three conditions: $H \cup K = G$, H and K have exactly n common vertices and H and K have at least n edges. A graph is said to be m-connected if it has no n-separations for any $n < m$.

4.1 Tutte Decomposition

The Tutte decomposition decomposes a 2-connected graph G into its set of "3-blocks". These blocks which are not exactly subgraphs of G, are either a 3-connected graph, a polygon (i.e., a cycle having a least 3 edges) or a bond (i.e., a connected loopless graph on two vertices but with at least 3 edges). An algorithm for computing

recursively the set of "3-blocks" is proposed in [7]. A tree can be associated with the Tutte decomposition: its vertices are the "3-blocks" and its edges correspond to a 2-separation of G. Figure 4 shows the tree associated with the decomposition of the circuit 2 of Figure 1. This circuit is decomposed into one 3-connected graph (H4), two polygons (H1 and H3) and two bonds (H2 and H5). The eight dotted edges are called virtual edges because they are not element of the initial graph. The edges of the tree associated with the decomposition are represented with bold segments. By merging and deleting virtual edges of 3-blocks adjacent in the tree, the graph G is rebuilt.
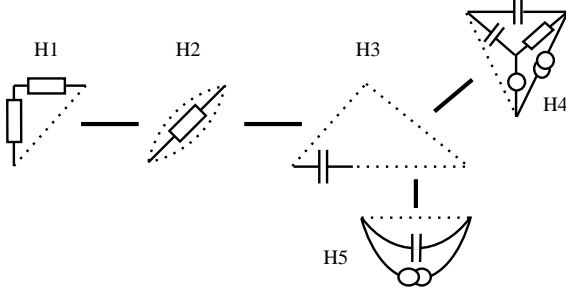

Figure 4: The Tutte decomposition of the circuit 2 of Figure 1 .

4.2 Bond Graph Compounds

Each "3-blocks" is then associated with a bond graph compound. The transformation depends on the type of the "3-blocks" as follows:

1. When dipoles are element of a polygon, they are in series. So, the bond graph compound associated with a polygon is a serial junction linked with the elements of the polygon.

2. When dipoles are element of a bound, they are in parallel. So, the associated bond graph compound is a parallel junction linked with the elements of the bound.

3. When dipoles are element of a 3-connected graph, the bond graph compound is obtained by applying the first step of the algorithm proposed by Karnopp.

Figure 5 shows a polygon, a bond, a 3-connected graph and their associated bond graph compounds.

4.3 Connection of the Bond Graph Compounds

In the last step, these bond graph compounds are connected to each other if their antecedents are adjacent in
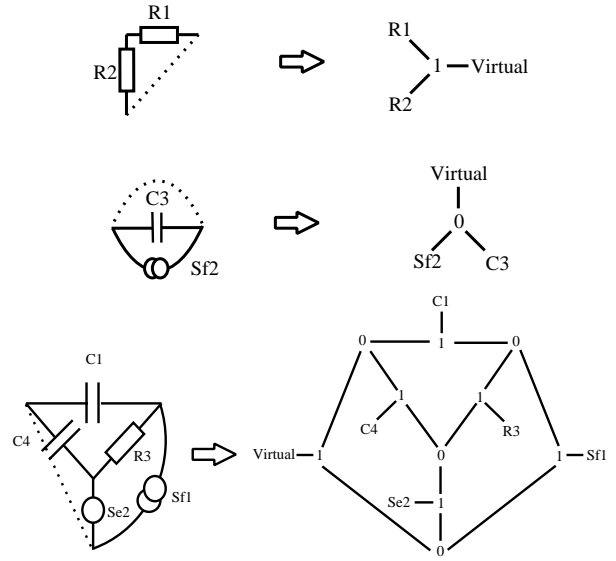

Figure 5: Three examples of graph transformation.

the tree associated with the Tutte decomposition. For that, the two vertices corresponding to virtual edges are merged and suppressed. Next, the two edges which were connected to these two vertices are merged. The acausal bond graph of Figure 6 is obtained from the circuit 2 of Figure 1.
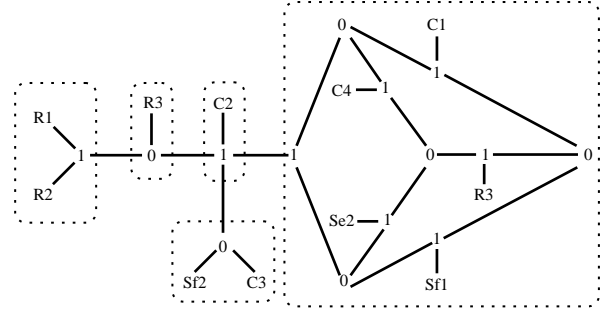

Figure 6: The acausal bond graph associated with circuit 2 of Figure 1.

4.4 Remark: Assignment of Causality

When a graph contains no 3-connected component, the procedure for assignment of causality can be directly applied. But when the circuit contains 3-connected subgraphs, the problem is slightly different: the bond graph compounds which are associated with 3-connected subgraphs are similar to bond graphs obtained with the first step of the straightforward algorithm. In the algorithm proposed by Karnopp, a parallel junction is sup-

pressed because it is supposed to be connected with a zero voltage source. Without using this artificial zero voltage source, the particular parallel junction is a parallel junction with no causal stroke pointing at it. In the bond graphs obtained with the algorithm described in 4, for each bond graph compound associated with a 3-connected subgraph, a parallel junction is first arbitrary chosen. Then, the causal strokes of the bonds connected to these junctions are chosen such that they do not point at the particular parallel junctions. At last, the procedure for assignment of causality is normally applied. This procedure applied to the acausal bond graph of Fig. 6 leads to the bond graph of Fig. 7.
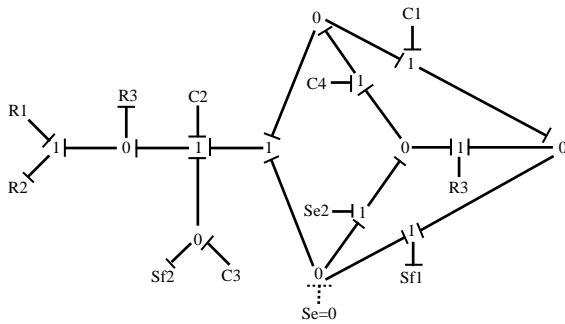

Figure 7: The bond graph of 6 after the procedure for assignment of causality.

4.5 Properties

As Tutte proved that his decomposition of graphs is unique, the algorithm proposed above associates a unique bond graph with a given circuit. Obviously, the reverse procedure described in [8] does not provide a unique circuit from a given bond graph. The algorithm presented above formalizes the inspection method because dipoles in series (resp. in parallel) in the circuit are systematically connected using a serial (resp. parallel) junction in the associated bond graph. The obtained bond graphs are fully adapted to the bond graph formalism. Therefore, they can be considered as canonical.

This inspection method was not yet formalized because it is neither equivalent to the algorithm proposed by Karnopp nor to the one proposed by Birkett. For example, by using the straightforward algorithm, it is not possible to obtain the bond graph of Figure 6 from the circuit 2 of Figure 1.

At last, the obtained bond graphs fully express some mathematical properties of Kirchhoff's laws because the information about the order of elements in series and in parallel is totally deleted. With this representation,

modifying the order of any pair of elements in series or in parallel in a circuit does not affect the associated bond graph. For example, the circuits 1 and 2 of Figure 1 are both associated with the bond graph of Figure 6 because they only differ from the order of their elements in series.

5 CONCLUSION

In this paper, the two procedures proposed by Karnopp to construct a bond graph from a circuit have been compared. The more suitable one, called the inspection method, has been formalized. With this algorithm, a representation which fully expresses some mathematical properties of Kirchhoff's laws is obtained. Based on this representation, an efficient method for generating the set of electrical circuits has been proposed in [9]. It must be generalized to circuits composed of oriented dipoles and to networks which contain n-port elements.

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