A solution procedure for an integrated lot-sizing and scheduling problem

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Mots-clés : *lot-sizing, scheduling, Lagrangian heuristic, integrated approach*

1 Introduction

In practice and in theory, planning and scheduling decisions are still most often taken sequentially. Mathematical planning models take into account aggregate capacity constraints, and thus do not guarantee that the proposed production plan is feasible when it is forwarded to the scheduling level, i.e. that there exists a schedule which allows lots of the production plan to be produced on time (see [2]). Lot-sizing problems have been treated extensively in the literature. However, most lot-sizing problems consider aggregate capacity constraints. Dauzère-Pérès and Lasserre [2] and Ouenniche [8] study the impact of sequencing decisions on a multi-item lot-sizing and scheduling problem. Small time-bucket lot-sizing problems consider short time periods and the sequencing of lots. A basic small-bucket problem is the Discrete Lot Sizing and scheduling Problem (DLSP) [5]. The main drawback of the DLSP is that only one item may be produced per period and, in that case, the production quantity is equivalent to using full capacity. This drawback is overcome in the Continuous Setup Lot Sizing Problem (CSLP), but still only one item per period may be produced. In the Proportional Lot Sizing and scheduling Problem (PLSP) [3], the remaining capacity in a given period is used for a second item. These models allow simultaneous lot sizing and scheduling but limit the number of products to be manufactured in one period. The General Lot Sizing and scheduling Problem (GLSP) [6] takes into account multiple products but features a single machine. An extension of these problems to multiple machines is proposed by Kimms [7] for the PLSP and Fandel and Stammen-Hegene [4] for the GLSP. Another class of problems considers lot sizing and scheduling with sequence-dependent setup costs and/or setup times.

2 Solution procedure

We propose a new model where all paths of the conjunctive graph associated to a fixed sequence of operations of lots on machines are modeled. A conjunctive graph is a representation of a scheduling problem, where nodes correspond to operations and arcs to precedence constraints between two operations. Precedence constraints are between two successive operations in the routing of an item, or between two successive operations on a resource in the sequence. In order to meet deadlines, the last operation of each path must be completed before its due date. Hence, the sum of processing and setup times of all operations in a path must not exceed the due date of the last operation of this path. And this must be true for all paths of the graph. These constraints can be seen as capacity constraints.

Lagrangian relaxation aims at decomposing an optimization problem into a number of easy-to-solve subproblems by dualizing some complicating constraints. Applying Lagrangian relaxation to solve our problem is justified by the fact that our mathematical model has an exponential number of capacity constraints, since these constraints correspond to paths of a conjunctive graph. These constraints are relaxed but, as their number is very large, it is not possible to consider them simultaneously. We thus initialize all multipliers to zero and choose among the set of the most violated constraints a set of multipliers to increase. This method allows us to only handle the most relevant constraints, and the number of these constraints is never very large in our numerical experiments.

Since there is no guarantee that the Lagrangian solution satisfies the relaxed capacity constraints, we designed a procedure which modifies the solution to satisfy the violated constraints. In classical lot-sizing problems, increasing or decreasing lot sizes in one period has no impact on the feasibility in other periods. Thus, most smoothing heuristics in Lagrangian relaxation approaches proposed in the literature successively transfer product quantities between lots in two different periods until a feasible solution is found. For each transfer, only the impacts on the two periods need to be measured. In our integrated problem, increasing or decreasing lot sizes in one period may influence the feasibility in all successive periods since the schedule considers lots on the entire horizon. Hence, we propose different and more complex smoothing heuristics.

Finally, using the Lagrangian heuristic developed for the lot-sizing problem with a fixed sequence of lots on the machines, several heuristics have been proposed to solve the integrated lot-sizing and scheduling problem. The first one is based on the iterative approach introduced in [1]. Two other heuristics are developed, based on Simulated Annealing and Tabu Search, that aim at improving both the sequence and the lot sizes. These two heuristics are better integrated to the Lagrangian heuristic, in particular information from the Lagrangian solution is used to improve the sequence.

3 Conclusions and perspectives

An effective approach is proposed to solve a general integrated lot-sizing and scheduling problem. The results of numerical experiments show that the proposed algorithms are effective. We are working on improving these heuristics, but also on extending the approach to consider multi-level lot sizing and additional constraints.

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