MRI sequence denoising using gaussian processes

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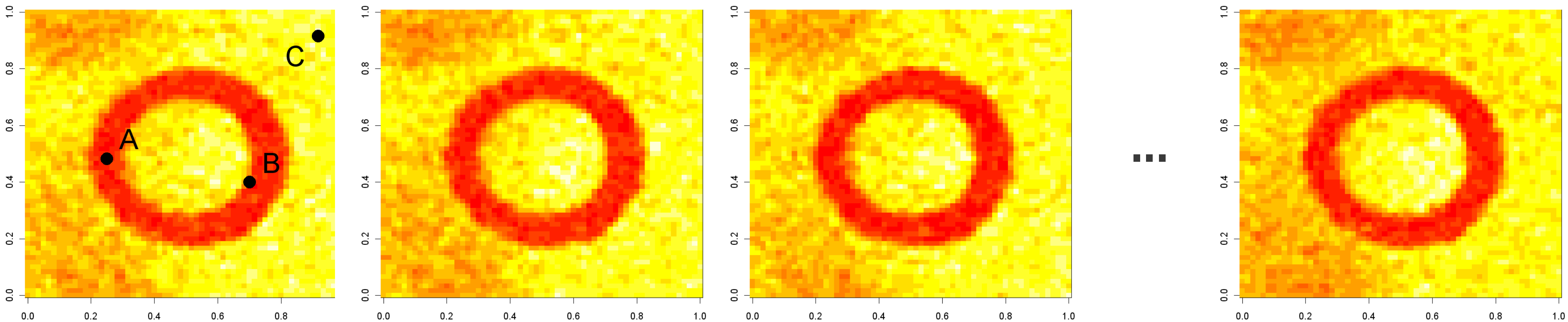
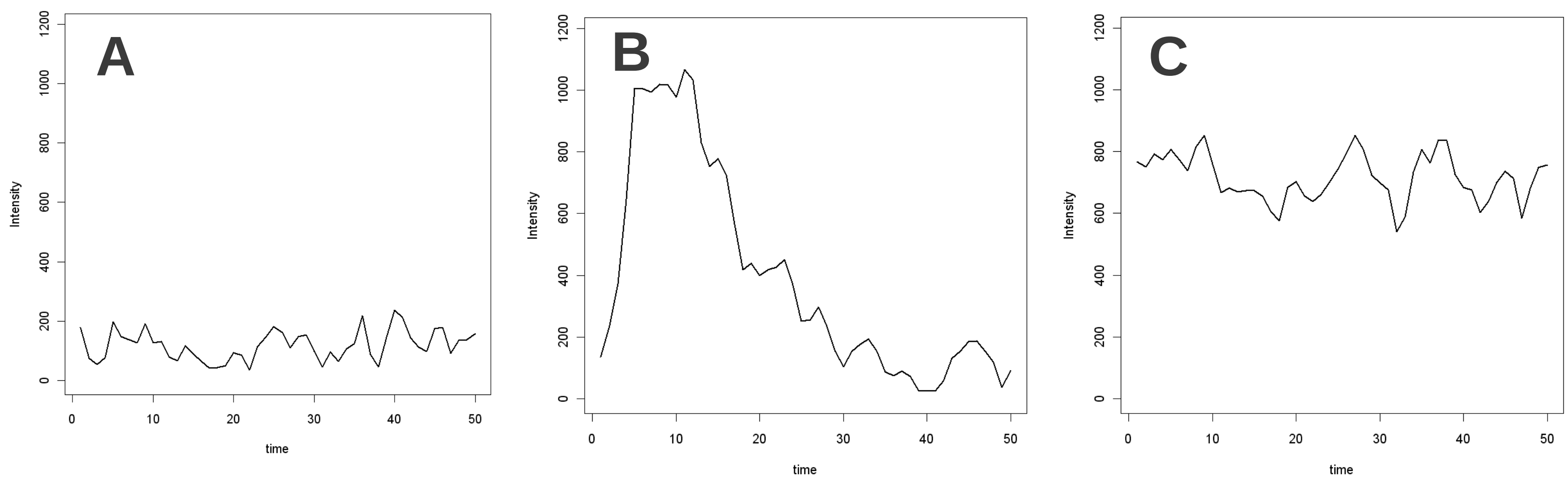
Presentation of the data

50 images of a carotid artery obtained by PC-MRI :





→ **The images** $I(x,t)$, $x \in \Re^2$, $t \in \Re$ **are noisy**

→ **The 50 time steps correspond to one cardiac cycle. 1 px ~ 0.1mm**

Talk outline

Goal : smooth a sequence (time) of images (space)
Approach : statistical (Gaussian processes and POD)

In order to reduce the complexity of the space and time statistical description of our data (artery), two steps

1. Spatial smoothing
2. Time smoothing

and

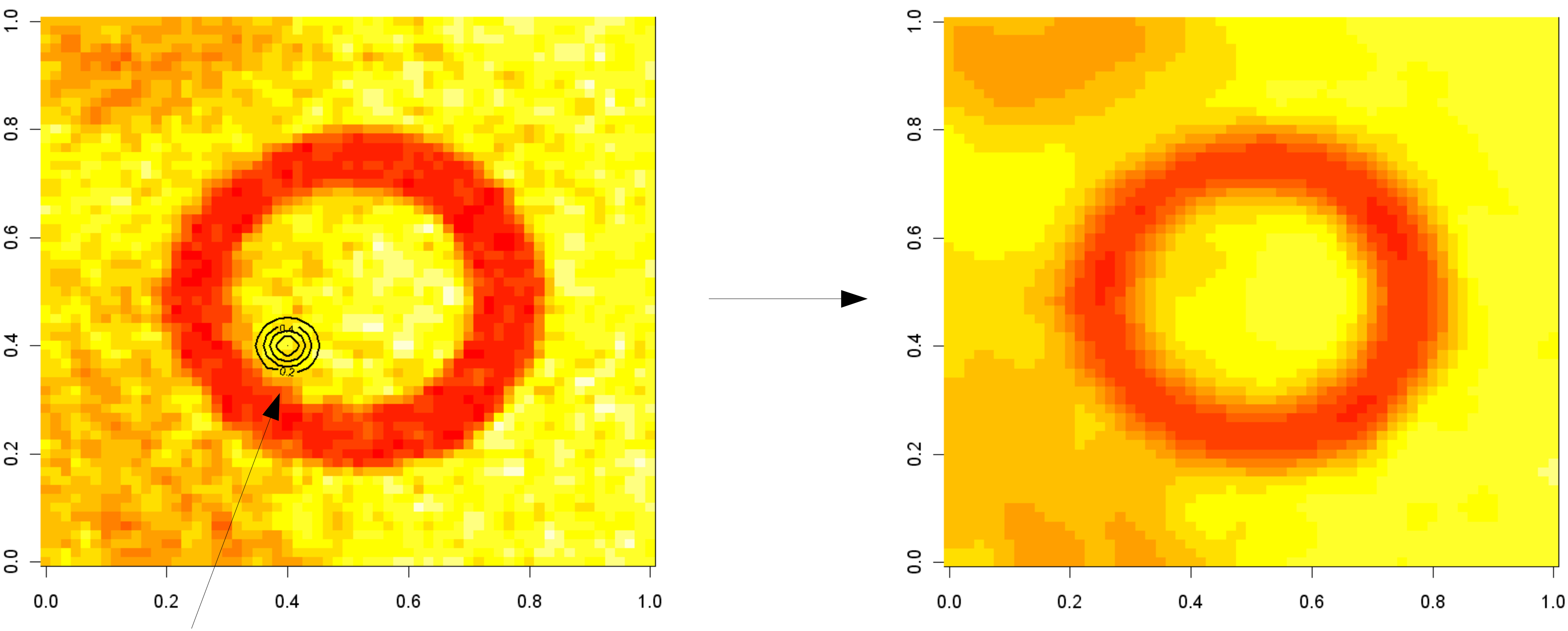
3. Conclusions

Spatial smoothing (1/6)

A basic approch is to smooth the data with a Gaussian Smoother

→ local average at each pixel

$$I_F(x,t) = \sum_{x',t'} w_x(x',t')\; I(x',t')$$



Contour lines of *w* a Gaussian smoother @ px (25,25)

The nature of the data is not respected :

The smoother has to be adapted

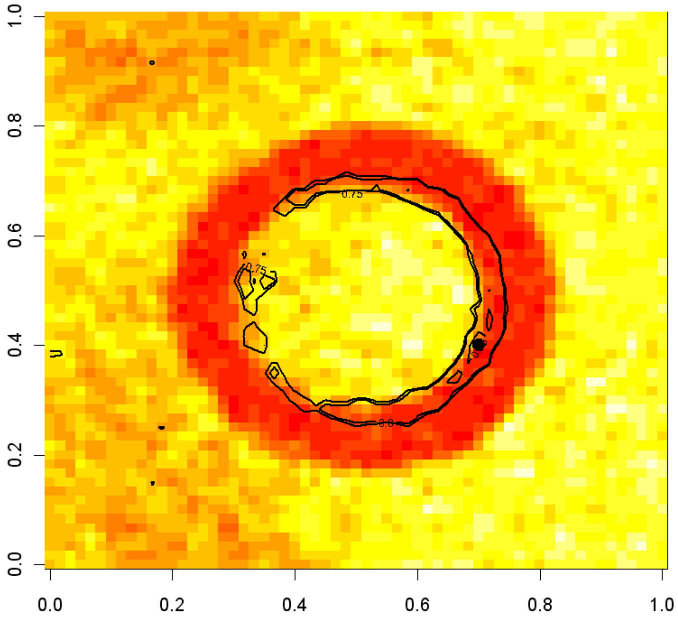
Related work

- Local filters : the standard procedure.
- PDE – based filters : Lysaker et al., 2003.
- Non local filters : Buades et al. (2005), Manjon et al. (2008).
- Bayesian + Rician noise + Markov random field : Awake and Whitaker (2007)

- Contribution of this work :
  - Non local filter
  - Based on Gaussian processes + space-time decomposition : may be not the most physical, but computationally and mathematically simple with a clear probabilistic interpretation.

Spatial smoothing (2/6)

The basic idea is to adapt the neighbourhood of each pixel to the problem at hand
→ we consider neighborhoods based on the empirical covariance matrix
The intensity at each pixel is seen as a random variable and each time step gives us a realization of this RV.



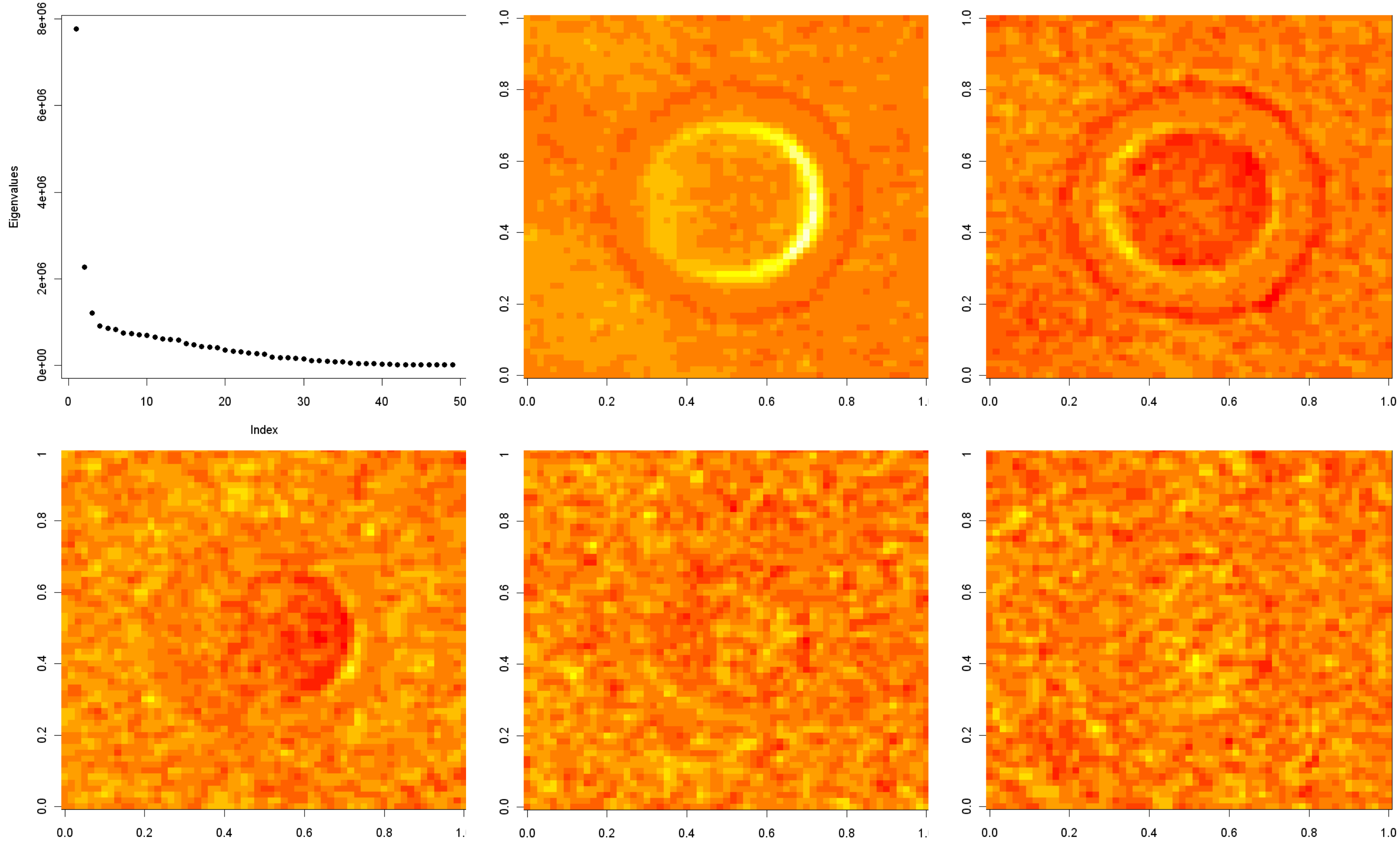
$$C = \frac{1}{N}\sum_{i=1}^{N}\left(I(.,t_i)-\bar{I}(.)\right)\left(I(.,t_i)-\bar{I}(.)\right)^T$$

Correlation between the pixel (43,25) – black dot – and the other pixels

This covariance matrix catches the physical partition of the space.

Spatial smoothing (3/6)

To denoise, we study the diagonalization $C = P\,D\,P^t$



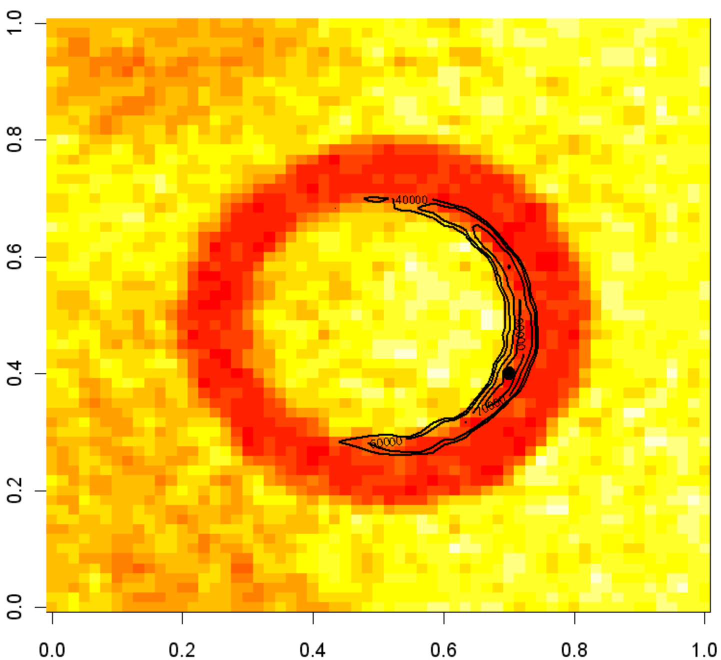
The first three eigenvectors seem meaningful.
From the 4th on, we consider that they represent noise.

Spatial smoothing (4/6)

We thus split the covariance matrix in two groups

$$
\begin{aligned}
C &= \begin{pmatrix} P_1 & \dots & P_p \end{pmatrix} \begin{pmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_p \end{pmatrix} \begin{pmatrix} P_1 & \dots & P_p \end{pmatrix}^T \\
&= P\left(D_s + D_n\right)P^T = C_s + C_n
\end{aligned}
$$

And we look at the neighborhood given by C_s



The use of covariance for smoothing is called *kriging* (in statistics).

Here, as the covariance structure is learnt empirically, *kriging* is equivalent to *Proper Orthogonal Decomposition (POD)*.

Spatial smoothing (5/6)

The smoothed image at t is given by the kriging average (statistical model) :

$$I(x,t) = I_s(x,t) + I_n(x,t) \quad \text{(signal + noise , all Gaussian processes with Cov. } C_s \text{ and } C_n \text{)}$$

Filtered I : $$I_F(.,t) = E[I_s(.,t) \mid I(.,t)] = C_s (C_s + C_n)^{-1} I(.,t)$$

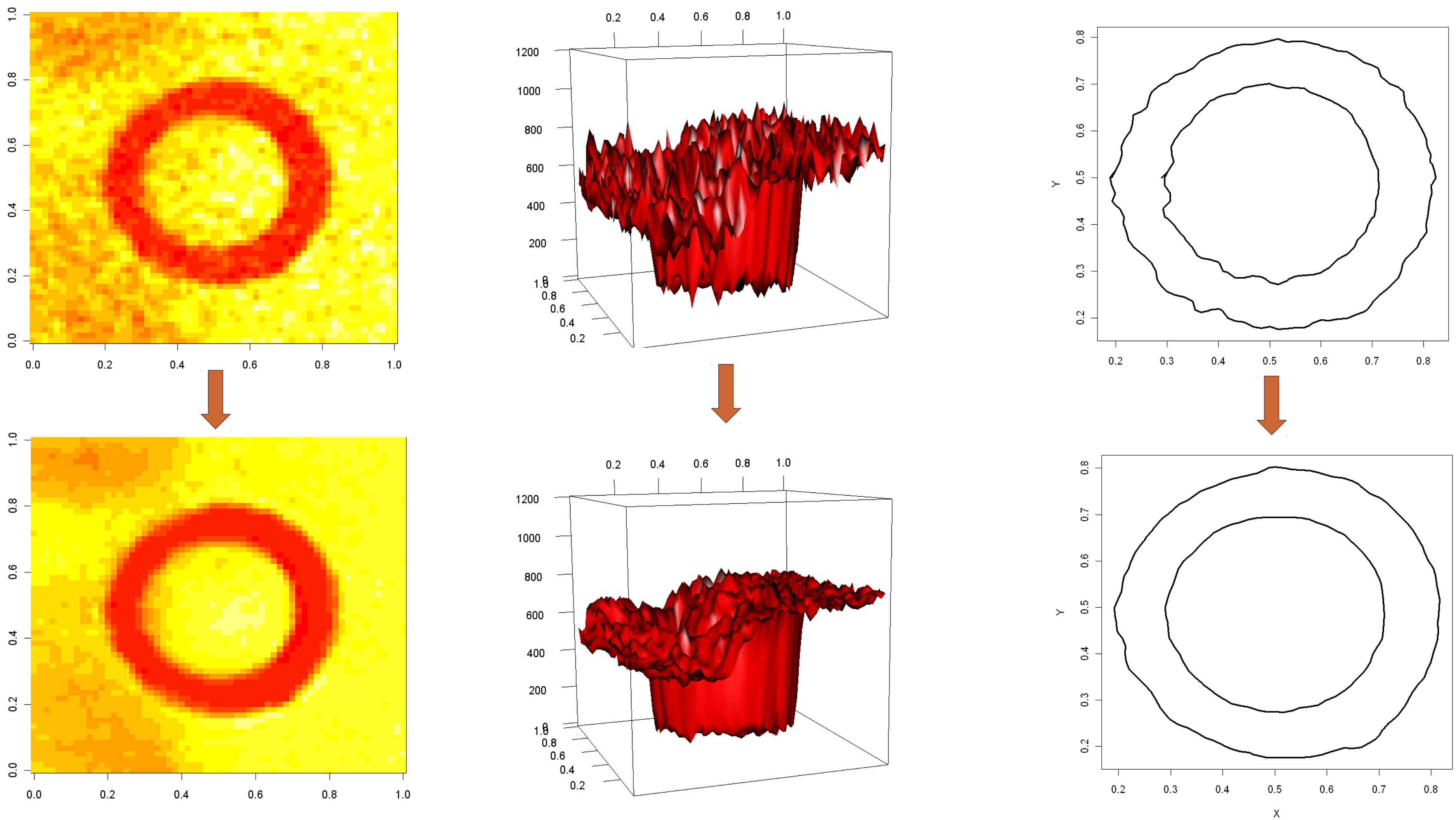
Kriging (stat model) and POD (projection on C eigenvectors) are equivalent here :

$$I_s(t) = C_s (C_s + C_n)^{-1} I(t) = (P D_s P^T)(P D^{-1} P^T) I(t) = P D_s D^{-1} P^T I(t)$$

$$= P \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 0 & \\ & & & & \ddots \end{pmatrix} P^T I(t)$$

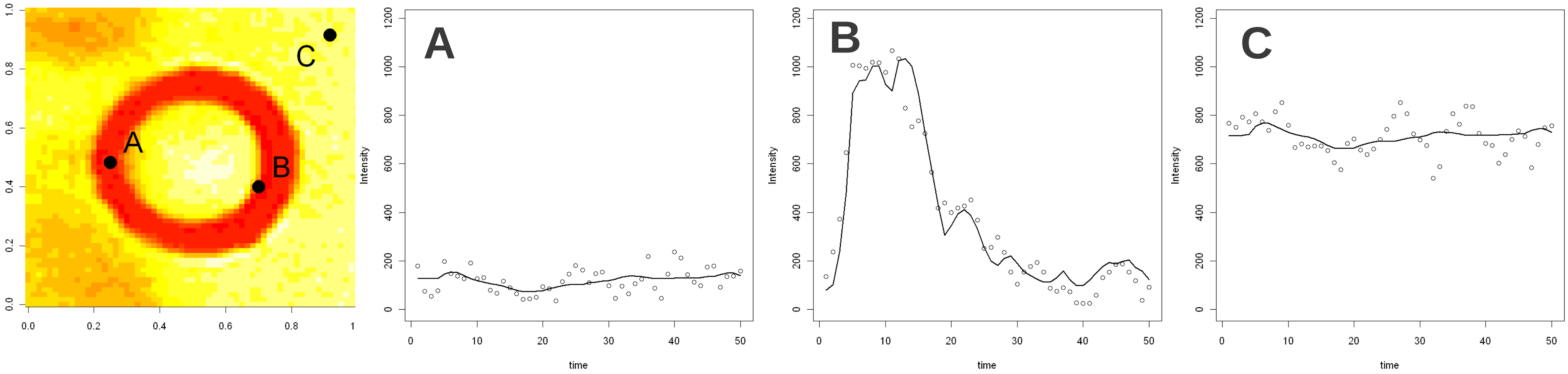
Spatial smoothing (6/6)

Result with effect on contour (right)



Time smoothing

Effect of spatial smoothing (previous slides) on time response :



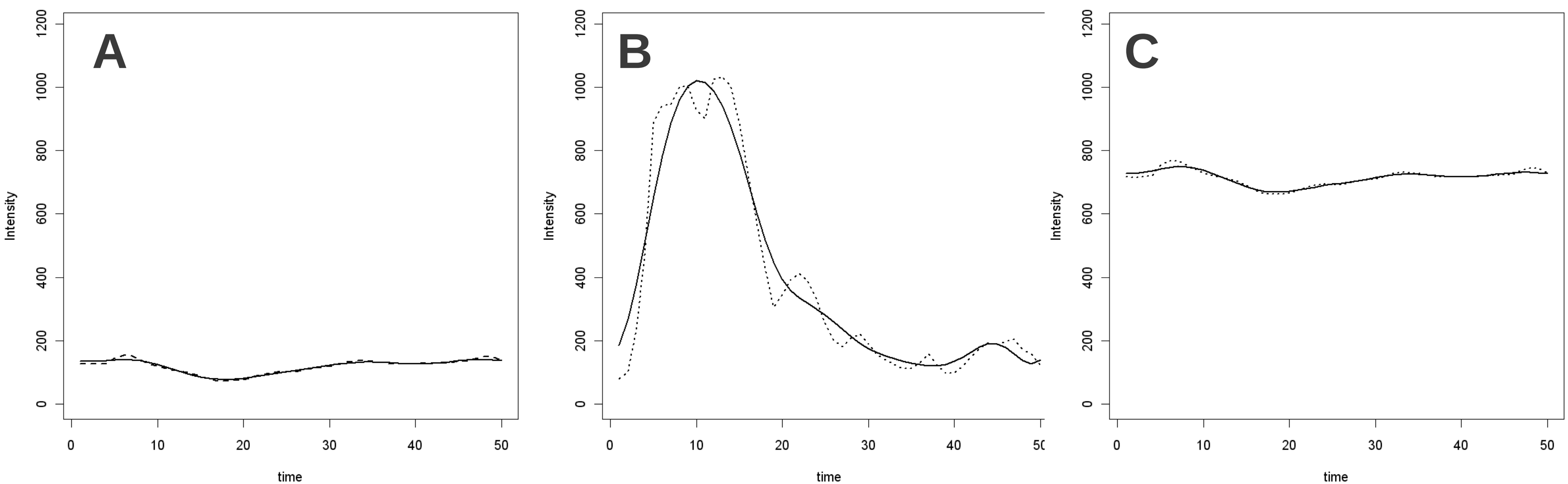
→ The curves are much less noisy
→ The periodicity is not guaranteed (observed here because data is periodic but closely look at B)

Time smoothing

→ The artery movements are periodic: we apply a time smoothing (covariance in time) which is periodic.

$$C_x(t_1,t_2) = Cov(I(x,t_1),I(x,t_2)) = \exp\left(-\lambda \sin^2\left(\pi \frac{t_1-t_2}{50}\right)\right)$$

Note : any periodical Cov function does not provide a valid covariance. Here, it is the restriction of a classical Gaussian kernel to the circle.

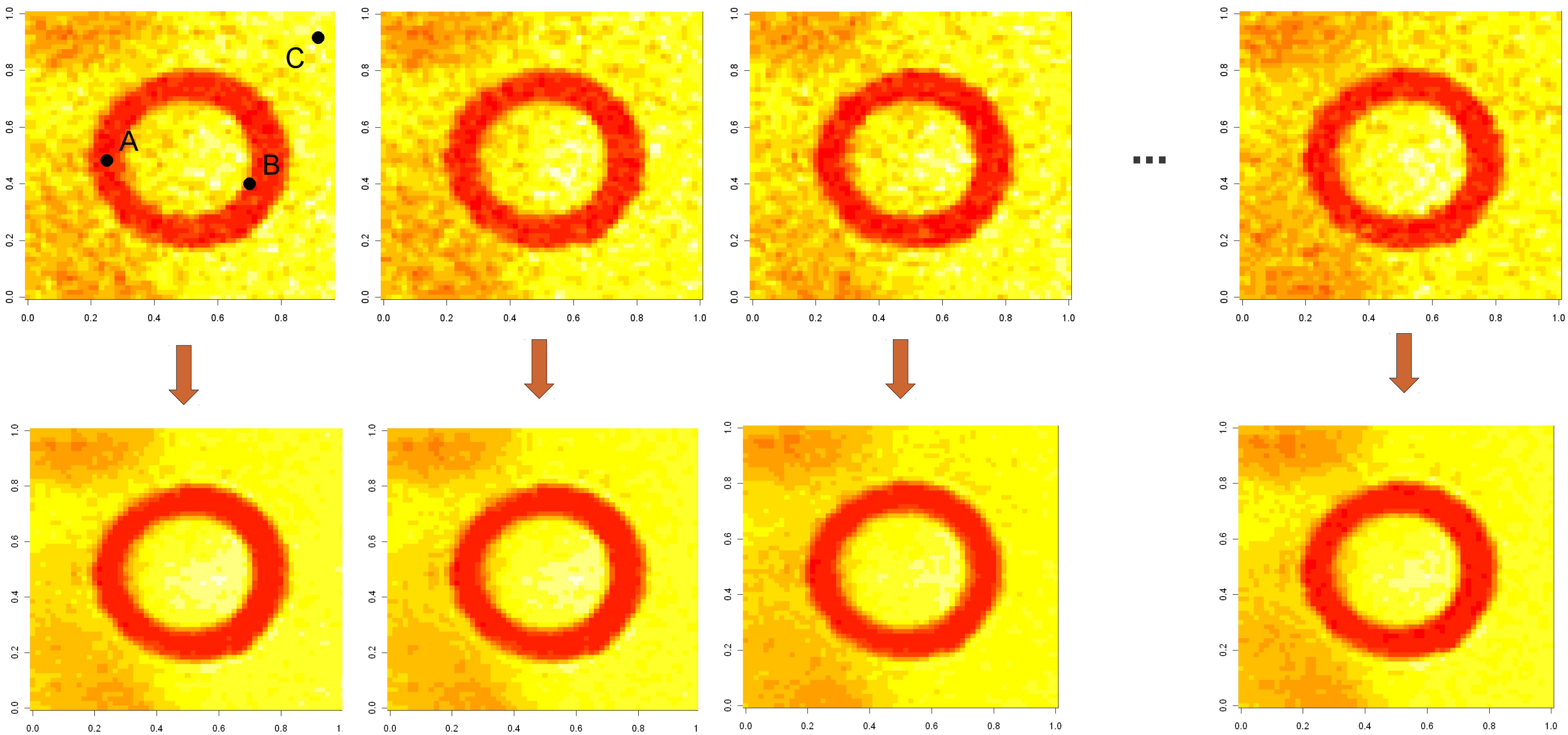


Both smoothings are krigings since they are based on covariance.

Final results

The sequence (space then time) allows to reduce the complexity

Space smoothing:	inversion of a 3721 x 3721 matrix
Time smoothing:	inversion of a 50 x 50 matrix
Time and space:	inversion of a 186050 x 186050 matrix !!



Conclusions : summary

• We have proposed an approach for denoising MRI signals based on Gaussian processes.

• Space and time have been handled in the following way

1. Space treatment, time taken as a random event
2. Time treatment, independently at each point in space

⇨ never construct the complete time-space covariance matrix (186050^2 here).

• **Some physical features are accounted for : spatial neighborhoods based on data and a priori that time neighborhoods are periodical**.

Conclusions : perspectives

• Advantage over regression-based filtering : thanks to covariance learning, possibility to sample *I* or $I_s(x,t)$ with *very little functional assumptions* (infinite number of basis functions in the RKHS). But there are assumptions in the covariance structure (i.e., the RKHS).

• Perspective 1 : such probabilistic framework is well-adapted to noisy data. Use it to describe the uncertainties associated to the measures (e.g., confidence interval on $I_s(x,t)$).

• Perspective 2 : use the decomposed *I(x,t)* to segment images with associated uncertainties.