Challenges and future trends in Uncertainty-Based Multidisciplinary Design Optimization for space transportation system design

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Loïc Brevault[(a,b)], *Mathieu Balesdent*[(b)], *Nicolas Bérend* [(b)], *Rodolphe Le Riche* [(c)]
[(a)] *CNES, Launchers Directorate, Paris, France*
[(b)] *Onera, The French Aerospace Lab, Palaiseau, France*
[(c)] *CNRS LIMOS and Ecole des Mines de St-Etienne, St-Etienne, France*

Abstract

The design of a space transportation system is a complex multidisciplinary optimization problem. Uncertainty existing at the early phases of the design process influences the final system performances and reliability. Uncertainty-based Multidisciplinary Design Optimization (UMDO) methodologies allow to determine the optimal design solution with respect to the performance criteria while ensuring the reliability and the robustness of the final system to uncertainty. They enable to handle the disciplinary couplings and to facilitate the search of compromises between conflicting objectives. This paper provides a survey of the UMDO process. Different techniques such as uncertainty modeling, uncertainty propagation or optimization problem formulation are presented and a qualitative comparison of the UMDO methodologies is performed in order to provide general guidelines. A literature review of case studies of UMDO process applied to aerospace systems is provided. Moreover, open issues that need to be addressed in order to develop an efficient UMDO framework for the design of space transportation systems are discussed and future trends for UMDO are highlighted.

1. Introduction

Space transportation system designs are long term projects involving important budgets. NASA, in 2012 [147], stressed the need to reduce the cost and to improve the effectiveness of space missions and satellite launches. Improving the design process for aerospace vehicles is essential to obtain low cost, high reliability, and effective launch capabilities [19]. The design of aerospace vehicles is a complex multidisciplinary optimization process. Designing a space transportation system consists in finding the configuration which provides, by multidisciplinary analyses, the optimal performances under various constraints [67]. The performance estimation results from coupled multidisciplinary analyses. The different disciplinary analyses may induce to make trade-offs due to antagonist disciplinary effects on launcher performances in order to achieve a balance among competitive objectives including safety, reliability, performance and cost.

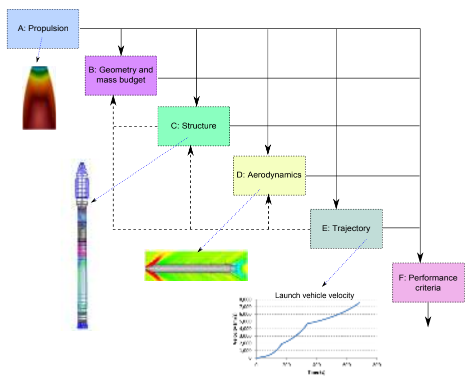

Figure 1: Multidisciplinary design of a launch vehicle

Figure 1 illustrates an example of multidisciplinary design of a launch vehicle. The simulation of the disciplines and the evaluation of the performance criteria can be computationally intensive. Moreover, the disciplines are coupled. For example, the outputs of the trajectory discipline are inputs of the structure discipline, but outputs of structure discipline are also inputs of trajectory discipline. Consequently, the design of space transportation system needs dedicated methodologies to handle the complexity of the problem to solve. Adapted tools, called Multidisciplinary Design Optimization (MDO), have been developed to help solving this problem and are introduced in the next section.

1.1 Multidisciplinary Design Optimization (MDO)

MDO is a set of engineering methods to handle complex multidisciplinary design optimization problems. It provides an informed decision framework for system designers. MDO relies on three aspects: multidisciplinary design, analysis and optimization [5].

MDO methods take advantage of the inherent synergies and couplings between the disciplines involved in the design process to decrease the computational cost and/or to improve the quality of the global optimal design [118]. Unlike the sequential disciplinary optimizations performed with classical design methods, the interactions between the disciplines are directly incorporated in the MDO methods [12]. However, the complexity of the problem is significantly increased by the simultaneous handling of all the disciplines.

To subdue this complexity, various MDO formulations have been developed. In the 90's, several surveys classed MDO formulations into two general types of architecture: single level methods [14, 31], and multi level methods [5, 74]. The single level methods (*Multi Discipline Feasible* (MDF) [14], *Individual Discipline Feasible* (IDF) [14] and *All At Once* (AAO) [14]) are characterized by one optimizer at the system level. The multi-level methods (*Collaborative Optimization* (CO) [20], *Concurrent SubSpace Optimization* (CSSO) [116], *Bi-Level Integrated Systems Synthesis* (BLISS) [117], *Analytical Target Cascading* (ATC) [7], *etc.*) present optimizers at the system and subsystem levels. Several papers have detailed MDO formulations, compared them qualitatively highlighting advantages and drawbacks [13, 124]. In the recent years, different surveys present new architectures MDO developed in the last decade (*Exact and Inexact Penalty Decomposition* [33], *MDO of Independent Subspaces* [113], *etc.*) [84, 127].

MDO methods have been applied to a large panel of case studies in various fields such as: aircrafts [62], automotives [86], ships [99], buildings [29], *etc.* Moreover, they have been applied to launch vehicle design in a large number of case studies summarized in [13]. Launch vehicles are complex systems and their design involves several coupled disciplines requiring computationally expensive simulations. For these reasons, MDO methodologies are particularly suited to solve multidisciplinary design problems.

1.2 Uncertainty in MDO methodologies

Incorporating uncertainty in MDO methodologies for aerospace vehicle design has become a necessity to offer important improvements in terms of [146]:

- Reduction of design cycle time, cost and risk,
- Robustness of launch vehicle design to uncertainty along the development phase,
- Increasing system performances while, meeting of reliability requirements.

In aerospace industry, a new system follows a development process involving several specific phases (Conceptual design, Preliminary design, Detailed design, Manufacturing) [19]. Uncertainty at the early phases of the development exist due to the limited knowledge concerning the system characteristics and due to the low fidelity simulations and analyses performed. UMDO methods are recent, still under development and partially applied in conceptual phases [146]. Incorporating uncertainty in MDO formulations in the early design phases would provide a set of optimal designs robust to the uncertainty and reliable with respect to the imposed constraints. The robustness of the set of optimal design alternatives would ensure the satisfaction of the requirements and constraints even if future modifications in the design occur. Moreover, providing a degree of confidence in the architecture found by optimization under uncertainty would enable the decision maker to perform trade-offs between the expected performances and the confidence in the achievement of the architecture chosen at the final design stage. Nevertheless, incorporating uncertainty in MDO methodologies rises a number of challenges that need to be addressed. In this paper we propose an overview of the challenges and future trends in Uncertainty-based Multidisciplinary Design Optimization (UMDO) for the design of space transportation system. Several reviews of UMDO exist [137, 146], and are focused on the existing methods to solve UMDO problems.

Yao *et al.*, [137] details a general process for UMDO methods. The process relies on two steps: the uncertain system modeling and the UMDO procedure (Figure 2). The first step consists of the modeling of the considered system (a space transportation system in our case) and the mathematical representation of uncertainty. The modeling of uncertainty corresponds to the identification and representation with the appropriate mathematical formalism of the uncertainty involved in the system design. The second step, the UMDO procedure, includes the optimization under uncertainty, the uncertainty propagation, reliability and robust analyses.

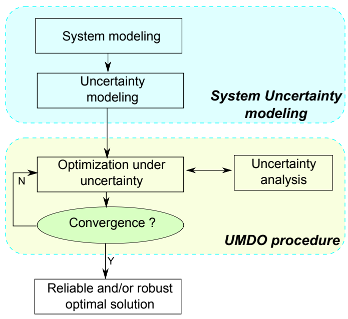

Figure 2: MDO process [137]

The UMDO process is the leading thread of the paper and a presentation of each step is provided in the following. In Section 2, the first step of the UMDO, the system uncertainty modeling, is introduced. It presents the uncertainty definitions, the different sources of uncertainty, the identification of the most important sources of uncertainty and their modeling with the appropriate mathematical formalism. Section 3 and 4 introduce the second step of the UMDO process: the UMDO procedure. Section 3 outlines the different methods to propagate uncertainty in the models simulating the design of a space system transportation. The UMDO strategies and formulations are described in Section 4. Section 5 provides a review of UMDO methods applied to the design of space transportation system. Eventually, Section 6 discussed the challenges and future trends in UMDO for the design of space transportation systems.

2. System uncertainty modeling

2.1 Uncertainty definition and classification

Firstly, in this Section we detail the definition and classification of uncertainty. Engineering problems are solved with physical and mathematical models. A model is a representation of the reality through a set of simulations and/or experimentations under appropriate assumptions [72]. Due to simplification hypotheses, lack of knowledge and inherent uncertain quantities, models represent reality with uncertainty. Two different meanings can be distinguished for uncertainty [70]:

- Uncertainty as a state of mind,
- Uncertainty as a physical property of information.

The 1^{st} meaning describes the lack of knowledge and information of an agent to make a decision. The 2^{nd} meaning refers to a physical property, representing the limitation of perception systems (for instance measurement uncertainty). Different taxonomies and classifications of uncertainty have been proposed and a consensus has been established on two main categories: *aleatory* and *epistemic* uncertainties, reflecting the two meanings of uncertainty (Table 1), [126]. Aleatory uncertainty is an inherent physical property of information and cannot be reduced by collecting more information or data. Epistemic uncertainty results from a lack of knowledge and can be reduced by increasing our knowledge or collecting more information. The

Table 1: Aleatory and epistemic uncertainty

Aleatory Uncertainty	Epistemic Uncertainty
• Definition: Inherent variability of the physical system and the environment under consideration. Aleatory uncertainty cannot be reduced by collecting more information or data [126].	• Definition: Any lack of knowledge or information about fundamental phenomena. It encompasses the model uncertainty which is associated to the precision of the chosen simplified mathematical models to represent the real physical phenomena [126].
• Also referred to as: Variability, stochastic uncertainty, randomness, irreducible uncertainty [126]	• Also referred to as: Ignorance, incertitude, subjective uncertainty, reducible uncertainty [126]
• Examples: The presence, the direction and the amplitude of a wind gust during a rocket launch, the exact mass of propellant introduced in the rocket before the launch, *etc.*	• Examples: The choice of a compressible or not, with boundary layer or not, with turbulence or not, flow modeling in aerodynamics represents with more or less accuracy of the air flow around a launch vehicle during the launch.
• Adapted mathematical framework for uncertainty modeling: Probability theory	• Adapted mathematical framework for uncertainty modeling: Imprecise probability, Dempster-Shafer theory, Possibility theory, Probability boxes

distinction between the two types of uncertainty is important because appropriate mathematical framework for uncertainty modeling depends on the type of uncertainty. Uncertainty can be classified into two categories but sources of uncertainty in the modeling of a system are multiples and detailed in next Section.

2.2 Sources of uncertainty

Several sources of uncertainty can be distinguished. We consider a parametrized model representing a real physical system. The model involves a set of input variables represented by a set of uncertain variables $\mathbf{U} = (U_1, ..., U_k)$. Moreover, the model outputs are given by the mapping: $\mathbf{Y} = f(\mathbf{U}, \mathbf{P})$, with $\mathbf{P}$ parameters of the physical model. In this context, we can identify the following non exhaustive sources of uncertainty [72]:

- Uncertainty due to the inherent randomness of the input variables $\mathbf{U}$ (*e.g.* on-board propellant mass),
- Uncertainty due to the choice of modeling of the uncertainty of the input variables $\mathbf{U}$, used to describe the available information to the designer (*e.g.* probability, evidence theory, possibility theory),
- Uncertainty due to the choice of modeling of the physical phenomena represented by $f(\mathbf{U}, \mathbf{P})$ (*e.g.* Ideal gas modeling, Van der Waals modeling, *etc.*),
- Uncertainty due to the parameters $\mathbf{P}$ (*e.g.* wind gust),
- Uncertainty in the computation of $\mathbf{Y}$, due to the numerical approximations, errors (*e.g.* numerical integration),

- Uncertainty due to the non modeling of interactions between different disciplines involved in the launch vehicle system, *etc.*

The interaction uncertainty [126] is specific to multidisciplinary systems and consequently is interesting in the context of MDO. It refers to the unknown and non modeled interactions between the disciplines involved in the conception of launch vehicles. For instance, it can be assumed that the drag coefficient is not a function of the angle of attack and therefore a coupling between the trajectory discipline and the aerodynamics discipline is not modeled, introducing uncertainty on the drag coefficient of the launch vehicle.

In a MDO framework, the identification of the sources of uncertainty has to be conducted with experts of each discipline and with experts of aerospace vehicle design to identify all the potential sources of uncertainty. Once the potential sources of uncertainty have been found, an identification of the most important and influential sources of uncertainty has to be performed based on Sensitivity Analysis.

2.3 Uncertainty identification and sensitivity analysis

For a complex system such as a launch vehicle, especially at the conceptual design phase, a large number of uncertainty sources can be identified. The computation burden introduced by the propagation and the optimization under uncertainty is dependent of the problem dimensions [66]. All the sources of uncertainty cannot be taken simultaneously into account. A prescreening of the most important sources of uncertainty is required before simulation based optimization can be applied. Sensitivity Analysis (SA) is the study of how the variation in the model output can be apportioned, qualitatively or quantitatively between variations in model inputs [108]. The purpose is to identify and rank the most influential model inputs. Two approaches of SA can be distinguished: local (around a design) and global (on the entire input variation domain). The use of global SA methods allows to filter out the uncertainty factors with negligible effects on the output. It decreases the computational cost and the complexity of the model. Furthermore, it enables to allocate resources on the modeling of the most influential uncertainty on the outputs. The screening of the most important effects allows for the decision maker to settle on which factors should be considered as uncertain or deterministic for the optimization problem. Several global SA methods exist: variance decomposition methods (Sobol [119], ANalyse Of VAriance [76]), differential analysis [92], linear relationship measures (Correlation Coefficients, Partial Correlation Coefficients, Standardized Regression Coefficient, [66]). A detailed comparative study of the SA techniques can be found in [21]. Once the most important sources of uncertainty are identified through SA, a mathematical modeling of these sources is essential to represent uncertainty according to the available information. Different mathematical formalisms exist and are detailed in the next Section.

2.4 Uncertainty modeling

The uncertainty modeling is a key step in the UMDO process and touch on it could result in non robust solution to the real uncertainty [148]. Mathematical representation of uncertainty relies on two elements: the set and the measure described by the set theory and the theory of measure. The generalization of set theory and measure theory enables to diversify uncertainty mathematical formalisms through various theories representing different approaches for uncertainty modeling. Five theories to represent uncertainty are presented in the next paragraphs. The choice between the different theories depends on the available information.

2.4.1 Imprecise theory

Imprecise probability theory, developed by Walley, [133], is a generalization of the existing uncertainty theories. Let (Ω, Υ) a measurable space , with Υ an algebra of measurable subsets of the universe Ω and Π a set of monotone measures on the measurable space. It is possible to define two measures such that:

- Upper probability: $\overline{P}(A) = \underset{P\in\Pi}{sup}[P(A)]; \ \forall A \subseteq \Omega$
- Lower probability: $\underline{P}(A) = \underset{P\in\Pi}{inf}[P(A)]; \ \forall A \subseteq \Omega$

$\underline{P}$ and $\overline{P}$ define two measures that are respectively the inferior and the superior bounds over all the monotone measures that could be defined on the measurable space [16]. $\overline{P}$ is a superadditive measure and $\underline{P}$ is a subadditive measure. Superadditive and subadditive measures are defined such that for any pair $A_1 \subseteq \Omega$ and $A_2 \subseteq \Omega$ such that $A_1 \cap A_2 = \emptyset$:

$$\underline{P}(A_1 \cup A_2) < \underline{P}(A_1) + \underline{P}(A_2); \ \ \overline{P}(A_1 \cup A_2) > \overline{P}(A_1) + \overline{P}(A_2) \tag{1}$$

The two measures encode a family of measures. The introduction of two measures enables to quantify the confidence in the uncertainty modeling, and the difference between the two measures reflects the incomplete nature of the knowledge [16]. Indeed, available information about uncertainty in performances of a future complex systems are often partial [34] and uncertainty in the available knowledge needs to be quantify.

Nevertheless, reasoning on a family of measures can be very complex. Further details on imprecise theory can be found in [133]. We focus on four formalisms for representing special sets of measures which allow practical engineering applications: Evidence theory [111], Probability theory [73], Possibility theory [143] and Probability boxes [50]. Next, an overview of the four uncertainty theories is presented, highlighting advantages and drawbacks to represent uncertainty with each formalism.

2.4.2 Evidence theory

The theory of evidence, developed by Dempster [35] and Shafer [111], is based on two monotonic non additive measures: the Belief (Bel) and the Plausibility (Pl). These measures are determined from known evidences. Evidences are available through the mass assignment function m:

$$\begin{cases} m : P_\Omega \to [0,1] \\ \sum\limits_{E \in P_\Omega} (m(E)) = 1 \end{cases} \tag{2}$$

which assigns positive mass to the element of the power set of Ω, P_Ω. If $m(E) \neq 0$, E is called a focal element. The mass assignment function can encode a family of measures such that: $\Pi(m) = \{P | \forall A \in P_\Omega,\ Bel_m(A) \leq P(A) \leq Pl_m(A)\}$ [111]. The upper and lower probabilities are defined by [111]:

$$\overline{P}(A) = Pl(A) = \sum_{\substack{(E_j \cap A) \neq \emptyset \\ E_j \in P_\Omega}} (m(E_j));\ \underline{P}(A) = Bel(A) = \sum_{\substack{E_j \subseteq A \\ E_j \in P_\Omega}} (m(E_j)) \tag{3}$$

The mass is assigned to an element of the power set of Ω without possible discernment of the mass repartition inside this element. Evidence theory allows combination of evidences from different sources and experts through rules of combination of evidence (Dempter's rule, Yager's rule, *etc.*) [110].

Interval formalism for uncertainty modeling, referred to as interval analysis, models an uncertain variable as an interval with a minimum and a maximum values that the uncertain variable might take. An introduction to the interval analysis theory and application can be found in [90]. Interval analysis can be seen as a special case of Evidence theory in which only one focal element exists for the uncertain variable with a mass assignment of 1.

2.4.3 Probability theory

The probability theory has been formalized by Kolmogorov [73]. In probability theory, the upper and lower probabilities are equal and collapse to a single additive measure: $P = \overline{P} = \underline{P}$. The additive characteristic of the probability measure implies that: for any pair $A_1 \subseteq \Omega$ and $A_2 \subseteq \Omega$ such that $A_1 \cap A_2 = \emptyset$ then, $P(A_1 \cup A_2) = P(A_1) + P(A_2)$. Uncertainty information are available through the Probability Distribution Function (PDF): $p : \Omega \to [0,1]$ which is defined over the universal set Ω and not over the power set, P_Ω. Probability theory can be seen as a particular case of Evidence theory in which all the focal elements are singletons. Probability theory is the formalism traditionally used to represent uncertainty. It is appropriate for the representation of aleatory uncertainty [61]. However, probability theory, as it uses a single measure, does not faithfully represent epistemic uncertainty [16]. Because of limited knowledge or no possibility to collect more information, it can be difficult to use classical probability modeling and to define accurately the input PDFs. Bayesian approach of probability offers the possibility to update prior distributions and to combine expert opinions according to Bayes rule by gathering more information [56]. However, the choice of the prior distribution is a challenge [133]. As probability theory is defined as a single measure, it does not enable to quantify the confidence in the uncertainty modeling based on the available uncertainty information.

2.4.4 Probability boxes (Pbox)

Consider X a random variable on (Ω, Υ) and a pair $(\underline{F}, \overline{F})$ of non intersecting cumulative distributions. The Pbox $[\underline{F}, \overline{F}]$ encodes a class of probability measures whose Cumulative Distribution Functions (CDF) F (defined in the classical probabilistic sense) are bounded by the pair of cumulative distribution functions $\underline{F}$ and $\overline{F}$ such that: $\underline{F}(x) \leq F(x) \leq \overline{F}(x); \forall x \in \Omega$. Moreover, $\underline{F}(x) = \underline{P}(]-\infty, x])$ and $\overline{F}(x) = \overline{P}(]-\infty, x]); \forall x \in \Omega$, [51].

Pbox incorporates both imprecision on the available knowledge and probabilistic characterization [11]. It is possible to distinguish two types of Pbox, the parametrized Pbox and the general Pbox. In the case of parametrized Pbox, the distribution is known but the parameters characterizing the distribution are unknown, only intervals can be identified. General Pbox are distribution-free representation methods using only statistical or experimental data. There exists different rules to construct Pboxes based on the available information, [51,52]. Relations between Pboxes and Dempster-Shafer Structures (DST) exist, for instance to one DST corresponds a single Pbox but the reciprocal is not true [51].

2.4.5 Possibility theory

Possibility theory has different origins in its development and diverse interpretations exist. Zadeh [143] extended the concept of fuzzy set to a mathematical framework to represent uncertainty. Furthermore, possibility theory can be interpreted as a particular case of Evidence theory in which the focal elements are nested. Information are available through the possibility distribution r defined such that: $r : \Omega \rightarrow [0, 1]$ and $\underset{x \in A}{sup}(r(x)) = 1$ with A a subset of Ω. The possibility distribution encodes two measures, the Possibility (Pos) and the Necessity (Nec) such that: $Pos(A) = \underset{x \in A}{sup}(r(x))$ and $Nec(A) = 1 - \underset{x \in A^c}{sup}(r(x)) = \underset{x \in A^c}{inf}(1 - r(x))$ [143]. A^c refers to the complementary set of A. *Pos(A)* measures the amount of information that does not refute that A contains the instantiation x of the uncertain variable X and *Nec(A)* measures the amount of uncontradicted information that supports the affirmation that A contains the instantiation x of the uncertain variable X [16].

2.4.6 Comparison of uncertainty formalism

Table 2 summarizes the key characteristics for each uncertainty theory. Each uncertainty modeling theory has advantages and drawbacks (Table 3). Several comparisons of the uncertainty formalisms exist in literature, outlining the expressibility of each formalism [60, 94, 120, 128]. To choose the appropriate uncertainty modeling, it is important to take into account the available information. The appropriate choice of uncertainty modeling is crucial because it will influence the rest of the UMDO process [148]. Indeed, the choice of uncertainty propagation, the formulation of the MDO process and the optimization algorithms depend on the uncertainty modeling framework. The key steps of the first level of UMDO process, the uncertainty system modeling, have been detailed in this Section. We have highlighted the potential sources of uncertainty and presented methods to identify the most influential sources on the system performances by SA. Then, according to the available information and evidences, uncertainty modeling formalisms have been presented. The next sections detail the second level of UMDO process, the UMDO procedure. Firstly, Section 3 describes uncertainty propagation methods adapted for each uncertainty modeling formalism. Then, in Section 4, the UMDO strategies and formulations are introduced.

Table 2: Key characteristics for uncertainty theories

Uncertainty theory	Distribution function	Measures	Relations
Imprecise probability	$\emptyset$	$\begin{cases} \overline{P}(A) = \underset{P \in \Pi}{sup}[P(A)] \\ \underline{P}(A) = \underset{P \in \Pi}{inf}[P(A)] \end{cases}$	$\underline{P}(A) = 1 - \overline{P}(A^c)$
Evidence theory	$\begin{cases} m : P_\Omega \rightarrow [0, 1] \\ \sum_{E \in P_\Omega} (m(E)) = 1 \end{cases}$	$\begin{cases} Bel(A) = \sum_{\substack{E_j \subseteq A \\ E_j \in P_\Omega}} (m(E_j)) \\ Pl(A) = \sum_{\substack{(E_j \cap A) \neq \emptyset \\ E_j \in P_\Omega}} (m(E_j)) \end{cases}$	$Bel(A) = 1 - Pl(A^c)$
Probability theory	$p : \Omega \rightarrow [0, 1]$	$P(X \in A) = \int_A (p(x)dx)$	$P(A) = 1 - P(A^c)$
Possibility theory	$\begin{cases} r : \Omega \rightarrow [0, 1] \\ \underset{x \in \Omega}{sup}(r(x)) = 1 \end{cases}$	$\begin{cases} Pos(A) = \underset{x \in A}{sup}(r(x)) \\ Nec(A) = 1 - \underset{x \in A^c}{sup}(r(x)) \end{cases}$	$Pos(A) = 1 - Nec(A^c)$
Probability boxes	$\emptyset$	$F \in [\underline{F}, \overline{F}]$	$\underline{F}(x) \leq F(x) \leq \overline{F}(x);\ \forall x \in \Omega$

3. Uncertainty propagation

In uncertainty propagation, we can distinguish two objectives: computing the uncertainty in the outputs of a discipline due to the uncertainty in the inputs or evaluating the uncertainty of failure in reliability analysis which corresponds to the uncertainty that a constraint is not satisfied. Several uncertainty propagation methods exist depending on the uncertainty mathematical modeling framework used. First, we detail the propagation of uncertainty from the inputs to the outputs of a function. In a second step, reliability analysis is discussed.

3.1 Uncertainty propagation input/output

3.1.1 Uncertainty propagation within the probability framework

To compute the uncertainty in the output Y of a discipline represented by the function f such that: $Y = f(\mathbf{U}) = f(U_1, ..., U_k)$, with $U_j \in \Omega,\ j \in \{1, ..., k\}$, k random variables, different methods exist. Five categories of uncertainty propagation to compute the uncertainty in the output can be distinguished [77]:

Table 3: Advantages and drawbacks of uncertainty mathematical modeling framework

Uncertainty theory	Advantages	Drawbacks
Probability theory	• Adapted to represent aleatory uncertainty • Methods in place, engineers familiar with it • Combination of information with Bayesian approach • Parameter estimation methods if not enough information	• Needs information on each singleton of the subset • Does not faithfully represent epistemic uncertainty • No measure of the confidence in the uncertainty modeling information
Evidence theory	• Needs less information than probability theory • Makes easier to combine expert opinion • Handles aleatory and epistemic uncertainties • Expresses confidence in uncertainty modeling information	• Might be difficult to interpret • Might be difficult to get information from experts • Computation burden for the propagation • Discontinuous measures
Possibility theory	• Needs less information than probability theory • Concept of membership function and fuzziness • Expresses confidence in uncertainty modeling information • Handles aleatory and epistemic uncertainties	• Might be difficult to interpret • Might be difficult to get information from experts • Computation burden for the propagation • Discontinuous measures
Probability boxes	• Can use the probability or evidence theory frameworks to built the Pboxes • Handles both aleatory and epistemic uncertainties • Express confidence in uncertainty modeling information	• Computation burden for the propagation • Needs information on each singleton of the subset in case of parametrized Pboxes • Makes easier to interpret due to the link with probability theory

- Simulation-based methods: Monte Carlo simulation [112], importance sampling [88], adaptive sampling [23], importance splitting [91],
- Local expansion-based methods: Taylor series method [38],
- Numerical integration-based methods: Full factorial numerical integrations [79], sparse grid approach [135],
- Functional expansion-based methods: Polynomial chaos expansion [46], stochastic collocation [85],
- Surrogate-based methods: Kriging [45, 122], quadratic polynomial approximation [49], neural network [65], Support Vector Machine [114], Radial Basis Function [98], *etc.*

In the probabilistic framework, two results are interesting: the PDF of the output or the statistical moments of the output. To compute the j^{th} centred statistical moment, a multidimensional integral has to be calculated:

$$M_j(Y) = \int_{\Omega} (\mathbf{U} - \mu_Y)^j f(\mathbf{U}) p(\mathbf{U}) d\mathbf{U} \tag{4}$$

with p the joint probability distribution of the input variables and μ_Y the mean of the output.

Simulation-based methods: They rely on repeated sampling in the input space Ω to evaluate the multidimensional integral (Eq. 4). Monte Carlo Simulation (MCS) [112] is the most common uncertainty propagation method because it can reach any level of accuracy if enough samples are calculated. For instance the mean of the output can be estimated by: $E[Y] \simeq \frac{1}{N}\sum_{i=1}^{N} f(\mathbf{U_i})$. The convergence of MCS to evaluate the mean is in order of $\frac{1}{\sqrt{N}}$ with N the number of samples [44]. MCS is easy to implement but, when the function f is computationally intensive as in the case of launch vehicle design, the method is computationally prohibitive. Other sampling methods such as Latin Hypercube Sampling (LHS), [59], have been developed to decrease the computation burden by a better sampling scheme.

Local expansion-based methods: Taylor series expansion locally approximates the function and the statistical moments. For instance, with a first order Taylor series expansion, the function f is approximated around the local point $\mathbf{U_0}$ by: $f(\mathbf{U}) \simeq f(\mathbf{U_0}) + \sum_{i=1}^{k} \frac{\partial f(\mathbf{U_0})}{\partial U_i}(U_i - \mathbf{U_0}(i))$. Based on this local decomposition, the mean and standard deviation of the output are estimated as:

$$\mu_Y \simeq f(\mathbf{U_0});\ \sigma_Y \simeq \sqrt{\sum_{i=1}^{k}\left(\frac{\partial f(\mathbf{U_0})}{\partial U_i}\right)^2 \sigma_{U_i}^2 + \sum_{i=1}^{k}\sum_{j=i+1}^{k}\left(\frac{\partial f(\mathbf{U_0})}{\partial U_i}\right)\left(\frac{\partial f(\mathbf{U_0})}{\partial U_j}\right) cov(U_i, U_j)} \tag{5}$$

with $cov(U_i, U_j)$ the covariance between the input variables U_i and U_j, and σ_{U_i} the standard deviation of the random variable U_i. Taylor series expansion is only valid locally and requires the computation of partial derivatives that can be difficult for complex simulation models [10]. The first order approximation is only accurate for linear functions. Higher order expansion can be used for non linear functions but requires the computation of the Hessian of the function. The estimation accuracy of the statistical moments decreases as the coefficient of variation (ratio of the standard deviation over the mean) of the input random variable increases [10].

Numerical integration-based methods: They rely on an approximation of integrals and multidimensional integrals by appropriate quadrature formula to propagate the statistical moments [48]. They can evaluate any statistical moment. For a single random variable U, distributed according to $p(U)$, the expected value of the output $Y = f(U)$ is given by:

$$E(Y) = \int_{\Omega} f(U)p(U)dU \simeq \sum_{i=1}^{N} f(U^i)w_i \tag{6}$$

with U^i a quadrature point (also called collocation point) and w_i the quadrature coefficient. There are two main ways to choose the quadrature points:

- randomly and the quadrature coefficients are computed according to [1]:

$$w_i = \int_{\Omega} p(U) \prod_{j=0\ j\neq i} \frac{U - U^j}{U^i - U^j} dU \tag{7}$$

- according to Gaussian quadratures, *i.e.* as the roots of a polynomial q which is orthogonal to the probability density function $\int_{\Omega} q(U)p(U)w(U)dU = 0$ [1]. For classic density distributions, orthogonal polynomial families exist [46]. Gaussian quadratures allow to increase the accuracy of the estimation of the statistical moments. Gaussian quadratures approach is the optimal quadrature method as it offers the highest precision with respect to the integration order [78]. Compared to Monte Carlo, for classical distribution (Gaussian, uniform, beta, *etc.*), only three to five calls to the function are necessary to estimate the mean of the output of an univariate function [69].

In the case of a multivariate function, a multivariate integral has to be evaluated to compute the statistical moments of the output. The Gaussian quadrature can be employed but the increasing number of variables may result in prohibitive computations. The use of sparse grids [135], such as Smolyack sparse grid [115], allows to decrease the number of function evaluations while preserving an acceptable accuracy.

Functional expansion-based methods: They rely on an approximation of the function on his entire domain of definition and not just locally such as Taylor series expansion. Two main functional expansion methods can be distinguished: Polynomial Chaos Expansion (PCE) and Stochastic Collocation (SC) [46].

PCE consists of an expansion of the function f over a polynomial orthogonal basis [63]:

$$Y = f(\mathbf{U}) = a_0 + \sum_{i=1}^{\infty} a_i P_1(U_i) + \sum_{i=1}^{\infty} \sum_{j=1}^{i} a_{i,j} P_2(U_i, U_j) + ... \tag{8}$$

with $\{P_1, P_2, ..., P_i\}$ a basis of orthogonal polynomials, with P_i of degree i. The choice of the polynomial family is made consistently with the input distribution of the random variables [46]. The polynomial basis is orthogonal to the weighting function. In practice, the expansion is truncated to a degree d and by re-organizing the expansion (Eq.8) to have a one-to-one correspondence between the coefficients and the polynomials, we have: $f(\mathbf{U}) \simeq \sum_{j=0}^{d} \alpha_j \Psi_j(\mathbf{U}) = \tilde{Y}(\mathbf{U})$. The difficulty in PCE is the estimation of the polynomial coefficients [101]. Two methods can be employed: by orthogonal spectral projection or by regression [134]. The orthogonal spectral projection consists in projecting the output $\tilde{Y}$ against each polynomial basis function using the orthogonality [46]:

$$\alpha_j = \frac{< \tilde{Y}, \Psi_j >}{< \Psi_j^2 >} = \frac{1}{< \Psi_j^2 >} \int_{\Omega} \tilde{Y}(\mathbf{U}) \Psi_j(\mathbf{U}) p(\mathbf{U}) d\mathbf{U} \tag{9}$$

The multivariate integral can be estimated using sampling or with the numerical integration-based methods presented in the previous paragraph. The regression method relies on a least square method. Given N sample points $\{\mathbf{U^1}, ..., \mathbf{U^N}\}$, the polynomial chaos coefficients $\alpha = \{\alpha_0, ..., \alpha_d\}$ are determined by [47]:

$$\underset{\alpha}{min} \sum_{i=1}^{N} \left(Y(\mathbf{U^i}) - \tilde{Y}(\mathbf{U^i}) \right)^2 \tag{10}$$

In [134], the authors propose to use the roots of the orthogonal polynomial family as the sample points and to use a weighted least square regression to represent the higher contribution of sample points in the higher frequency region of the input random variable distributions.

Stochastic Collocation (SC) is another functional expansion-based method, but instead of expanding the function over an orthogonal polynomial family, the expansion is based on the Lagrange interpolation polynomial [85]. For an univariate function:

$$Y(U) \simeq \sum_{j=1}^{m} f(U^j)L^j(U) \text{ with: } L^j = \prod_{i=1, i\neq j}^{m} \frac{U-U^i}{U^j-U^i} \tag{11}$$

and in the multivariate case, $Y(\mathbf{U}) = f(U_1, ..., U_k)$:

$$Y(\mathbf{U}) \simeq \sum_{j_1=1}^{m_1} \sum_{j_2=1}^{m_2} ... \sum_{j_k=1}^{m_k} f(U_1^{j_1}, ..., U_k^{j_k})(L_1^{j_1}(U_1, U_1^{j_1}) \otimes ... \otimes L_k^{j_k}(U_k, U_k^{j_k})) \tag{12}$$

with $L_1^{j_1}$ the ${j_1}^{th}$ Lagrange polynomial term for the interpolation in the U_1 direction and $\otimes$ the tensor product. In [46], Eldred proposes to improve the accuracy of the interpolation by choosing the collocation points as the roots of the orthogonal polynomial family according to the input random distributions. The computation of the tensor product can be intractable for a high number of input random variables, and Smolyak sparse grid method [115] may be used to decrease the number of calls to the function.

The functional expansion methods (PCE and SC) allow to create surrogate models of the function depending on the input random distribution. They efficiently propagate statistical moments and create the output PDF by MCS on the surrogate model [46]. The two first statistical moments are computed by:

$$E[Y_{PCE}] \simeq \sum_{j=0}^{d} \alpha_j < \Psi_j(\mathbf{U}) >= \alpha_0;\ \sigma^2_{Y_{PCE}} \simeq \sum_{j=0}^{d} \alpha_j^2 < \Psi_j(\mathbf{U})^2 > \tag{13}$$

$$E[Y_{SC}] \simeq \sum_{i=1}^{N} f(\mathbf{U^i})w_i;\ \sigma^2_{Y_{SC}} \simeq \sum_{j=1}^{N} f^2(\mathbf{U^i})w_i - E[Y_{SC}]^2 \tag{14}$$

Surrogate modeling: To overcome the computational burden introduced by the Monte Carlo method which requires a large number of calls to the function, a surrogate model of the computationally expensive function can be created [97, 100]. For complex systems, such as launch vehicles, the involved disciplines in the design are expensive to evaluate. The creation of such metamodels for the disciplines allows to call the inexpensive surrogate models instead of the complex and expensive function. Sampling methods on the surrogate model are applied to propagate the uncertainty and to limit the computation effort. Surrogates are built from a suitable reasonably-sized set of design points known as a Design of Experiments (DoE)(for instance with Latin Hypercube Sampling [121] or low discrepancy sequences [132]). Several families of surrogate models may be chosen depending on the purpose of the study. Among them are: Polynomial response surfaces [49], Support Vector Regressors and Classifiers (SVR, SVC) [114], Neural Networks [65], Radial Basis Function [98], Kriging (or Gaussian process) predictors [68].

At the difference of PCE and SC (with efficient interpolation methods), the surrogate modeling approaches do not depend directly on the input random variable distributions [122]. Table 4 summarizes the advantages and drawbacks for each probabilistic uncertainty propagation method. We have detailed uncer-

Table 4: Probabilistic uncertainty propagation

Uncertainty propagation	Advantages	Drawbacks
Simulation (Monte Carlo)	Simplicity of implementation, independent of the input distribution, independent of the number of input variables	Very slow convergence, intricate for complex expensive function to evaluate
Local expansion (Taylor series)	Simplicity of implementation, less expensive than Monte Carlo	Limited to linear or approximated linear functions, requires differentiability properties
Numerical integration	Less expensive than Monte Carlo, uses of sparse grid methods (Smolyak)	Only computes statistical moments, Gaussian quadrature depends on input distributions, curse of dimensionality
Functional expansion	**Polynomial chaos**: provides a surrogate model, allows to compute statistical moments and the PDF, less expensive than Monte Carlo	**Polynomial chaos**: Difficult to compute coefficients, choice of the polynomial order for decomposition, depends on input distribution, curse of dimensionality
	Stochastic collocation: provides a surrogate model, allows to compute statistical moments and the PDF, less expensive than Monte Carlo, makes easier to implement than PCE	**Stochastic collocation**: Possible divergence due to Lagrange polynomial interpolation, curse of dimensionality, accuracy of the interpolation depends on the input distributions
Surrogate	Computationally efficient, surrogate model faster to evaluate, does not depend on input distributions, allows to compute statistical moments and the PDF	Quantification of uncertainty introduced by the surrogate model, creation of the surrogate can be complex

tainty propagation in the probabilistic framework. Non probabilistic uncertainty propagation methods are detailed in [22] for the Pboxes and in [4] for the Evidence theory.

In the presence of optimization constraints, in an uncertainty framework, it is necessary to quantify the possible non satisfaction of the constraints. Propagation of uncertainty for reliability analysis is detailed in the next section.

3.2 Propagation of uncertainty for reliability analysis

In the context of reliability analysis, we need to compute: $\Lambda[g(\mathbf{U}) \leq 0]$, with Λ an uncertainty measure (probabilistic, evidence theory, Pbox, *etc.*) and g an inequality constraint function (also referred to as limit state function). The inequality constraint function defines the boundaries between feasible and unfeasible domains corresponding to violation of constraints. We denote $\mathbf{U} = (U_1, ..., U_k)$. Within the probability framework, (Λ is the probability measure P), the probability of failure is defined by:

$$p_f = \int_\Omega I(\mathbf{U})p(\mathbf{U})d\mathbf{U}; \text{ with } I(\mathbf{U}) = \begin{cases} 1 & \text{if } g(\mathbf{U}) \leq 0 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

with p the joint probability distribution. The probability of failure can be estimated by MCS. However, if the probability of failure is low, a large number of samples and function evaluations are required to compute accurately the probability of failure [44]. Furthermore, g can be a complex and expensive function to evaluate resulting in intractable MCS.

Improved sampling based methods have been developed to overcome the computational burden. Importance sampling [45] consists in replacing the original joint probability distribution p by a more adapted distribution h:

$$p_f = \int_\Omega I(\mathbf{U})p(\mathbf{U})d\mathbf{U} = \int_\Omega I(\mathbf{U})\frac{p(\mathbf{U})}{h(\mathbf{U})}h(\mathbf{U})d\mathbf{U} = E_{\mathbf{Z}}\left[\frac{I(\mathbf{Z})p(\mathbf{Z})}{h(\mathbf{Z})}\right] \tag{16}$$

with $\mathbf{Z}$ the uncertain vector distributed according to h. In [107], the authors demonstrate that the optimal instrumental distribution h^* is defined by: $h^* = \frac{I(\mathbf{U})p(\mathbf{U})}{p_f}$. The optimal instrumental distribution is impossible to implement directly because it depends on the unknown quantity of interest p_f. However, several methods have been developed to approximate this optimal instrumental distribution.

The First Order Reliability Method (FORM) [102] is one of the most applied reliability analysis method [44]. It consists in a local linearization of the state function g at the Most Probable Point (MPP) of failure. It is assumed that the MPP concentrates a large amount of the probability of failure [44]. The initial input random variables $U_i,\ i \in \{1, ..., k\}$ are transformed into uncorrelated standard Gaussian random variables $\mathcal{U}_i,\ i \in \{1, ..., k\}$ using Rosenblatt transformation [105]. The inequality constraint function in the Gaussian standard space is noted: $\mathcal{G}$. In the standard Gaussian space, the MPP is defined by a constrained optimization problem (also referred to as Reliability Index Approach (RIA)) [131]:

$$\begin{cases} \beta = \min\limits_{\mathcal{U}} \parallel \mathcal{U} \parallel \\ s.t.\ \mathcal{G}(\mathcal{U}) = 0 \end{cases} \tag{17}$$

the MPP is the closest failure point to the origin in the standard space. Then, the probability of failure is computed by approximating the inequality constraint function $\mathcal{G}$ by a first order Taylor series. It is approximated by [131]: $p_f \simeq \Phi(-\beta)$ with Φ the standard normal cumulative distribution function. Several limitations exist in FORM. For inequality constraint functions which cannot be approximated accurately by a first order approximation, the Second Order Reliability Method (SORM) have been developed [102]. However, the second order approximation requires the computation of the Hessian matrix of $\mathcal{G}$ at the MPP which can be expensive to evaluate [9]. Moreover, it is assumed that there exists only one MPP, however, FORM and SORM can lead to inaccurate results when several MPP exist [44]. In [131], the authors highlight that FORM/SORM do not provide an estimator of the error introduced by the approximation of g at the MPP to compute p_f.

More advanced methods based on surrogate modeling and approximation of the optimal instrumental distribution such as subset sampling have been developed [44]. Subset sampling (also referred to as splitting simulation) is a variance reduction technique in which the initial reliability analysis is split into a sequence of reliability analyses [106]. Considering a sequence of s nested subsets indexed by $r_1 > r_2 > ... > r_s = 0$, the reliability analysis is decomposed into a sequence of reliability analyses, $P[g(\mathbf{U}) < r_i]$, for $i \in [1, s]$ [91]. The probability of failure of the initial problem is obtained by: $p_f = \prod_{k=1}^{s} P[g(\mathbf{U}) < r_i | g(\mathbf{U}) < r_{i-1}]$. Efficient choice of sequences enable to decrease the computation cost required to evaluate p_f compared to direct MCS [91]. Reliability analysis methods have been developed for non probabilistic modeling of uncertainty (Evidence or Fuzzy theories) based on FORM [37, 53, 125]. Few implementations of reliability analysis with non probabilistic modeling of uncertainty have been applied in an UMDO framework [139].

The first level of the UMDO process, the system uncertainty modeling, have been detailed in Section 2. In Section 3, the second level of UMDO process, the UMDO procedure have been introduced by detailing the uncertainty propagation methods for reliability analyses and for input/output uncertainty propagation. In the next section, the UMDO strategies and formulations are detailed.

4. Uncertainty Multidisciplinary Design Optimization (UMDO) Framework

The UMDO procedure formalizes the multidisciplinary design optimization problem under uncertainty according to the strategy adopted (Reliability-Based Multidisciplinary Design Optimization (RBMDO), Robust Multidisciplinary Design Optimization (RMDO), Reliability Based Robust Multidisciplinary Design Optimization (RBRMDO)). The UMDO procedure needs to be adapted depending on the uncertainty mathematical formalism (Probabilistic, Evidence theory, Possibility theory, Pbox, *etc.*), the criteria of optimization (single objective, presence of several objectives) and the decomposition of the problem according to coordination procedures (MDF approach of MDO or MDO decomposition approaches) [146]. In this section we detail the key characteristics in the formulation of an UMDO problem.

4.1 UMDO strategies

An UMDO problem can be solved according to different strategies resulting in diverse formulations depending on the designer preferences for the optimized system. The two most classical requirements in UMDO problem formulations are reliability (RBMDO) and robustness (RMDO). In RBMDO, the objective is to maintain the violation of the design constraints under an acceptable range. As illustrated in Figure 3, a reliable optimum stays inside the feasible region despite the uncertainty. The deterministic optimum might provide a better performance, however, due to uncertainty, the optimum might be unfeasible. In RMDO, the objective is to minimize the loss of performances due to uncertainty while optimizing the system performances. As illustrated in Figure 4, a robust optimum ensures that the loss of performances is acceptable for the designer. The deterministic optimum provides a better performance (minimizing the performance in Figure 4), however due to uncertainty, the possible variations in the performance may be too high for the designer. Eventually, the designer could be interested in both a reliable and robust optimized system with respect to uncertainty [139]. In the general case, if the designer is interested in both a reliable and robust optimized system, the UMDO formulation is given by [4]:

$$\begin{aligned}
&\textbf{Optimize} && \Xi\left[\mathbf{F}(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})\right] = \left(\Xi_1\left[f_1(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})\right],...,\Xi_n\left[f_n(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})\right]\right) && (18)\\
&\textit{With respect to} && \mathbf{Z} && (19)\\
&\textit{Subject to} && \Lambda\left[g_j(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U}) \leq 0\right] - \Lambda_{reqd_j} \geq 0;\ j = 1,...,m && (20)\\
& && \Lambda\left[h_k(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U}) = 0\right] - \Lambda_{reqd_k} \geq 0;\ k = 1,...,l && (21)\\
& && \mathbf{Z}_{min} \leq \mathbf{Z} \leq \mathbf{Z}_{max} && (22)
\end{aligned}$$

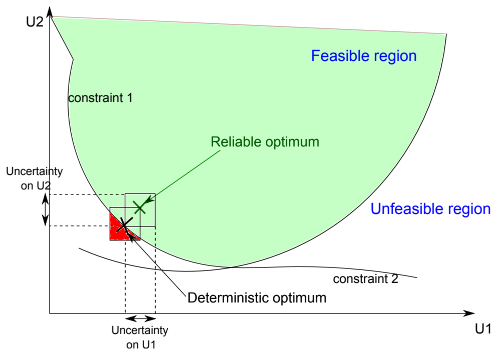

Figure 3: Reliable optimum

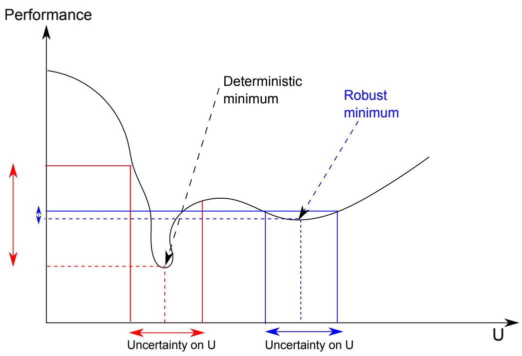

Figure 4: Robust optimum

with $\mathbf{F}$ a multi objective function and n the number of objective functions. Ξ is a performance measure. Each objective function f_i is measured by Ξ_i, which is a measure of either performance criteria or performance variation criteria (for instance the standard deviation, the percentile difference of the performance, *etc.*). Performance variation measures are used to design robust optimized system. Objective functions are functions of design variables $\mathbf{Z}$, uncertain variables $\mathbf{U}$, state variables $\mathbf{X}$, and coupling variables $\mathbf{Y}$. The design variables $\mathbf{Z}$ are deterministic. In the case a design variable is uncertain, then, it is decomposed into a deterministic design variable Z and an uncertain variable U. Λ refers to a measure of uncertainty that can rely on probability theory but also on evidence theory, possibility theory, interval theory, *etc.* Λ_{reqd} requires that the uncertainty measure of the constraint expressed through $\Lambda\left[g_j(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U}) \leq 0\right]$ has to be at least equal to Λ_{reqd} [4]. It translates the requirement of reliability for the optimized system to ensure that the system optimum always lies in feasible region under uncertainty. $g_j(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})$ is the j^{th} inequality constraint function (also referred to as state limit function) which delimits the feasible from the unfeasible region for the system and $h_k(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})$ is the k^{th} equality constraint function (a discussion about equality constraints in UMDO framework is provided in Section 6). First, two single discipline strategies are detailed: Reliability Based Design Optimization (RBDO) and Robust Design Optimization (RDO).

4.2 Reliability Based Design Optimization problem

The reliability of an optimized system refers to find an optimum lying within the feasible domain defined by the inequality constraint functions g_j $\{\forall j = 1, ..., m\}$. It ensures that despite the existence of aleatory and epistemic uncertainties at the current design phase, the optimum system will stay in the safe bounds even if several occur in further design phases or if aleatory events happen. Several approaches in terms of reliability design formulation exist [9]:

Nested approach (Figure 5): It consists in a non linear constrained optimization loop in which a reliability analysis is performed to ensure the feasibility of the system under uncertainty based on the methods described in Section 3.2 [9]. For each optimization iteration, reliability analyses are performed (Figure 5). In order to improve the computational cost of the reliability analysis, two formulations have been proposed: the Reliability Index Approach (RIA) [93] and the Performance Measure Approach (PMA) [130]. Both methods rely on FORM. The difference is in the way of solving the optimization problem to find the MPP in the case of RIA (Eq. 17) and the Minimum Performance Target Point (MPTP) for PMA defined by:

$$\begin{cases} \underset{\mathcal{U}}{min}\ \mathcal{G}(\mathcal{U}) \\ s.t.\ \ \parallel \mathcal{U} \parallel = \beta_T \end{cases} \tag{23}$$

with β_T the target reliability. In PMA, the probability constraints are replaced by quantile constraints approximated by FORM [44].

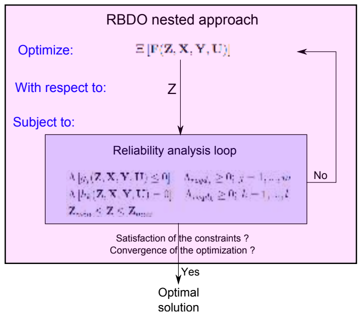

Figure 5: Nested approach of RBDO strategy

centered In presence of multiple inequality constraints, some constraints might never be active and the calculation of the probability of failure of these constraints dominates the computation effort due to the high reliability (approaching 1.0). PMA performs the reliability assessment only up to a necessary level materialized by the target reliability [130]. In [83, 130], a comparison of RIA and PMA for FORM is performed. According to the authors, PMA is more robust than RIA as it is easier to minimize a complex function subjected to a simple constraint (PMA) than to minimize a simple function subjected to a non linear constraint (RIA).

Decoupling approach: It consists in separating the optimization loop from the reliability analysis loop. Two approaches exist relying either on sequential (Sequential Optimization and Reliability Assessment (SORA) [40], Sequential Approximate Programming (SAP) [27]) or decoupled methods (RBDO based on Karush-Kuhn-Tucker optimality conditions [75], Single Loop Approach (SLA) [81]). The idea of sequential optimization is to transform the RDBO problem into a sequence of deterministic optimization problems while updating the admissible design space by reliability analyses.

SORA method: To decouple the problem in sequence of deterministic optimization problems, SORA [40] replaces the probabilistic reliability constraints by equivalent deterministic reliability constraints evaluated at the MPP (or MPTP). Four steps are distinguished [41]:

- Step 1: the first deterministic optimization problem is solved with the uncertain variables fixed at their mean value. In the next iterations, the deterministic optimization problem is solved with the uncertain variables fixed at the MPTP.
- Step 2: at the optimal design found in step 1, reliability analysis is performed to identify the MPTP of all the inequality constraints by PMA.

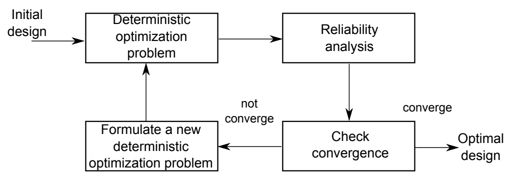

Figure 6: SORA procedure [41]

- Step 3: the convergence is checked. If the inequality constraints are verified and the optimization has converged, the optimal reliable solution is found.
- Step 4: if the convergence is not reached, or inequality constraints are violated, a new deterministic optimization problem is formulated based on the MPTP information of Step 3. A shifting vector is defined to push the next optimal design in the feasible region [41].

The sequence is performed until convergence (Figure 6). In [26], the authors highlight the importance of finding optimal shifting vectors at each iteration to decrease the number of sequential iterations of optimization and reliability analysis.

SAP method: In SAP, instead of computing the MPTP based on PMA as in SORA, a recurrence formula involving the gradient of the reliability constraints is performed [27].

SLA method: In SLA, the probabilistic constraints are approximated by deterministic constraints relying on the KKT optimality conditions of the PMA resulting on approximation of the MPTP [81].

Each approach relies on FORM to compute the probability of failure. According to [28], which is the most recent paper used for this review, no proof of convergence has been established for the decoupled methods. Table 5 compares the advantages and drawbacks of the different RDBO formulations.

Table 5: Reliability Based Design Optimization methods

RBDO methods	Advantages	Drawbacks
Nested approach	**RIA**: Simplicity of implementation	**RIA**: Intractable for complex model, FORM limitations
	PMA: Simplicity of implementation, more robust than RIA	**PMA**: Limited to low number of design and uncertain variables, FORM limitations
Decoupling approach	**Sequential (SORA, SAP)**: Less computationally intensive than nested approach and able to deal with complex system, SORA is fully deterministic, more robust and accurate than SAP	**Sequential (SORA, SAP)**: Approximation of objective function and constraints (by linearization for SAP), FORM limitations
	Decoupled, KKT, SLA: SLA is efficient and simple to implement with no reliability analysis calculations	**Decoupled, KKT, SLA**: KKT: weak stability, more expensive than nested approach and not adapted at several inequality constraint functions, suffer of increase of number of equality constraints and differentiability problems; SLA: difficulties with non-linear inequality constraint functions

4.3 Robust Design Optimization problem

The robustness of an optimized system refers to find an optimum such that the performance criteria are as insensitive as possible to variations due to aleatory and epistemic uncertainties. The sensitivity of a solution to uncertainty is quantified by the performance variation measure criteria. First, it is necessary to define a performance variation measure criterion for robust optimization.

4.3.1 Performance variation measure criteria

Several performance variation measure criteria can be defined depending on the uncertainty formalism (probability, evidence theory, possibility theory, *etc.*) and on the designer interests [15].

- Criterion based on expectation [129]: $\Xi_i = E[f_i(\mathbf{X}, \mathbf{Z}, \mathbf{U})]$, it corresponds to the expected value of the objective function over the uncertain variables, smoothing the initial objective function f_i.
- Criterion based on variance [96]: $\Xi_i = Var[f_i(\mathbf{X}, \mathbf{Z}, \mathbf{U})]$ allowing to quantify the importance of the variation of the objective function around potential optimal solutions.
- Criterion based on quantiles [18]: it allows to ensure a desired level of performance for the robust optimal solutions. The worst case approach is based on a quantile criterion where 100% of the points in an area around an optimal robust solution are with a better performance than the desired level of performance. The worst case approach is very conservative, different values (*e.g.* 99%, 95%, *etc.*) can be used as quantile orders depending on the problem specificities. The performance measure criterion based on the quantile order k, Q_k, is defined by $\Xi_i = Q_k[f_i(\mathbf{X}, \mathbf{Z}, \mathbf{P})] = \inf\{r \in \mathbb{R} | P\left(f_i(\mathbf{X}, \mathbf{Z}, \mathbf{P}) \leq r\right) \geq k\}$. Quantile criteria allow to have a control on the robust approach desired, however, discontinuities in the derivative of Ξ_i can complicate the optimization [15]. The dispersion of the distribution can also be evaluated by difference of quantiles.

In [15], the author explains that the performance variation measure criteria presented here are adaptable to the formalism of interval. Once a performance variation measure criterion is chosen, different robust design optimization formulations exist. They are detailed in the next Section.

4.3.2 RDO formulation

Taguchi established the foundation of the robust design to achieve high quality product based on the six sigma approach [123]. Since Taguchi first developments in robust design, several approaches have been suggested to design and optimize complex robust systems [18]:

- Multi objective approach: the objective function f_i is function of the performance criteria, and another objective function f_j is function of the performance variation measure criteria. The problem is therefore a multi objective problem, one to optimize the performance criteria and one to optimize the performance variation measure criteria [145].
- Weighted Aggregation approach: the objective function f_i is a weighted relation of the performance and performance variation measures [18]. The problem becomes a single objective optimization problem. However, the choice of the weighted relation is often hard to justify [18]. Within the probability formalism, engineering robust problem formulations often involve a weighted relation between the performance criteria to overcome the multi objective approach [54]. The objective function is a relation between the mean of the performance and the standard variation of the performance: $\Xi_{i1}[f_i(\mathbf{X},\mathbf{Z},\mathbf{U})] + w_i\Xi_{i2}[f_i(\mathbf{X},\mathbf{Z},\mathbf{U})] = \mu_{f_i}(\mathbf{X},\mathbf{Z},\mathbf{U}) + w_i\sigma_{f_i}(\mathbf{X},\mathbf{Z},\mathbf{U})$. w_i is a weight that reflects a trade-off in the relative importance of the performance criteria with respect to the robustness of the performance criteria.

Table 6 summarizes the advantages and drawbacks of each robust formulation.

Table 6: Robust Design Optimization methods

RDO methods	Advantages	Drawbacks
Multi objective approach	• Optimization of both performance criteria and performance variations	• Multi objective computation cost, no single optimal solution
Weighted Aggregation	• Less computationally expensive than multi objective approach	• Arbitrary choice of weighted relation

4.4 Multidisciplinary aspects of UMDO

In the previous paragraphs, we have detailed single discipline optimization approaches under uncertainty. However, aerospace vehicles are complex coupled multidisciplinary systems. UMDO strategies relying on a generalization of the RBDO and RDO formulations have been recently developed for multidisciplinary systems [139]. Compared with single disciplinary reliability based design optimization, the multi disciplinary problem is more complex due to the couplings between the disciplines [57]. The couplings make the uncertainty propagation and the optimization problem formulation more complex to handle.

A coupled MDA example is presented in Figure 7, consisting in 3 disciplines A, B and C. $\mathbf{Z_s}$ are shared design variables. They are shared either by all the disciplines or by a subset of the disciplines. $\mathbf{Z_i}$, $i \in \{A, B, C\}$, are disciplinary design variables. We denote $\mathbf{Z} = [\mathbf{Z_s}, \mathbf{Z_A}, \mathbf{Z_B}, \mathbf{Z_C}]$, the design variables vector. $\mathbf{U_s}$ are shared uncertain variables. They are shared either by all the disciplines or by a subset of the disciplines and $\mathbf{U_i}$, $i \in \{A, B, C\}$, are disciplinary uncertain variables. We denote $\mathbf{U} = [\mathbf{U_s}, \mathbf{U_A}, \mathbf{U_B}, \mathbf{U_C}]$, the uncertain variables vector. $\mathbf{Y_i}$ with $i \in \{A, B, C\}$ are disciplinary outputs and $\mathbf{Y^i}$ refers to all the outputs of the other disciplines that are inputs in the discipline i. $\mathbf{Y_{ij}}$ with $\{i, j\} \in \{A, B, C\}^2$, $i \neq j$ are coupling variables between disciplines i and j. $\mathbf{Y_{ij}}$ are outputs of discipline i and inputs of discipline j. Due to the uncertainty variables, the coupling variables $\mathbf{Y_{ij}}$ are also uncertain. To evaluate the system performances $\mathbf{P}$, a propagation of uncertainty through all the disciplines is necessary. Let us assume that the uncertain variables $\mathbf{U}$ are modeled with probability formalism and their PDF are known. Considering a design point, the propagation of uncertainty in a multidisciplinary environment consists in finding either the statistical moments of $\mathbf{P}$ or its PDF. To do so, most of the reviewed case studies (*cf* Section 5.1) propagate the uncertainty on the entire MDA ensuring the couplings between all the disciplines for all the possible instantiations of $\mathbf{U}$. In practice, a set of instantiations of $\mathbf{U}$ is defined because the number of instantiations taken by the uncertain variables can be infinite. However, this approach is computationally expensive as it is necessary to perform a MDA for the entire chosen set of instantiations of $\mathbf{U}$ at each design point $\mathbf{Z}$ during the optimization resulting in a prohibitive computational cost.

Decoupling the MDA consists in replacing the coupling variables $\mathbf{Y_{ij}}$ by their estimates $\mathbf{Y^*_{ij}}$ as shown in Figure 8. It allows to perform disciplinary analyses separately. To ensure the system feasibility, constraints on the coupling variables are imposed: $\hat{\mathbf{Y}}_{\mathbf{ij}} = \mathbf{Y^*_{ij}}$, with $\hat{\mathbf{Y}}_{\mathbf{ij}}$ the outputs of discipline i and inputs of discipline j. At the convergence of the optimization process, the feasibility of the solution is ensured by the consistency of the interdisciplinary couplings of the MDA.

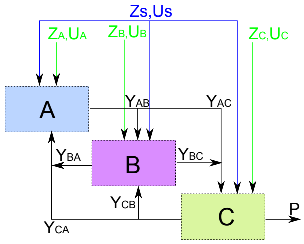

Figure 7: Coupled MDA

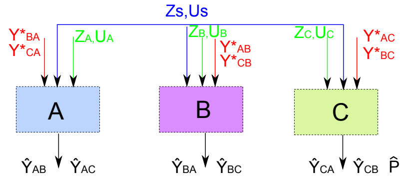

Figure 8: Decoupled MDA

$$
\begin{aligned}
\textbf{Optimize} \quad & \Xi\left[\mathbf{F}(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})\right] = \left(\Xi_1\left[f_1(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})\right],...,\Xi_n\left[f_n(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})\right]\right) & (24)\\
\textit{With respect to} \quad & \mathbf{Z},\mathbf{Y}^*_{\mathbf{ij}} & (25)\\
\textit{Subject to} \quad & \Lambda\left[g_k(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U}) \le 0\right] - \Lambda_{reqd_k} \ge 0;\ k = 1,...,m & (26)\\
& \hat{\mathbf{Y}}_{\mathbf{ij}} - \mathbf{Y}^*_{\mathbf{ij}} = 0\ \forall\{i,j\} \in \{A,B,C\}^2,\ i \neq j & (27)\\
& \mathbf{Z}_{min} \le \mathbf{Z} \le \mathbf{Z}_{max} & (28)
\end{aligned}
$$

The equality constraints h_k have been removed from the optimization problem (Eq.21) and are discussed in Section 6. In the case of deterministic multidisciplinary optimization problem, decomposition formulation have been developed to take advantages of inherent couplings between the disciplines. Several papers attempted to adapt these deterministic decompositions to UMDO framework [28, 41]. In the next paragraphs, existing UMDO methods are introduced.

4.4.1 RBMDO formulation

RBMDO formulation is the generalization of RBDO formulation for a multidisciplinary system. The essential difference between RBMDO and RBDO is the handling of the coupling variables. Two categories of RBMDO are distinguished [139]: the single level procedure and the decomposition and coordination procedure. A focus on the single level procedure is presented as it is the most common UMDO formulation [28, 41].

Single level procedure: The uncertainty analysis and the optimization are either combined into an equivalent deterministic optimization problem [3, 25], or decoupled in a sequence of deterministic optimization problems [41, 149]. This transformation into a single level procedure is the same as the RBDO approach for a single discipline. Once the optimization loop and the reliability loop are decoupled, existing MDO methods are used to solve the deterministic multidisciplinary design optimization problem (AAO, SAND, IDF, *etc.*).

In Step 1 of SORA, the uncertain variables $\mathbf{U}$ are fixed at their MPTP values $\mathbf{U_{MPTP}}$ and the optimization is performed on the design variables. In Step 2, the design variables are fixed at the optimal value found in Step 1 and reliability analysis is performed on uncertain variables to find the MPTPs. In [41], the authors introduce the MDF and IDF formulations to solve an UMDO problem with SORA. In the IDF approach, the interdisciplinary consistency is ensured by two sets of equations:

$$
\overline{\mathbf{Y^*}}_{\mathbf{ij}} - \hat{\mathbf{Y}}_{\mathbf{ij}}(\mathbf{Z},\overline{\mathbf{X}},\overline{\mathbf{U}},\overline{\mathbf{Y^{*i}}}) = 0;\ \{i,j\} \in \{A,B,C\}^2,\ i \neq j \tag{29}
$$

$$
\mathbf{Y}^*_{\mathbf{ijMPTP}} - \hat{\mathbf{Y}}_{\mathbf{ij}}(\mathbf{Z},\mathbf{X_{MPTP}},\mathbf{U_{MPTP}},\mathbf{Y}^{*\mathbf{i}}_{\mathbf{MPTP}}) = 0;\ \{i,j\} \in \{A,B,C\}^2,\ i \neq j \tag{30}
$$

$\overline{\mathbf{U}}$ stands for to the mean value of the uncertain variables $\mathbf{U}$. Eq.(29, 30) are system consistency constraints for the coupling variables at the MPTPs and the mean values of the uncertain variables. These equations need to be satisfied at the convergence of the UMDO problem. In the IDF formulation, the coupling variables, like the design variables, are provided by the optimizer. The deterministic MDO problem solved in the q^{th} cycle of SORA is given by:

$$
\begin{aligned}
\textbf{Optimize} \quad & \Xi\left[\mathbf{F}(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})\right] = \left(\Xi_1\left[f_1(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})\right],...,\Xi_n\left[f_n(\mathbf{Z},\mathbf{X},\mathbf{Y},\mathbf{U})\right]\right) & (31)\\
\textit{With respect to} \quad & \mathbf{Z},\mathbf{Y}^*_{\mathbf{ijMPTP}},\overline{\mathbf{Y}^*_{\mathbf{ij}}} & (32)\\
\textit{Subject to} \quad & g_k(\mathbf{Z},\mathbf{X}^{\mathbf{q-1}}_{\mathbf{MPTP}},\mathbf{Y}^{*\mathbf{i(q-1)}}_{\mathbf{MPTP}},\mathbf{U}^{\mathbf{q-1}}_{\mathbf{MPTP}}) \le 0;\ k = 1,...,m & (33)\\
& \overline{\mathbf{Y}^*_{\mathbf{ij}}} - \hat{\mathbf{Y}}_{\mathbf{ij}}(\mathbf{Z},\mathbf{X}^{\mathbf{q-1}}_{\mathbf{MPTP}},\mathbf{Y}^{*\mathbf{i(q-1)}}_{\mathbf{MPTP}},\mathbf{U}^{\mathbf{q-1}}_{\mathbf{MPTP}}) = 0;\ \forall\{i,j\} \subset \{A,B,C\}^2,\ i \neq j & (34)\\
& \mathbf{Y}^{*\mathbf{(q-1)}}_{\mathbf{ijMPTP}} - \hat{\mathbf{Y}}_{\mathbf{ij}}(\mathbf{Z},\mathbf{X}^{\mathbf{q-1}}_{\mathbf{MPTP}},\mathbf{Y}^{*\mathbf{i(q-1)}}_{\mathbf{MPTP}},\mathbf{U}^{\mathbf{q-1}}_{\mathbf{MPTP}}) = 0; \forall\ \{i,j\} \subset \{A,B,C\}^2,\ i \neq j & (35)\\
& \mathbf{Z}_{min} \le \mathbf{Z} \le \mathbf{Z}_{max} & (36)
\end{aligned}
$$

The reliability analysis in IDF formulation with SORA is based on PMA [41]. The system consistency is ensured by equality constraints in the optimization problem to find the MPTP in the standard Gaussian space:

$$\min_{(\mathcal{U},\mathbf{Y}^*_{\mathbf{ij}})} \quad \mathcal{G}(\mathbf{Z},\mathcal{U},\mathbf{Y}^*_{\mathbf{ij}},\mathbf{X}) \tag{37}$$

$$s.t. \quad \parallel \mathcal{U} \parallel = \beta_T \tag{38}$$

$$\mathbf{Y}^*_{\mathbf{ij}} - \hat{\mathbf{Y}}_{\mathbf{ij}}(\mathbf{Z},\mathcal{U},\mathbf{Y}^{*\mathbf{i}},\mathbf{X}) = 0;\ \forall\{i,j\} \in \{A,B,C\}^2,\ i \neq j \tag{39}$$

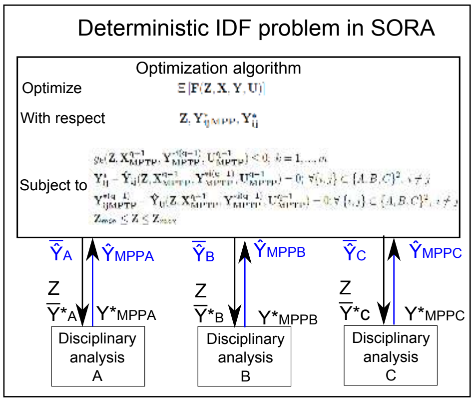

Figure 9: IDF formulation for SORA [41]

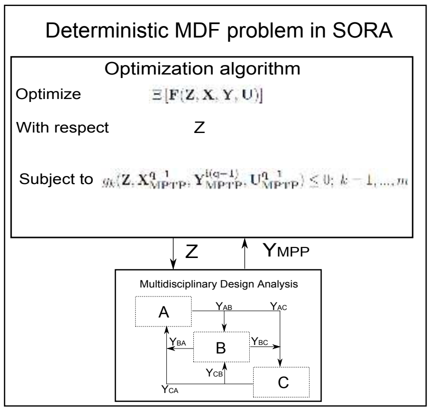

Figure 10: MDF formulation for SORA [41]

The MPTP reliability, β_T, refers to the imposed constraints on the reliability by Λ_{reqd_k} in the standard Gaussian space. The solution of this optimization problem (Eq.37) is the MPTP value for the uncertain variables and the coupling variables used in the next SORA cycle. In [41], the authors outline that, in presence of many inequality constraints and coupling variables, the number of variables handled by the optimizer becomes too large and may lead to convergence issue. The authors suggest to employ MDF with SORA for the UMDO problem (Figure 10). In MDF, the interdisciplinary consistency is verified by performing a MDA to obtain the solutions of Eq.(29, 30). Several iterative methods such as: Jacobi method [2], Newton methods [36], Gauss-Seidel methods [95] can be implemented to solve the system of equations. For each value of the design variables, interdisciplinary feasibility is verified during the optimization with MDF.

In [28], the authors compare three MDO formulations with SORA to solve an UMDO problem: MDF, IDF and AAO. In AAO, the state variables $\mathbf{X}$ are given by the optimizer, like the coupling variables and the design variables. Instead of disciplinary analyses like in IDF, disciplinary evaluations are performed on the values provided by the optimizer. The authors conclude on the similarity of results for probabilistic MDO and deterministic MDO formulations in terms of computational effort and efficiency (divergence, stop on non feasible designs). The authors highlight that AAO and IDF require less computational effort than MDF. However, in case of an inaccurate estimate of the probability of failure, IDF and AAO may diverge when integrated with SLA. SLA is interesting because it overcomes the nested RBMDO approach limitations and reduces the computational cost, however, divergence and initialization problems might affect the efficiency [28].

Decomposition and coordination procedures: The idea is to decompose the initial RBMDO problem into several sub-problems, solved under coordination strategies, either level by level or simultaneously. The reliability analysis and the optimization are not decoupled like in the single level procedure. Several approaches have been developed based on probabilistic CSSO [140], probabilistic CO [64] or probabilistic ATC [136].

Engineering applications of RBMDO are detailed in [41]. The authors highlight that most of the computation time is used to perform reliability analysis and the efficiency of the reliability analysis conditions the overall RBMDO efficiency. This remark might explain the large use of single level procedure with FORM to solve RBMDO. Most of the cited approaches for RBMDO are based on the probabilistic approach. Some papers focus on combination of probabilistic and evidence theory approaches to represent uncertainty and solve the UBMDO based on a single level procedure with FORM [139].

4.4.2 RMDO formulation

RMDO formulation is the generalization of RDO formulation for multidisciplinary systems. RMDO formulation shares some features with RBMDO in the way of propagating uncertainty information (mean, standard deviation) but this time applied to the objective function. Two categories of methods are distinguished for RMDO [6]:

- MDF approach: The robustness of the system performance is estimated by propagation of uncertainty on the multidisciplinary system treated as a whole. It requires the simulation of the entire MDA for the entire chosen set of instantiations of the uncertain variables $\mathbf{U}$ to allow the quantification of the performance variation measure criteria [145]. This approach ensures the feasibility of the system for each value of **U**, but the computation cost introduced by the repetition of the MDA is prohibitive.

- Decomposition approach: The robustness of the entire system is evaluated based on the robust analysis performed on each discipline. In [55], the authors perform worst case propagation analysis and robust optimization. Taylor series enable to perform first order sensitivity analysis to compute the worst case variation in the system performances. In [39], the authors develop a probabilistic uncertainty propagation framework to estimate the mean and the standard deviation of coupling variables are approximated by Taylor series based on Concurrent SubSystem Uncertainty Analysis (CSSUA) methods. This method has been adapted in [43], in case of the presence of both probabilistic uncertain variables and epistemic uncertainty represented by intervals. Probabilistic Analytic Target Cascading (PATC) has been developed by [82] in which the design consistency is ensured by the match of the first two statistical moments of coupling variables.

In this Section, we have detailed UMDO strategies according the designer preferences (robustness, reliability) and the different existing formulations to solve the problem. UMDO methods have been applied to the design of space transportation systems. A review of the methods employed in this field is presented in the next section.

5. UMDO applied to space transportation vehicle

In literature, several papers solve space transportation system design problem under uncertainty. The purpose of this section is to synthesize and to compare the UMDO approaches implemented. We focus on three topics: the uncertainty modeling frameworks, the uncertainty propagation methods and the UMDO formulations.

5.1 Uncertainty modeling framework

The probability formalism to represent uncertainty is the most employed (Table 7). Probability theory is used to represent aleatory uncertainty in all the referenced papers. In [144], the authors use the Johnson parametric family distributions to treat epistemic interval data with the probability formalism. In [32] and [71], the authors employed the Evidence theory to represent epistemic uncertainty in space transportation design. As the design experts are more familiar with probability framework, evidences and data for Evidence theory can be difficult to obtain. To design a Reusable Launch Vehicle (RLV), in [17], the authors model aleatory uncertainty with the probability formalism and the epistemic uncertainty with intervals. The propagation of mixed aleatory and epistemic uncertainties results in Pboxes for the vehicle takeoff gross weight. In [141] and [145] , the authors represent epistemic uncertainty by interval data however they used different propagation methods, discussed in the next section.

Table 7: Different uncertainty mathematical modeling frameworks applied to aerospace vehicle design

Uncertainty modeling framework	References
Probability theory	[24], [87], [89], [103], [138], [141], [142], [144], [145]
Evidence theory	[32], [71]
Possibility theory	/
Probability boxes	[17]

Table 8: Different uncertainty propagation methods applied to aerospace vehicle design

Uncertainty propagation methods	References
Input/output	
Polynomial chaos	[17],
Taylor series	[144], [145],
Surrogate model	[87], [138], [141], [142],
Monte Carlo	[24], [89], [103]
Reliability analysis	
Monte Carlo	[138], [141]
FORM	/

5.2 Uncertainty propagation methods

MCS and Taylor series approximation are the most employed methods to propagate input/output uncertainty. MCS is used directly in [24, 103], or on surrogate models in [87, 89, 141, 142]. In [89], the authors create a surrogate model for aerodynamics discipline based Artificial Neural Network. Response surfaces based on Discrete Probability Optimal Matching Distribution are created in [87] to propagate uncertainty. In this method, continuous probabilities are discretized and DoEs are performed on the discrete points to build the surrogate model. Taylor series are implemented in [144, 145] for the interdisciplinary propagation. The mean and the standard deviation of coupling variables are approximated by Taylor series based on Concurrent SubSystem Uncertainty Analysis methods developed for multidisciplinary uncertainty propagation in [39]. In [17], the authors propagate uncertainty with polynomial chaos expansion. The coefficients of the PCE are computed based on random collocation points and least square method. Legendre polynomials are used for epistemic uncertain variables given by intervals. Vertex combinatorial method is used in [71] to propagate evidences, assuming the system response is continuous and monotonic with respect to every uncertain variables. For reliability analysis, only one method is employed: MCS. MCS is used by Yao *et al.*, [138, 141], to compute the probability of failure.

5.3 UMDO formulation

In the literature reviewed for the present survey, only few papers formulate a complete UMDO problem for space transportation system design. In [17, 24], the authors quantify the uncertainty in the performances of the system but no optimization is performed. Heuristic optimization methods are compared in [103] to design and optimize deterministically a launch vehicle. The robustness of the found deterministic solutions is studied with the probabilistic modeling of uncertain parameters to select the most robust solution. Most of the case studies focus on the robustness of the optimal space transportation system design and not on the reliability. Moreover, except in Yao *et al.*'s papers , [138, 141], MDF is employed. The UMDO formulations used in aerospace vehicle design case studies are discussed in the next paragraphs.

Robust approaches of UMDO. ANOVA analyses on surrogate models are performed in [142] to determine the value of the design variables providing the minimal variance of the performance subjected to a constraint on the performance. MDF optimization on the 95^{th} percentile of a launch vehicle dry weight under aleatory uncertainty is performed in [87]. In [144], the authors perform a weighted aggregation robust MDF optimization of the mean and standard deviation of the total power consumption of a satellite. Six design variables and three parameters are considered with epistemic uncertainties. In [145], the upper stage of a two stage to orbit vehicle is optimized. A weighted aggregation robust MDF optimization is performed on the gross weight of the upper stage. Most of the robust optimizations avoid the multi objective optimization by aggregating the objective functions (corresponding to the mean and the standard deviation of the studied performance) into a single objective function. In [89], a multi objective robust MDF optimization is implemented of the means and variances of the maximum heat flux and the maximum internal temperatures of an unmanned space vehicle for re-entry operations.

Reliable approaches of UMDO. In [71], the weight of the wing of a RLV is optimized and the reliability constraints are quantified by Plausibility measure. A MDF approach is implemented to optimize the performance based on six epistemic design variables. Evidence theory optimization and Evidence theory with global response surface are compared. According to King [71], the response surface enables to decrease the computational cost by 98% while maintaining accuracy.

Reliable and robust approaches of UMDO. Yao *et al.*, [138, 141] formulate a reliable and robust optimization problem to design a satellite. The robustness is expressed by a weighted aggregation objective function of the mean and the standard deviation of the satellite mass. In [141], the MDO approach relies on Concurrent SubSpace Optimization (CSSO). The satellite system design is decomposed into different subsystem optimization problems corresponding to the disciplines. The system robustness and reliability are computed with MCS and the performances of the system are approximated by Taylor series. In [138], a MDF-CSSO decomposition is used. A first optimum is found by a sequence of surrogate construction and updates and MDF probabilist optimization. Then, once a global optimum is found, corresponding to a baseline, high fidelity simulations and analyses are performed and the optimization is based on the CSSO formulation allowing disciplinary autonomy.

This section presented space vehicle design with UMDO approach case studies. Most of the methods share common features such as a MDF approach of MDO to avoid the management of coupling variables, or Taylor series approximation for the propagation of uncertainty. However, to improve UMDO methods for the design of aerospace systems, several challenges have to be overcome. The next section highlights the issues introduced by the presence of uncertainty in the design of a multidisciplinary aerospace system.

6. Challenges induced by UMDO for space system transportation

Several categories of challenges can be distinguished in UMDO problems. The first type is the propagation of uncertainty in a multidisciplinary environment and the characterization of the uncertainty in the coupling variables. Moreover, developing an efficient mixed aleatory and epistemic framework to represent the available information is an issue. The use of surrogate models creates another challenge in the quantification of the uncertainty introduced by the metamodels.

6.1 Propagation of uncertainty in a multidisciplinary environment

One of the key issues to enable an efficient UMDO process is the development of uncertainty propagation in a multidisciplinary environment. Several problems arise in the propagation of uncertainty from one discipline to another.

To evaluate the system performances, a propagation of uncertainty through all the disciplines is necessary. Let us assume that the uncertain variables are modeled with probability formalism and their PDF are known. Considering a design point, the propagation of uncertainty in a multidisciplinary environment consists in finding either the statistical moments of the performance criteria either the PDF of the performance criteria. To do so, most of the reviewed case studies propagate the uncertainty on the entire MDA ensuring the couplings between all the disciplines for all the possible values taken by $\mathbf{U}$. However, engineering applications of this approach are limited due to its prohibitive computational cost. To overcome the computational burden introduced by uncertainty propagation over the entire system, decoupling of the multidisciplinary system have been proposed and discussed in the next section.

6.1.1 Decoupling for MDO formulations under uncertainty

In [39] the authors propose to use first-order Taylor series expansion to estimate the mean and the standard deviation of the coupled variables. The mean values of linking variables $\mathbf{Y_{ij}}$ and the performance criteria $\mathbf{Y_F}$ are approximated at the mean values of inputs as: $\mu_{\mathbf{Y_{ij}}} \simeq F_i(\mathbf{Z}, \mu_{\mathbf{U}}, \mu_{\mathbf{Y^i}})$ with F_i being the function to evaluate the discipline i. In the same way, the standard deviation is approximated by first-order Taylor series expansion. However, several problems arise from this approach:

- The approximation is only valid for function that can be locally approximated as linear function. However, some functions involved in aerospace vehicle design are non linear such as the function computing the aerodynamics coefficients of a launch vehicle function of the angle of attack, the geometry of the launch vehicle and the Mach number. In this case, $\mu_{\mathbf{Y_{ij}}} \simeq F_i(\mathbf{Z}, \mu_{\mathbf{U}}, \mu_{\mathbf{Y^i}})$ is not valid.
- In [8], the authors highlight that mean and standard deviation might not be enough to accurately describe the coupling variable and performance criterion distributions. The authors suggest to compute higher statistical moments as the skewness and the kurtosis based on a second order Taylor series expansion. However, this approach requires the computation of first and second order derivatives of the disciplinary functions which can be difficult and expensive to perform on black box function.

To decouple completely the multidisciplinary problem, it is necessary to estimate the coupling variables and to impose equality constraints at the optimization level. This approach is discussed in the next paragraph. The notations are same used in Section 4.4. In the literature reviewed for the present survey, no complete and clear definition of the satisfaction of equality constraints imposed on coupling variables at the convergence of a UMDO problem has been established. It is necessary to define the system compatibility requirements at the convergence for the coupling variables. In [104], the authors present the challenges imposed by equality constraints in robust design optimization. The authors distinguish two types of constraints:

- Equality constraints that may not be satisfied because of uncertainty,
- Equality constraints that must be satisfied regardless of uncertainty.

The first type of equality constraints expresses designer's or customer's preferences. For instance, the mission for a launch vehicle could require to insert the payload at a specific altitude, velocity and flight path angle. Therefore, it would impose equality constraints on uncertain quantities in the optimization of the launch vehicle. However, this type of equality constraints can be violated without losing the feasibility in the launch vehicle design. In practice, tolerances exist on the final orbit for the payload and the objective is to be inside the tolerances. Therefore, the mission constraints can be transformed into inequality constraints with the customer's tolerance to ensure the satisfaction of the mission. The equality constraint functions of the UMDO formulation, (Eq.21), is transformed into inequality equation functions with the customer's tolerance.

The second type of equality constraint is physical-based equality constraint and must always be satisfied. For instance, Newton's second law must always be satisfied no matter the uncertainty on the mass. In the decoupled approach of MDA, the system physical feasibility, is ensured by the equality constraints on the coupling variables : $\mathbf{\hat{Y}_{ij}} = \mathbf{Y^*_{ij}}$. These equations have to be satisfied regardless of uncertainty to ensure the feasibility of the aerospace vehicle design. In [42], the authors proposed two methods to solve RBDO problems under equality constraints:

- The first approach is to eliminate the equality constraints by variable substitutions. The k equality constraints induce dependencies between the n uncertain variables and the authors suggest to separate the uncertain variables $\mathbf{U}$ into two sets. One set is composed of n-k independent uncertain variables $\mathbf{U_{in}}$ and the other one of k dependent uncertain variables $\mathbf{U_d}$. The k equality constraints enable to express the dependent uncertain variables $\mathbf{U_d}$ as functions of the n-k independent uncertain variables by: $\mathbf{U_d} = \mathbf{H}(\mathbf{Z}, \mathbf{U_{in}})$. In our case, all the coupling variables $\mathbf{Y_{ij}}$ are uncertain variables and the equality constraints $\mathbf{\hat{Y}_{ij}} = \mathbf{Y^*_{ij}}$ allow to express some coupling variables as functions of others. However, in the presence of black box functions it is impossible to find the functional relationship $\mathbf{H}$ between the dependent and independent coupling variables.

- The second approach proposes to use FORM to formulate and solve a RBDO problem with equality constraints. The equality constraints are verified at the mean of the coupling variables and at the MPTPs of all the uncertain variables. However, ensuring equality constraints at the mean and MPTP does not guarantee the feasibility of the design for all the instantiations of the uncertain variables. To ensure the feasibility of a multidisciplinary system, at the convergence of the UMDO problem, equality constraints $\mathbf{\hat{Y}_{ij}} = \mathbf{Y^*_{ij}}$ need to be verified for each instantiation of the uncertain variables.

Different methods are proposed to ensure system feasibility and discussed in the next paragraphs.

- In [80], the authors propose an interval and interdisciplinary uncertainty propagation method. In this paper, the uncertain variables $\mathbf{U}$ are given by interval. The authors suggest to replace the equality constraints by: $\| \hat{Y}_{ij} - Y^*_{ij} \| \leq \epsilon$, with ϵ a positive tolerance. As the outputs of discipline i and inputs of discipline j, $\hat{Y}_{ij}$ are intervals, the authors suggest that the estimates Y^*_{ij} should be intervals enclosing the variation in $\hat{Y}_{ij}$. However, as outlined in [144] the system compatibility requirement should be satisfied at every single point value, and in [80] constraints are imposed on the interval bounds but not on the points inside the interval.
- In [136], the authors suggest to propagate uncertainty based on PCE and to satisfy the equality constraints by imposing constraints on the polynomial coefficients. Due to the nature of PCE, the polynomial coefficients can represent the entire probabilistic distribution of the coupling variables. The statistical moment information are implicitly included in the polynomial coefficients. The accuracy of the estimation of the statistical moments depends on the polynomial degree used for the decomposition [136]. The authors suggest to match the polynomial coefficients of PCEs instead of matching the two first statistical moments (or the fourth first statistical moments [8]). Y^*_{ij} and $\hat{Y}_{ij}$ are decomposed by PCE, assuming a degree d: $Y^*_{ij} \simeq \sum_{k=0}^{d} \alpha^*_k \Psi_k(\mathbf{Z}, \mathbf{Y^i})$ and $\hat{Y}_{ij} \simeq \sum_{k=0}^{d} \hat{\alpha}_k \Psi_k(\mathbf{Z}, \mathbf{Y^i})$. Then, constraints are imposed on the polynomial coefficients such that: $\| \alpha^*_k - \hat{\alpha}_k \| \leq \epsilon \; \forall k \in \{1, ..., d\}$. The authors applies this method on ATC decomposition of the optimization problem. However, to match the polynomial coefficients, it is necessary to know the basis in which the decomposition of the coupling variables has to be made. In general, the input distribution is unknown and the basis for the decomposition cannot be assumed. It raises the challenge of the propagation of uncertainty by PCE when the input distributions are unknown. The authors assume the same basis for the outputs as for the inputs. Moreover, matching all the polynomial coefficients does not guarantee that the feasibility is ensured for all the points of the input distributions at the convergence of the UMDO problem.
- Sankararaman *et al.* [109] develop an approach based on likelihood to satisfy interdisciplinary compatibility for each instantiation of the uncertain variables $\mathbf{U}$. For one instantiation of $\mathbf{U}$, couplings from discipline i to discipline j are satisfied if: $\mathbf{\hat{Y}_{ij}} = \mathbf{Y^*_{ij}}$. The authors consider the probability of satisfying the equality given a value of the design variables $\mathbf{Z}$, one instantiation of the uncertain variable $\mathbf{U}$ and one realization of the input coupled variables $\mathbf{Y^*_{ij}}$: $P(\mathbf{\hat{Y}_{ij}} = \mathbf{Y^*_{ij}} | \mathbf{Y^*_{ij}}, \mathbf{Z}, \mathbf{U})$. This conditional probability is proportional to the likelihood of $\mathbf{Y^*_{ij}}$ considered as an uncertain parameter with $\mathbf{Z}$ and $\mathbf{U}$ fixed. The authors derive the joint probability density of the input coupling variables from discipline i to discipline j:
$$f(\mathbf{Y^*_{ij}}) = \frac{L(\mathbf{Y^*_{ij}})}{\int L(\mathbf{Y^*_{ij}}) d\mathbf{Y^*_{ij}}} \tag{40}$$
with $L(\mathbf{Y^*_{ij}})$ the likelihood function of the coupling input variables. However, this approach has several limits. First, it is necessary to assess a prior density for $\mathbf{Y^*_{ij}}$ (assumed uniform in [109]) and the choice of the prior distribution influences the output distribution $\mathbf{\hat{Y}_{ij}}$. Moreover, it is assumed that for each value of the design variables and each instantiation of the uncertain variables there exists a value for $\mathbf{Y^*_{ij}}$ ensuring the interdisciplinary consistency and that this value is unique. However, for complex systems, the existence of such a value and its uniqueness are not guaranteed. Finally, the likelihood is a relative function and instantiation of $\mathbf{Y^*_{ij}}$ are compared to find the most likely value, but it is a relative measure.

To propagate uncertainties in a decoupled formalism, it is necessary to define the conditions required to ensure the feasibility of the system under uncertainty at the convergence of the optimization problem.

6.1.2 Dependencies in the coupling variables

Another issue introduced by the propagation of uncertainty in a multidisciplinary environment is due to shared variables. The presence of shared variables between several disciplines introduces a dependency between the coupling variables. Indeed, because of shared uncertain variables $\mathbf{U_s}$, the coupling variables that are inputs of discipline i, $\mathbf{Y^i}$, are dependent. Therefore, to compute the uncertainty of the outputs of discipline i, $\mathbf{Y_i}$ it is necessary to estimate the joint probability distribution of $\mathbf{Y^i}$. The straightforward method to compute the joint distribution of $\mathbf{Y^i}$ is by simultaneously simulating all the disciplines. Therefore, the estimation of the joint probability of the input coupling variables of discipline i goes against the decoupling approach of propagation of uncertainty in a multidisciplinary environment. A method to approximate the joint probability distribution of the coupling variables in a decoupled formalism is necessary to provide accurate uncertainty propagation methods.

6.2 Other important challenges in UMDO

Design variable variation domains in UMDO: In the early design phases, the design variable variation domains are large to cover all the possible architectures for the aerospace system. These large domains of variation introduce difficulties:

- In the uncertainty modeling: the modeling of the uncertain variables for different regions of the design space are not necessarily the same. For instance, the uncertainty on a propellant mass on-board of a launch vehicle might not be the same depending on the stage configuration. Due to heat loss and evaporation of liquid fuel during the tank filling on the launch pad, the propellant mass on-board is uncertain. However, the heat exchange depends on the tank surface (geometry, diameter, length, *etc.*). Therefore, the uncertainty modeling of the propellant mass on-board needs to be adapted for different stage configurations. For instance, if modelled by a normal distribution, the parameter distribution values are function of the design variables.
- In the uncertainty propagation: propagation methods need to be adapted to the large domains of variation in the design space. For instance, in PCE, due to the change in the input distribution depending on the design space region, for each value of the design variables provided by the optimization algorithm, the PCE coefficients need to be computed. This results in a prohibitive computational cost. In [30], the authors propose a moving least square surrogate model for the design variable conjugated with PCE surrogate model for the uncertain variables. A DoE in the design variable space is performed and for each value of the design variables, collocation points are used to compute PCE coefficients. Then based on moving least square method, the PCE coefficients are predicted for any design variable values, avoiding the call to original function to compute the PCE coefficients for each design variable values.

Reliability analysis: In the reviewed UMDO formulation, FORM is the most employed reliability analysis method. FORM is interesting for its simplicity of implementation and its computational cost [44]. However, as highlighted in Section 3.2, approximations (linearization, transformation in the Gaussian standard space, uniqueness of the MPP) introduce limitations. As outlined in [41], most of the computational cost to solve an UMDO problem comes from the reliability analysis. Research efforts on more advanced reliability methods are required in UMDO context to overcome the FORM limitations and to improve the reliability analysis accuracy while maintaining an acceptable computational cost.

Uncertainty formalism choice and efficient mixed aleatory epistemic uncertainty propagation: The referenced RBMDO methods in Section 4.4 are mostly applied to random uncertainties with probability framework. As highlighted in Section 2.4, it is suited to represent aleatory uncertainty but efficient epistemic uncertainty representation requires different formalisms than probability [61], due to the lack of information. An efficient framework incorporating both aleatory and epistemic uncertainties is essential to represent the available knowledge to the experts and designers. Most of the mixed aleatory and epistemic propagation methods rely on nested loops. For instance, with evidence theory to represent epistemic uncertainty for each possible focal element, probabilistic uncertainty is propagated resulting in a computational burden [58]. Furthermore, as the measures to represent aleatory and epistemic uncertainty are not defined in the same way (additive, superadditive and subadditive measures) as outlined in Section 2.4, it is difficult to develop a mixed uncertainty framework. In [139], the authors adapt existing deterministic MDO methods to incorporate mixed uncertainties in which reliability analysis is performed by FORM.

Surrogate model and approximation methods: Several challenges arise from the use of surrogate models instead of the original functions or approximation methods (FORM, numerical integration, *etc.*):

- Uncertainty is introduced through all the approximations along the UMDO process (propagation of uncertainty, computation of the uncertainty of failure, numerical integration, derivative calculation approximation, *etc.*). Uncertainty introduced by approximation methods are part of the design process and have to be incorporated to accurately optimized under uncertainty.
- Another challenge comes from the necessity to determine errors introduced by the use of surrogate models instead of the original computationally expensive functions.

In spite of all the improvements that have been achieved so far in the field of UMDO, all the challenges highlighted in this Section have to be addressed to provide an efficient UMDO framework for space system transportation design. One of the key challenge is the propagation of uncertainty in a multidisciplinary environment and the interdisciplinary coupling satisfaction at the convergence of the UMDO process.

7. Conclusion

This paper has provided an overview of nearly 30 years of developments in fields connected to UMDO (reliability analysis, uncertainty modeling, uncertainty propagation, *etc.*). UMDO methods are recent and derived from MDO methodologies. The progress achieved in UMDO over the last 10 years are the result of efficient strategies for assessing reliabilities and for formulating optimization problem. In this paper, the UMDO process has been detailed while highlighting the specificities of the design of space transportation systems. The key steps such as the uncertainty identification, modeling and propagation have been presented and different methodologies have been compared. The process to solve an UMDO problem has been detailed and existing UMDO formulations has been described and compared. A literature review of the UMDO case studies applied to space transportation system has been provided. In spite of all the improvements that have been achieved so far, there are still open issues. We have highlighted the issues and further research efforts that are required to develop an efficient UMDO framework to design space transportation system. One of the key issues to be addressed in the future is the propagation of uncertainty in a multidisciplinary environment and the interdisciplinary coupling satisfaction at the convergence of the UMDO problem.

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