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Abstract
Train path (infrastructure), rolling stock and crew scheduling are three critical planning decisions in railway transportation. These resources are usually planned separately in a sequential process that typically starts from planning (1) train paths and goes further on to (2) rolling stock and (3) train drivers. Such a sequential approach helps to handle the complexity of the planning process and simplify the underlying mathematical models. However, it generates solutions with higher cost because the decisions taken at one step can drastically reduce the set of feasible solutions in the following steps. In this paper, we propose a Lagrangian heuristic to solve an integrated problem which globally and simultaneously considers the planning of two railway resources: Rolling stock units and train drivers. Based on a mixed integer linear programming formulation, this approach has two important characteristics in an industrial context: i) It can tackle real-life integrated planning problems, and ii) The Lagrangian dual is solved by calling two proprietary software modules available at the SNCF. First numerical experiments on real-life instances are promising. Compared to a sequential approach, the Lagrangian heuristic leads to cost reductions and generates good solutions in a reasonable CPU time.

Keywords
Railway transportation, integrated planning, mixed integer programming, Lagrangian heuristic

1 Introduction and industrial context

Train path (defined as the part of infrastructure required to operate a train between two points of the railway network during a given time period), rolling stock and crew scheduling are three critical planning decisions in railway transportation. These resources are usually planned separately in a sequential process that typically starts from train paths and goes further on to rolling stock and train drivers:

1. Optimized Planning of Railway Timetables. The commercial offer is considered by elaborating a space-time graph where each train corresponds to a path. This graph supports the booking of train paths in the railway infrastructure. Constructing an

optimal railway timetabling requires to consider the major constraints of the railway system, but often not the detailed constraints on rolling stock units and train drivers.

2. Optimized Planning of Rolling Stock. This stage consists in planning the rolling stock to cover the train paths defined in Step 1. This is done based on the available number of rolling stock units of each type, and aims at satisfying all technical and functional constraints.

3. Optimized Planning of Train Drivers. The first step of this stage consists in defining a set of working days (shifts) for the train drivers to cover the requirements of the plan for the rolling stock, while respecting all legal constraints. The second step aims at combining these working days in a consistent roster for each driver.

Optimization tools are sometimes used at some point in each of these steps. In particular, SNCF has already developed optimization tools for Steps 2 and 3. Such a sequential approach helps to handle the complexity of the planning process and simplify the underlying mathematical models. However, it generates solutions with higher cost because the decisions taken at one step can drastically reduce the set of feasible solutions in the following steps.

In this context, studying an integrated planning approach is relevant to increase the quality of the production process of railway transportation plans. Several decision-making problems can be identified when considering integrated planning of railway transportation resources (see for example [1]). In this paper, we focus on the case of fixed timetables where only rolling stock units and train drivers are planned in an integrated way.

Furthermore, and as mentioned in [1], integrated planning approaches were already studied in the airline industry [5], [4], [6], and in public transportation [3]; where cost reductions ranging from 5 to 10% are reported. Although a lot of attention has been devoted to rolling stock and train driver scheduling in the railway literature, only few papers deal with the integration of rolling stock and train drivers when optimizing the transportation plan.

In this paper, we propose a mixed integer linear programming model to globally and simultaneously consider the planning of two railway resources: Heterogeneous rolling stock units and heterogeneous train drivers. We introduce a Lagrangian heuristic to solve the resulting problem. This approach has two important characteristics in an industrial context: i) It can tackle real-life integrated planning problems, and ii) The Lagrangian dual is solved by calling two proprietary software modules available at the SNCF.

In the remainder of this paper, we present in Section 2 a mixed integer linear programming model for our integrated planning problem of railway resources. The Lagrangian relaxation heuristic approach is sketched in Section 3 while Section 4 provides some industrial implementation details. We then discuss, in Section 5, preliminary computational experiments on a real-life instance extracted from the transportation plan of a French region (Bretagne). Some conclusions are finally drawn in Section 6.

2 Mathematical Formulation

Our modeling approach basically consists in formalizing separately the rolling stock and train driver planning problems with mixed integer linear programming (MIP) models. Coupling constraints are further introduced in order to:

- Model the fact that each train path requires one driver and one (or more) rolling stock unit(s) to be covered,

- And control the technical consistency of the rolling stock unit(s) and the driver assigned to each train path.

2.1 Basic notations

Throughout this paper, we use the following notations:

- $\mathcal{S}$ is the set of train paths that need to be covered with rolling stock and drivers,

- $\forall s \in \mathcal{S}, PAF_s$ is the rolling stock capacity (in terms of powers, seats or units) required to cover train path s,

- $\forall s \in \mathcal{S}, \mathcal{EC}_s$ is the set of driver duties necessary to cover train path s,

- $\mathcal{EC} = \cup_{s \in \mathcal{S}} \mathcal{EC}_s$ is the set of all driver duties that must be covered,

- $\mathcal{K}$ is the set of rolling stock types,

- $\forall k \in \mathcal{K}, CAP_k$ is the capacity (in terms of powers, seats or units) of a type k rolling stock,

- $\mathcal{D}$ is the set of driver depots,

- $\forall d \in \mathcal{D}, H_d$ is the number of train drivers available at depot d,

- $\forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall d \in \mathcal{D}, COMP_{s,k,d} \in \{0, 1\}$ is equal to 1 if type k rolling stock can be driven by drivers of depot d to cover train path s, and 0 otherwise,

- $\forall k \in \mathcal{K}, CA_k$ is the cost of assigning a type k rolling stock unit to a train path,

- $\forall k \in \mathcal{K}, CE_k$ is the cost of using a type k rolling stock unit,

- $\forall s \in \mathcal{S}, PENRS_s$ is the penalty incurred if train path s is not covered with a rolling stock unit(s),

- $\forall s \in \mathcal{S}, PENDR_s$ is the penalty incurred if train path s is not covered with a driver,

- $\forall s \in \mathcal{S}, PEN_s$ is the penalty incurred if train path s is not covered with a rolling stock unit(s) and a driver.

Figure 1 illustrates the decomposition of one train path into two drivers duties. To be completely covered, a rolling stock unit(s) must be assigned to train path s_i and both ec_a and ec_b must be covered by one train driver each.

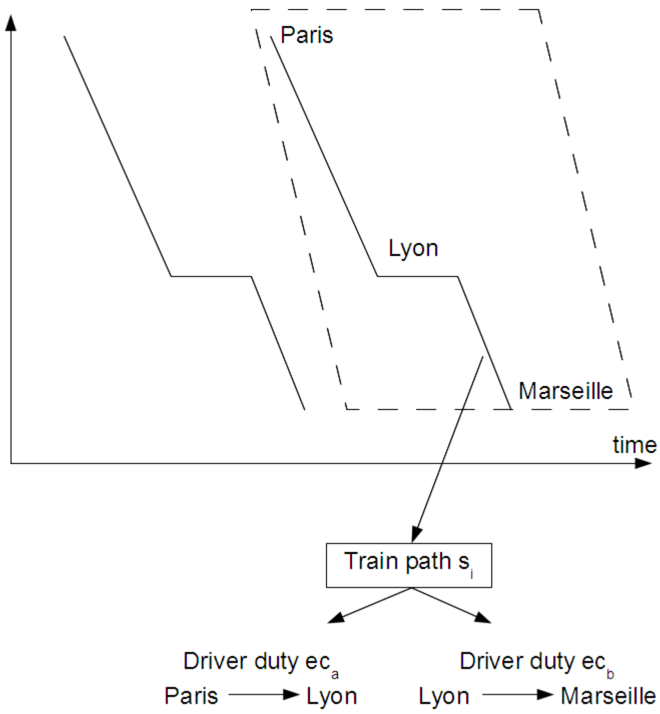

Figure 1: Decomposition of one train path into two driver duties

2.2 Rolling stock planning subproblem

Let $(MIP1)$ denote the rolling stock mixed integer programming model. $(MIP1)$ is a multi-commodity network flow problem derived from a space-time graph that models the data and parameters of the planning problem. $(MIP1)$ captures constrained flows (denoted by F below) that express the circulation of rolling stock units over each arc of the graph. Let us introduce the following basic decision variables:

- $\forall s \in \mathcal{S}, \forall k \in \mathcal{K}$, $Y_{s,k} \in \{0,1\}$ is equal to 1 if a type k rolling stock unit is assigned to train path s, and 0 otherwise,

- $\forall k \in \mathcal{K}$, N_k is the integer number of type k rolling stock units used in the transportation plan,

- $\forall s \in \mathcal{S}, \forall k \in \mathcal{K}$, $F_{s,k}$ is the integer number of type k units used in the transportation plan to cover train path s,

- $\forall s \in \mathcal{S}$, $ee_s \in \{0,1\}$ is equal to 1 if train path s is not covered with rolling stock, and 0 otherwise.

$(MIP1)$ can be expressed as follows:

$$\min \quad Z(MIP1) = \sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} CA_k \cdot F_{s,k} + \sum_{k\in\mathcal{K}} CE_k \cdot N_k + \sum_{s\in\mathcal{S}} PENRS_s \cdot ee_s \quad (1)$$

$$\forall s \in \mathcal{S} \qquad \sum_{k\in\mathcal{K}} Y_{s,k} + ee_s = 1 \quad (2)$$

$$\forall s \in \mathcal{S} \qquad \sum_{k\in\mathcal{K}} F_{s,k} \cdot CAP_k \geq PAF_s \cdot (1 - ee_s) \quad (3)$$

$$A_1 \cdot F + A_2 \cdot Y + A_3 \cdot N \leq b_1 \quad (4)$$

Constraints (2) link the fact that no rolling stock unit is assigned to a train path to the variable specifying that the train path is not covered by rolling stock. In practice, these constraints must be rewritten to allow heterogeneous flows (i.e. different types of rolling stock units assigned to a same train path) but this is not detailed in this paper. Constraints (3) ensure that, if a train path is covered by rolling stock, than there are enough units of this rolling stock type assigned to the train path. Constraints (4), not detailed in this paper, are the usual constraints for defining consistent flows over the space-time graph that underlies this MIP model. The objective function aggregates production costs (first two terms) and penalties when a train path is not covered with a rolling stock unit.

2.3 Train driver planning subproblem

The train driver planning problem is complex and therefore classically divided into a crew pairing problem and a crew rostering problem, while respecting all working constraints. In this paper, we restrict our attention to the crew pairing problem which consists in finding an optimized set of legal shifts. For each depot, we assume that the number of shifts is constrained by the number of available drivers. Let $(MIP2)$ denote this crew-pairing MIP model. $(MIP2)$ is a set covering problem derived from legal labor constraints and the driver workload that needs to be covered.

Let us introduce the following basic decision variables:

- $\forall d \in \mathcal{D}, \forall j \in \{1, \ldots, H_d\}, \forall ec \in \mathcal{EC}$, $X_{j,ec,p} \in \{0, 1\}$ is equal to 1 if driver duty ec is assigned to shift j (of depot d) in the p^{th} position of this shift, and 0 otherwise; here, P is assumed to be the maximal length of a legal shift,
- $\forall ec \in \mathcal{EC}$, $ea_{ec} \in \{0, 1\}$ is equal to 1 if driver duty ec is not covered with a driver, and 0 otherwise.

$(MIP2)$ can be expressed as follows:

$$\min \quad Z(MIP2) = \sum_{d\in\mathcal{D}}\sum_{j=1}^{H_d}\sum_{ec\in\mathcal{EC}}\sum_{p=1}^{P} j \cdot X_{j,ec,p} + \sum_{ec\in\mathcal{EC}} PENDR_{ec} \cdot ea_{ec} \quad (5)$$

$$\forall ec \in \mathcal{EC} \qquad \sum_{d\in\mathcal{D}}\sum_{j=1}^{H_d}\sum_{p=1}^{P} X_{j,ec,p} + ea_{ec} = 1 \quad (6)$$

$$A_4 \cdot X \leq b_4 \quad (7)$$

Constraints (6) are the driver duty covering constraints. Constraints (7) are legal labor constraints that are not detailed in this paper. Such constraints typically refer to:

- The maximal duration of a shift; this duration is different if the shift comprises a night period work (typically between 11:00 PM and 4:00 AM),
- The sequence of duties within a shift; this sequence must obviously be feasible in time and space (including driver deadheadings),
- The depots where a train driver starts or ends its duty; when these depots are different, this means that the shift is associated with an over-night rest for the driver.

The objective function (5) aggregates production costs (see the first term which captures the number of driver shifts and the densities of these shifts, i.e. the number of duties within a shift) and penalties when a driver duty is not covered.

2.4 Coupling constraints

Coupling constraints are required to capture the complete covering of train paths and to ensure the consistency of the rolling stock unit(s) and driver assigned to each train path. In other words, when a driver and a rolling stock unit(s) are assigned to a train path, we must ensure that the driver is qualified to drive the unit on the train path.

$\forall s \in \mathcal{S}$, let us denote es_s the binary variable which is equal to 1 if train path s is not completely covered with a rolling stock unit(s) and a driver, and 0 otherwise. The three types of coupling constraints are formalized below.

- The train path covering constraints related to rolling stock:

$$\forall s \in \mathcal{S}, ee_s \leq es_s \tag{8}$$

- The train path covering constraints related to drivers, i.e. any driver duty $ec \in \mathcal{EC}$ which is not covered implies that the related train path is not covered:

$$\forall s \in \mathcal{S}, \forall ec \in \mathcal{EC}_s, ea_{ec} \leq es_s \tag{9}$$

- The consistency constraints between the rolling stock unit(s) and the driver assigned to each train path:

$$\forall k \in \mathcal{K}, \forall d \in \mathcal{D}, \forall s \in \mathcal{S}, \forall ec \in \mathcal{EC}_s, \quad \sum_{j=1}^{H_d} \sum_{p=1}^{P} X_{j,ec,p} + Y_{s,k} \leq 1 + COMP_{s,k,d} \tag{10}$$

2.5 MIP model for the integrated planning problem

The global MIP model for our integrated planning problem, denoted by $(MIPGlob)$, merges the variables and constraints of $(MIP1)$ and $(MIP2)$ with the coupling Constraints (8), (9) and (10). Note that variables es_s only appear in Constraints (8) and (9) and in the global objective function. The latter consists in minimizing the sum of rolling stock costs, train driver costs and penalties when a train path is not covered with both a rolling stock unit(s) and a driver, and is written below:

$$\min \quad Z(MIPGlob) = Z(MIP1) + Z(MIP2) + \sum_{s \in \mathcal{S}} PEN_s \cdot es_s \tag{11}$$

3 A Lagrangian relaxation heuristic

Given its complexity, $(MIPGlob)$ cannot be directly solved with a standard MIP solver for real-life instances. We therefore proposed a Lagrangian relaxation approach in [1]. Lagrangian relaxation can be used to obtain very good lower bounds for integer linear programs and to design effective heuristics [7]. The idea is to relax some of the constraints that "make the problem difficult", and roll them in the objective function with positive Lagrangian multipliers. To relax Constraints (8), (9) and (10), the following Lagrangian multipliers are introduced to penalize the non-satisfaction of the relaxed constraints in the objective function: u_s for Constraints (8), $v_{s,ec}$ for Constraints (9) and $w_{k,d,s,ec}$ for Constraints (10).

The Lagrangian function can then be written:

$$\begin{aligned} Lag(MIPGlob) = Z(MIP1) + Z(MIP2) + \sum_{s\in\mathcal{S}} PEN_s \cdot es_s \\ + \sum_{s\in\mathcal{S}} u_s \cdot (ee_s - es_s) + \sum_{s\in\mathcal{S}} \sum_{ec\in\mathcal{EC}_s} v_{s,ec} \cdot (ea_{ec} - es_s) \\ + \sum_{k\in\mathcal{K}} \sum_{d\in\mathcal{D}} \sum_{s\in\mathcal{S}} \sum_{ec\in\mathcal{EC}_s} w_{k,d,s,ec} \cdot (\sum_{j=1}^{H_d} \sum_{p=1}^{P} X_{j,ec,p} + Y_{s,k} - 1 - COMP_{s,k,d}) \end{aligned} \tag{12}$$

$Lag(MIPGlob)$ can be rewritten as follows:

$$\begin{aligned} Lag(MIPGlob) = \sum_{s\in\mathcal{S}} \sum_{k\in\mathcal{K}} CA_k \cdot F_{s,k} + \sum_{k\in\mathcal{K}} CE_k \cdot N_k \\ + \sum_{s\in\mathcal{S}} (PENRS_s + u_s) \cdot ee_s + \sum_{d\in\mathcal{D}} \sum_{j=1}^{H_d} \sum_{s\in\mathcal{S}} \sum_{ec\in\mathcal{EC}_s} \sum_{p=1}^{P} (j + \sum_{k\in\mathcal{K}} w_{k,d,s,ec}) \cdot X_{j,ec,p} \\ + \sum_{s\in\mathcal{S}} \sum_{ec\in\mathcal{EC}_s} (PENDR_{ec} + v_{s,ec}) \cdot ea_{ec} + \sum_{s\in\mathcal{S}} (PEN_s - u_s - \sum_{ec\in\mathcal{EC}_s} v_{s,ec}) \cdot es_s \\ + \sum_{k\in\mathcal{K}} \sum_{s\in\mathcal{S}} (\sum_{d\in\mathcal{D}} \sum_{ec\in\mathcal{EC}_s} w_{k,d,s,ec}) \cdot Y_{s,k} \\ - \sum_{k\in\mathcal{K}} \sum_{d\in\mathcal{D}} \sum_{s\in\mathcal{S}} \sum_{ec\in\mathcal{EC}_s} (1 + COMP_{s,k,d}) \cdot w_{k,d,s,ec} \end{aligned} \tag{13}$$

The resulting problem can be solved separately by solving three subproblems. The first subproblem is very close to the rolling stock planning subproblem modeled as $(MIP1)$, except that there is now a cost associated to each variable $Y_{s,k}$. The second subproblem is very close to the train driver planning subproblem modeled as $(MIP2)$, except that there is now a more complex cost associated to each variable $X_{j,ec,p}$. The third and last subproblem on variables es_s can be solved trivially.

Numerical experiments performed using a simplified model were promising, but showed that the model had to be improved for implementing a Lagrangian relaxation framework to solve real-life integrated planning problems arising at SNCF [1]. Since two independent optimization modules already existed at SNCF for planning rolling stock and train drivers, we wanted to re-use these modules in our Lagrangian heuristic. Challenges related to industrially implementing the heuristic are discussed in Section 4.

The remaining of this section provides details on the Lagrangian relaxation heuristic developed for our integrated planning model of rolling stock and train drivers. The selected relaxation scheme is described in Section 3.1 and the construction of feasible solutions is discussed in Section 3.2.

3.1 The relaxation scheme

The first challenge was to select the right relaxation scheme. Four possible relaxation schemes were investigated depending whether the coupling Constraints (8) and (9) are relaxed or not (Constraints (10) are always relaxed): (1) Both Constraints (8) and (9) are relaxed (leading to the expression of $Lag(MIPGlob)$ in 13), (2) Only Constraints (8) are relaxed, (3) Only Constraints (9) are relaxed, and (4) Both Constraints (8) and (9) are not relaxed (Lagrangian decomposition). The relaxation schemes were compared using a simplified MIP model and small instances created from real-life data. The numerical results, not detailed in this paper, showed that not relaxing the train path covering Constraints (8) for rolling stock or not relaxing the train path covering Constraints (9) for drivers provide better lower bounds and thus better guides the Lagrangian heuristic than when both constraints are relaxed. Lagrangian decomposition, where both Constraints (8) and (9) are not relaxed, is not really dominating.

The train path covering Constraints (8) for rolling stock, together with the cost $\sum_{s \in S} PEN_s \cdot es_s$ in the objective function, could rather easily be incorporated in subproblem $(MIP1)$ solved by the optimization engine of the rolling stock planning tool available at SNCF. This is why we decided to use the corresponding scheme in the industrial implementation. Subproblem $(MIP2)$ is also solved with a driver planning tool available at SNCF.

3.2 Constructing feasible solutions

Two feasible solutions can be constructed from the solutions of the relaxed subproblems. The first feasible solution is determined by fixing the rolling stock plan and, using the consistency Constraints (10), constraining the set of drivers that can be assigned to each train path before solving the train driver planning subproblem. The second feasible solution is determined by performing the opposite, i.e. fixing the train driver plan and, using the consistency Constraints (10), constraining the rolling stock types that can be assigned to each train path before solving the rolling stock planning subproblem.

In our industrial implementation, only the first feasible solution is actually determined. This is very natural from a practical standpoint, since rolling stock units are more expensive than drivers, and are thus naturally prioritized.

4 Industrial implementation

The industrial implementation of the Lagrangian relaxation scheme previously described relies on an iterative process where a master module pilots the two proprietary software modules available at SNCF. The Lagrangian dual is solved by calling these two modules with cost parameters that are updated at each iteration according to the Lagrangian multipliers. Relaxed solutions are then periodically made feasible and upper bounds are computed. As already mentioned, in our industrial context, only one feasible solution is determined by computing a feasible train driver schedule when the decision variables of the rolling stock subproblem are fixed to their values in the relaxed solution (i.e., obtained when computing the Lagrangian lower bound). Since our model allows for a partial covering of the train paths (however with a high penalty), the convergence of this heuristic towards a feasible solution is therefore guaranteed.

Note that the implementation of our Lagrangian heuristic inherits the following properties:

- The feasible solution determined in iteration 1 corresponds to a sequential use of the existing rolling stock and train driver software modules,
- By construction, the best feasible solution found in the course of the iterations necessarily improves the feasible solution found at iteration 1.

When implementing our approach, several quality/cost trade-offs were studied with regard to the theoretical solving process. Indeed, the two existing proprietary software modules had not been designed for a coordinated use, as suggested by the theoretical process. When theory and implementation diverged, we had to choose if it was worth investing in modifying the mathematical models exploited by the two existing modules; for example, adding complementary terms in the objective functions, or modifying existing constraints. This analysis was supported by an identification of the theoretical features that were not natively supported in the modules; for example, the consistency of the dual multipliers with the objective functions in the modules. In practice, the return on investment (when comparing the development costs to the benefits for the quality of solutions) of all modifications was significant and we implemented all the necessary adjustements. The remaining of this section presents some modifications that were undertaken.

4.1 Modifications of the industrial rolling stock planning module

A simplified version of the objective function of the industrial rolling stock planning module can be formulated as follows:

$$\min Z(MIP1Indus) = \sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} CA_k \cdot F_{s,k} + \sum_{k\in\mathcal{K}} CE_k \cdot N_k + C_{RS}(F') + \sum_{s\in\mathcal{S}} PENRS_s \cdot ee_s \tag{14}$$

where

- F' is an extension of F that included rolling stock deadheading flows;
- $C_{RS}(F')$ is an additional cost that captures deadheading costs; in particular, when train paths corresponding to deadheadings are chosen in order to optimize the computed rolling stock circulations.

In order to implement our Lagrangian heuristic, it was therefore necessary to modify the existing rolling stock planning module. In particular, the element $\sum_{k\in\mathcal{K}}\sum_{s\in\mathcal{S}}(\sum_{d\in\mathcal{D}} \sum_{ec\in\mathcal{EC}_s} w_{k,d,s,ec})\cdot Y_{s,k}$ in the Lagrangian function (13) had to be taken into account when solving the rolling stock planning subproblem. From an implementation point of view, this consisted in:

- Adding $\sum_{s \in \mathcal{S}} \sum_{k \in \mathcal{K}} CA'_{s,k} \cdot Y_{s,k}$ to the existing objective function (14), where $CA'_{s,k}$ denotes an additional cost computed from the values of the dual multipliers $w_{k,d,s,ec}$ at each iteration. This cost is related to the attractiveness of type k rolling stock for covering train path s.

- Adding cost coefficients $CA'_{s,k}$ in the data model of the existing planning module.

Moreover, because the coupling Constraints (8) are not relaxed in the selected relaxation scheme (see Section 3.1), it was also necessary to consider these coupling constraints and the Lagrangian cost $(PEN_s - \sum_{ec \in \mathcal{EC}_s} v_{s,ec})$ for each variable es_s (see Lagrangian function (13) without Lagrangian multipliers u_s since Constraints (8) are not relaxed).

4.2 Modifications of the industrial train driver planning module

The objective function of the industrial train driver planning module is slightly different from the expression shown in (5). Indeed, the associated mathematical model is a column-based formulation where decision variables are directly associated to the choices of complete train driver shifts that are built (enumerated) beforehand (see also [2]). Moreover, on top of (i) classical production costs and (ii) penalties when driver duties are not covered, it also comprises penalties related to the production level of each depot. These penalties aim at smoothing the workload across the depots. A simplified version of the objective function of the industrial train driver planning module can be formulated as follows:

$$\begin{aligned} \min Z(MIP2Indus) = \quad & \sum_{d \in \mathcal{D}} \sum_{j=1}^{H_d} CostShift_j \cdot XShift_j + C_{TD}(X) \\ & + PENDR \cdot \sum_{ec \in \mathcal{EC}} ea_{ec} \end{aligned} \tag{15}$$

where

- The decision variable $XShift_j \in \{0,1\}$ is equal to 1 if a shift j, $j \in \{1,\ldots,H_d\}$, is chosen (with cost $CostShift_j$) in depot $d \in \mathcal{D}$, and 0 otherwise,

- $C_{TD}(X)$ is an additional cost that captures the smoothing of workload among the driver depots,

- $PENDR$ is a fixed penalty applied when a train path is not covered by a driver.

As mentionned previously, the term $\sum_{d \in \mathcal{D}} \sum_{j=1}^{H_d} \sum_{ec \in \mathcal{EC}} \sum_{p=1}^{P} j \cdot X_{j,ec,p}$ in (5) evaluates the number of shifts and the densities of these shifts in the train driver plan (this can be readily seen since this term can be rewritten as $\sum_{d \in \mathcal{D}} \sum_{j=1}^{H_d} j \cdot (\sum_{ec \in \mathcal{EC}} \sum_{p=1}^{P} X_{j,ec,p})$). It can therefore be identified to the term $\sum_{d \in \mathcal{D}} \sum_{j=1}^{H_d} CostShift_j \cdot XShift_j$ in (15). However, $CostShift_j$ allows more general shift costs to be modeled, for example by considering deadheadings and night working periods.

In order to implement our Lagrangian heuristic, it was therefore necessary to proceed with the following modifications in the industrial train driver planning module:

- Enrich the data model so that:

- The cost of a duty j also depends on the depot d where the duty j is assigned and is written as $CostShift_{d,j}$,
- The penalty for not covering a train duty ec by a driver depends on ec and is written as $PENDR_{ec}$,

- Modify consequently the existing objective function (15) so that:
  - The modified cost coefficient $CostShift_{d,j}$ replaces $CostShift_j$. As indicated in Section 3, the cost coefficient $CostShift_{d,j}$ is updated from the Lagrangian multipliers $w_{k,d,s,ec}$ at each iteration,
  - The element $PENDR \cdot \sum_{ec \in \mathcal{EC}} ea_{ec}$ becomes $\sum_{ec \in \mathcal{EC}} PENDR_{ec} \cdot ea_{ec}$. As indicated in Section 3, the cost coefficient $PENDR_{ec}$ is updated from the Lagrangian multipliers $v_{s,ec}$ at each iteration.

5 Preliminary computational experiments

5.1 A real-life instance

We tested our approach on several real-life instances extracted from the transportation plan of a French region (Bretagne). This paper focuses on one of these instances; Table 1 summarizes its main characteristics.

Table 1: Characteristics of the industrial instance

Characteristics	**Instance**
Time horizon	1 week
Number of train paths ($\lvert\mathcal{S}\rvert$)	416
Number of rolling stock types ($\lvert\mathcal{K}\rvert$)	7
Number of rolling stock units ($\sum_{k\in\mathcal{K}} CAP_k$)	73
Number of driver depots ($\lvert\mathcal{D}\rvert$)	7
Number of drivers ($\sum_{d\in\mathcal{D}} H_d$)	81

Computational experiments were performed on a regular PC, with 3.42GB of RAM and using IBM ILOG CPLEX 12.2 for solving the MIP models. Runs were arbitrarily limited to 50 iterations. We observed that the average CPU time per iteration was 2 minutes.

5.2 Numerical results

As indicated earlier, note that the first iteration corresponds to a sequential use of our two software planning modules. The automated connection of these modules within our prototype is therefore a first added value of this work. Moreover, the feasible solution can only be improved as more iterations are performed, which is the main contribution of this work.

Figure 2 shows the evolution over the iterations of the Lagrangian lower bound and upper bound. The upper bound corresponds to the cost of the feasible solution of the Lagrangian heuristic. According to this figure, the Lagrangian process converges as expected because i) both the Lagrangian lower bound and upper bound decrease over the iterations, and ii) the upper bound is always larger than the lower bound at any given iteration. Furthermore, we can see that there is no strict monotony in the evolution of the upper bound and lower bound in the course of the iterations. Indeed, this property cannot be satisfied since the MIP sub-models are often not solved to optimality when computing the lower and upper bounds at each iteration in our industrial context.

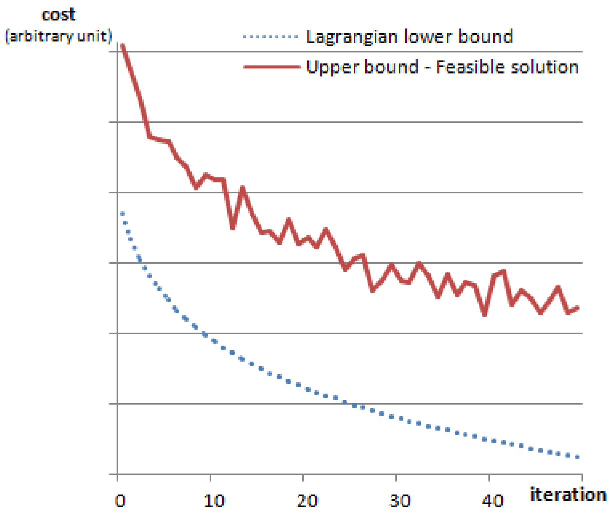

Figure 2: Results of the Lagrangian heuristic

Table 2 provides complementary insights on the objective function and on the improvements that are achieved in the course of the iterations. Four representative criteria of the objective function are listed and sorted by order of importance. The first one is the number of uncovered train paths (because no rolling stock or driver could be allocated); it cannot be compensated by any other criterion. The next two criteria are rolling stock costs and are in practice much larger than driver costs. Only the fourth criterion corresponds to train driver costs. The optimization of the objective function therefore leads to cover the maximum number of train paths while minimizing (then) i) the rolling stock costs and ii) the train driver costs. Compared to the first iteration, the solution of the best iteration covers 3 additional train paths (main non-compensatory objective) with the same number of rolling stock units and 79 fewer deadhead kilometers, but requires 3 more driver shifts.

Table 2: Numerical results for the industrial instance

Criteria to minimize (in order of importance)	**Iteration 1**	**Best iteration (40/50)**
Number of uncovered train paths	25/416	22/416
Number of rolling stock units	22	22
Rolling stock deadhead kilometers	1482	1403
Number of shifts of train driver plan	200	203

6 Conclusions

Based on a mixed integer linear programming formulation for the integrated planning problem of railway production resources, a Lagrangian heuristic was proposed. Characteristics of this heuristic were discussed, in particular related to its industrial implementation. The first numerical experiments on real-life instances are promising. Our current research includes the integration of additional levels of integration, such as the possibility of slightly changing the times of train paths.

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