

# A LAGRANGIAN HEURISTIC FOR A REAL-LIFE INTEGRATED PLANNING PROBLEM OF RAILWAY TRANSPORTATION RESOURCES

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# AGENDA

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- **Industrial Context**
  - Railway Production
  - Integrated planning problem: An example
  - Objectives
  - Literature Review
- **Modeling Problem**
  - Mathematical Model
- **A Lagrangian relaxation heuristic**
  - Presentation
  - General scheme
  - Relaxation schemes
  - Industrial implementation
  - Preliminary computational results
- **Conclusions and Perspectives**

# INDUSTRIAL CONTEXT

# INDUSTRIAL CONTEXT

## RAILWAY PRODUCTION

### Resources to Schedule



### Current Process

Forecast demand + commercial offers

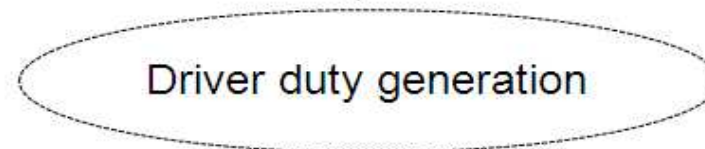


**Planning of railway Timetables**

↓ Train paths



**Rolling stock (locomotive) planning**

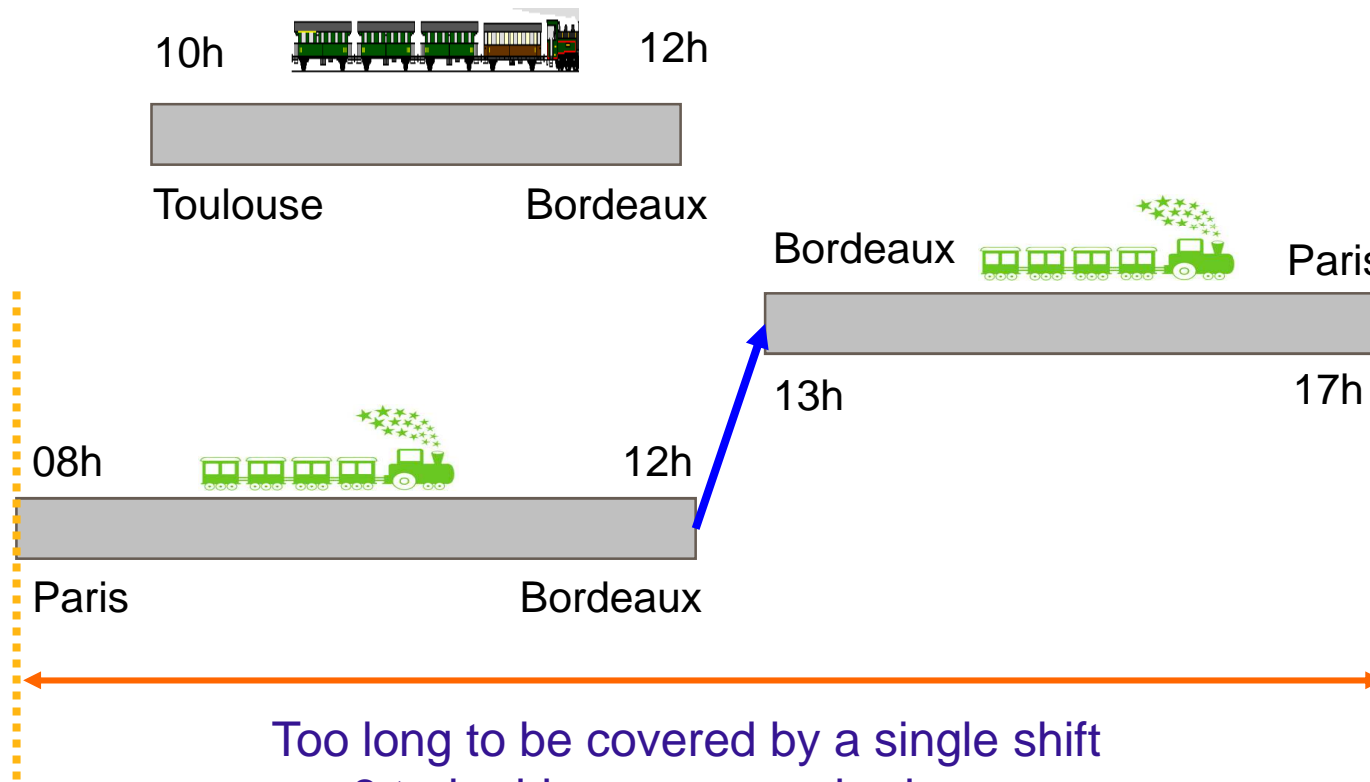


**Planning of train Drivers**

→ **Set of working days**

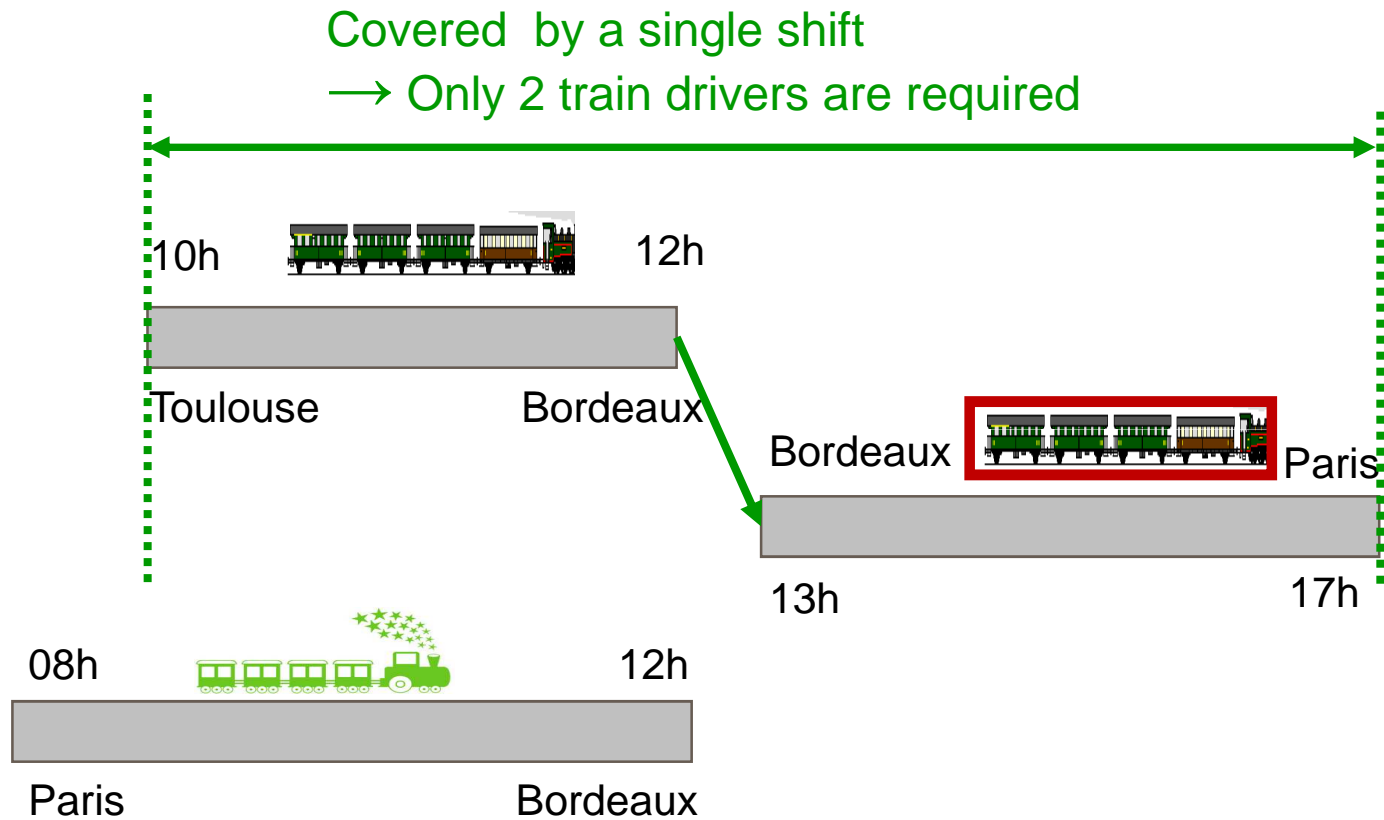
# INDUSTRIAL CONTEXT

## INTEGRATED PLANNING PROBLEM: AN EXAMPLE



# INDUSTRIAL CONTEXT

## INTEGRATED PLANNING PROBLEM: AN EXAMPLE



# INDUSTRIAL CONTEXT

## OBJECTIVES

- **Study the feasibility of an integrated planning process**
  - Definition of the planning problem(s)
  - Models / Algorithms / Performances
  - Software prototyping
- **In order to increase**
  - Competitiveness
    - Better use of resources
    - Cost minimization
  - Quality of service
    - Consistent transportation plans



# INDUSTRIAL CONTEXT

## LITERATURE REVIEW

### Airline

- **J-F. Cordeau, G. Stojkovic, F. Soumis, J. Desrosiers.** Benders decomposition for simultaneous aircraft routing and crew scheduling. *Transportation Science*, 35: 375-388, 2001.
- **M. Haouari, K.H. Bae.** Integrated airline schedule design and fleet assignment: Polyhedral analysis and Benders' decomposition approach. *European Journal of Operational Research*, 39(4): 491-502, 2010.
- **M. Lohatepanont, C. Barnhart.** Airline schedule planning: Integrated models and algorithms for schedule design and fleet assignment. *Transportation Science*, 38(1): 19-32, 2004.
- **A. Mercier, F. Soumis.** An integrated aircraft routing, crew scheduling and flight retiming model. *Computers & Operations Research*, 34: 2251-2265, 2005.
- **N. Papadakos.** Integrated airline scheduling: decomposition and acceleration techniques, 2006.
- **N. Papadakos.** Integrated airline scheduling. *Computers & Operations Research*, 36: 176-195, 2009.
- **R. Sandhu, D. Klabjan.** Integrated airline fleet and crew-pairing decisions. *Operations Research*, 55(3): 439-456, 2007.

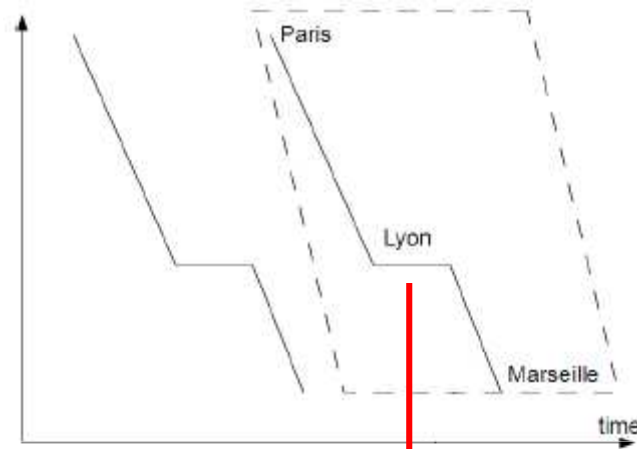
### Public transport

- **D. Huisman, R. Freling, A. P. M. Wagelmans.** Multiple-depot integrated vehicle and crew scheduling. *Transportation Science*, 39(4): 491-502, 2005.

# MODELING PROBLEM

# MODELING PROBLEM

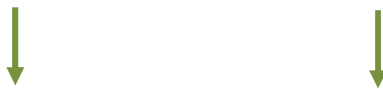
## MATHEMATICAL MODEL



**train path s**



**Rolling stock model**



**Coupling constraints**

**Driver duty generation**



**Train driver model**

# MODELING PROBLEM

## MATHEMATICAL MODEL

- Rolling stock planning subproblem
  - Constraints
    - Covering constraints of a train-path by rolling stock *only*
    - Capacity constraints...
  - Objectives
    - Production costs (number of units, total distance...)
    - Penalties when train paths are not covered by rolling stock units
- Train driver planning subproblem (*crew pairing problem*)
  - Constraints
    - Driver duty covering constraints
    - Capacity constraints
    - Legal labor constraints...
  - Objectives
    - Production costs (number of drivers, density of the shifts)
    - Penalties when driver duties are not covered by drivers

# MODELING PROBLEM

## MATHEMATICAL MODEL

- **Coupling constraints**
  - Train-path covering constraints related to rolling stock
  - Train-path covering constraints related to drivers (*i.e. if any driver duty of a train-path is not covered, the train-path is not covered*)
  - Consistency constraints between rolling stock unit(s) and the driver assigned to each train-path
- **An additional term in the global objective function**
  - Penalties when train-paths are not covered with both rolling stock unit(s) and a driver

# A LAGRANGIAN RELAXATION HEURISTIC

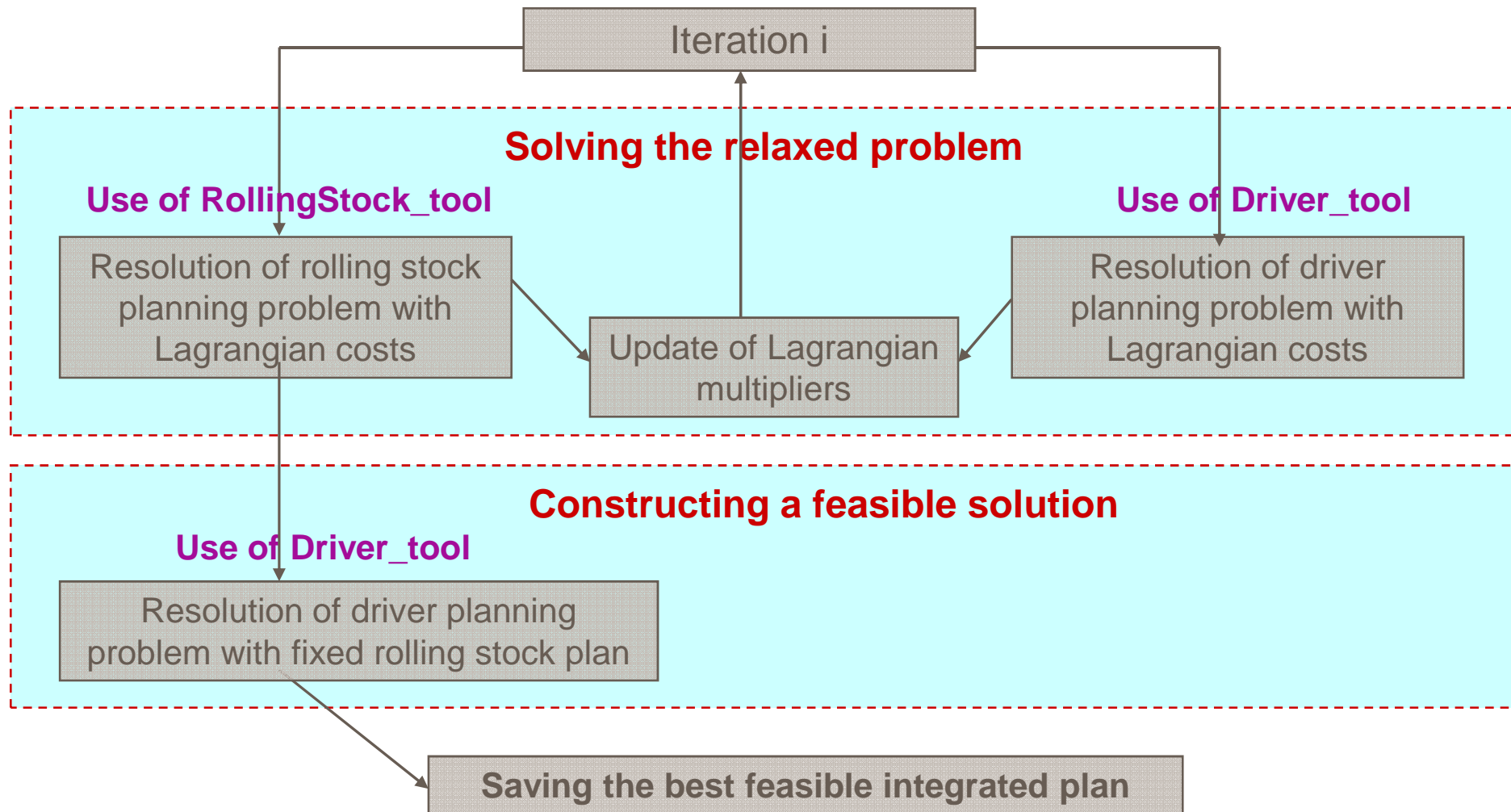
# A LAGRANGIAN RELAXATION HEURISTIC

## PRESENTATION

- New approach to solve the integrated planning model: Lagrangian Relaxation
  - Relaxation of the coupling constraints
    - **Different possible relaxation schemes**
  - Roll these constraints in the objective function with multipliers
  - **Use of dedicated industrial tools** to solve the relaxed rolling stock planning problem and driver planning problem
  - **Lagrangian heuristic** by using industrial tools to determine a feasible integrated rolling stock and driver plan

# A LAGRANGIAN RELAXATION HEURISTIC

## GENERAL SCHEME





# A LAGRANGIAN RELAXATION HEURISTIC

## RELAXATION SCHEMES

Relaxation schemes	Coupling constraints		
	<i>Covering constraints (rolling stock)</i>	<i>Covering constraints (drivers)</i>	<i>Consistency constraints</i>
<i>Total relaxation</i>	Relaxed	Relaxed	Relaxed
<i>Partial driver relaxation</i>	<b>Not relaxed</b>	Relaxed	Relaxed
<i>Partial rolling stock relaxation</i>	Relaxed	<b>Not relaxed</b>	Relaxed
<i>Lagrangian decomposition</i>	<b>Not relaxed</b>	<b>Not relaxed</b>	Relaxed

# A LAGRANGIAN RELAXATION HEURISTIC

## RELAXATION SCHEMES

- **Experimental validation of the 4 relaxations schemes** on 4 selected small instances based on industrial data.
  - First prototype software developed with IBM ILOG OPL Studio and CPLEX for a simplified MIP model

	Instance 1	Instance 2	Instance 3	Instance 4
Rolling stock	40	30	20	10
Drivers	20	10	10	20
Train paths	32	32	32	32
Driver duties	32	32	32	32
Rolling stock types	12	12	12	12
Driver types	6	6	6	6

# A LAGRANGIAN RELAXATION HEURISTIC

## RELAXATION SCHEMES

The scheme for the industrial implementation was selected to ensure a trade-off between performance and complexity of implementation.

Relaxation schemes	(UB-LB)/UB			
	Instance 1	Instance 2	Instance 3	Instance 4
<i><b>Total relaxation</b></i>	0.05%	1.40%	3.15%	10.03%
<i><b>Partial driver relaxation</b></i>	0.00%	0.21%	0.23%	0.01%
<i><b>Partial rolling stock relaxation</b></i>	0.01%	0.09%	0.20%	8.51%
<i><b>Lagrangian decomposition</b></i>	0.00%	0.85%	0.03%	0.08%

# A LAGRANGIAN RELAXATION HEURISTIC

## INDUSTRIAL IMPLEMENTATION

Properties of our Lagrangian heuristic:

- The feasible solution determined in the first iteration corresponds to a sequential use of the two existing rolling stock and train driver software,
- By construction, the best feasible solution found by the iterative heuristic necessarily improves the feasible solution found at the first iteration.

**The two existing proprietary software had not been designed for a coordinated use:**

- Additional terms in the objective functions were required,
- Existing constraints had to be modified.

The return on investment (development costs vs. quality of solutions) of all modifications was significant and **all the necessary adjustments were implemented.**

# MODELING PROBLEM

## INDUSTRIAL IMPLEMENTATION

- **Modifications of the industrial **rolling stock** planning module**
  - Add an additional term in the objective function, computed from the values of the dual multipliers corresponding to the relaxation of the consistency constraints
- **Modifications of the industrial **train driver** planning module**
  - Specify some costs and penalties
    - The cost of a shift also depends on the depot (*not only on its density*)
    - The penalty for not covering a train duty is specific (*not a constant*)
  - Update two terms in the objective function, according to the values of the dual multipliers corresponding to the relaxation of the covering constraints by drivers) and the consistency constraints

# A LAGRANGIAN RELAXATION HEURISTIC

## PRELIMINARY COMPUTATIONAL RESULTS

**A real-life instance is used** (*the transportation plan of a French region*).

Because data are not generated, collecting industrial data was time-consuming and tricky, in particular the parameters that connect rolling stock and drivers.

Different scenarios (different costs) were created from the real-life instance.

Characteristics	Instance
Time horizon	1 week
Train paths	416
Rolling stock types	7
Rolling stock units	73
Driver depots	7
Drivers	81

# A LAGRANGIAN RELAXATION HEURISTIC

## PRELIMINARY COMPUTATIONAL RESULTS

Results are promising and significant gains are obtained.

Results for one scenario	Iteration 1	Iteration 23	Iteration 24	Iteration 53
Rolling stock units	23	23	24	24
Driver shifts	208	217	218	228
Uncovered train paths	25	13	7	2

- The industrial implementation has to be finalized, in particular :
- The choice of the initial Lagrangian multipliers
  - The choice of the initial costs and penalties of the objective functions of the two modules, in order to reach a *good* trade-off between rolling stock and drivers costs

# CONCLUSIONS AND PERSPECTIVES



# CONCLUSIONS AND PERSPECTIVES

- **Conclusions**
  - Development of a **Lagrangian relaxation heuristic** for an integrated planning problem of railway transportation resources
  - **Industrial implementation** of the Lagrangian heuristic, with different modifications.
  - **First numerical experiments on industrial instances** are promising. Several additional industrial instances from other French regions are being created.
- **Perspectives**
  - **Tuning** the Lagrangian relaxation approach
  - Including the integration of **additional levels of integration** ; e.g. the possibility of slightly changing the times of train paths

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