Integration of additional purchase cost to reduce the lead time uncertainty for one level assembly system
Faicel Hnaien, Hasan Murat Afsar, Alexandre Dolgui

To cite this version:
Faicel Hnaien, Hasan Murat Afsar, Alexandre Dolgui. Integration of additional purchase cost to reduce the lead time uncertainty for one level assembly system. 7th IFAC Conference on Manufacturing Modelling, Management, and Control, 2013, Jun 2013, St Petersburg, Russia. pp. 383-388, 10.3182/20130619-3-RU-3018.00561. emse-00881563

HAL Id: emse-00881563
https://hal-emse.ccsd.cnrs.fr/emse-00881563
Submitted on 22 Mar 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Integration of additional purchase cost to reduce the lead time uncertainty for one level assembly system

F. Hnaien* H.M. Afsar* A. Dolgui**

*ICD-LOSI, University of Technology of Troyes, UMR STMR-CNRS. 12 rue Marie Curie, 10010 Troyes, France
**Ecole Nationale Superieure des Mines, FAYOL-EMSE, CNRS UMR 6158 LIMOS, 158, cours Fauriel, 42023 Saint-Étienne Cedex 2, France

Abstract: We consider the supply planning for one level assembly system under uncertainty of lead times. The finished product need several components for the assembly operation. The lead times can be uncertain for different reasons such as machine breakdowns, transport delay, strike, etc. In literature, this type of problem was studied in the case of random lead times that follow continuous distributions as well as in the case of any discrete laws with known distribution. We consider the case of discrete distributions to be closer to the industrial reality because often in real life situations the lead time is expressed as the number of periods. Typically, this is an inventory control problem where the objective is to minimize the component holding and backlog costs for the finished product due to the uncertainty of lead times. Indeed, as the finished product is assembled by using several types of components, the assembly is stopped even if a single type of component is delayed. The other components are stored between their arrival and the arrival of the latest component. To reduce stock and backlog costs, we propose to pay the supplier more, if he/she agrees to decrease his lead time uncertainty. A model describing this choice is suggested.

Keywords: Assembly systems, Random lead time, Replenishment, Pricing, Purchase cost.

1. INTRODUCTION AND RELATED WORKS

The additional difficulty of the assembly systems is the dependence among the necessary components for the assembly operation (Dolgui and Proth [2010]). In the following, the assembly systems with stochastic lead times are reviewed. Liberatore [1979] tried to extend directly the EOQ model so as to treat stochastic lead time, but no closed-form solution was given for optimal order size. Yano [1987] considered the problem of two-level assembly systems with only two types of components at level 2 and one type of components at level 1. The lead times of the three components are stochastic Poisson and negative binomial distribution. The problem was to find planned lead times minimize the sum of inventory holding costs and tardiness costs. An algorithm was developed which exploits properties of the objective function to find optimal solutions. Computational results indicate that optimal solutions often have negative safety times, there are two situations. First, where the component holding and storage cost are both relatively high. Second case, the situation in which one component lead time is much longer than the other. Negative safety times do not have any practical signification. But, in our opinion since that safety time is the difference between planned and expected lead times, then this difference can be negative if the planned lead time is shorter than the expected lead time. Evidently, this result depends also in the values of holding and tardiness cost. Kumar [1987] presented a generic study of inventory control in an assembly system of several different components where the component procurement lead times are stochastic and the assembly date and quantity are fixed. The problem consists in determining the timing of each component’s order so that the total cost, composed of the component holding and the tardiness of the assembly costs is minimized. Many of her results are based on exact analysis which is only possible to carry out for special types of distributions (exponential, uniform, and normal).

Another interesting single-period model of this type was developed in Chu et al. [1993]. Their model deals with a fixed demand for one finished product. To assembly this product several types of components are needed. The lead times of components are random variables. It is necessary to determine the order date for each type of component. The criterion considered is the mathematical expectation of the sum of the holding cost for the components and the backlogging cost for the finished product. The authors prove the convexity of the expected average cost and propose an iterative algorithm to minimize it.

In manufacturing systems, the demand is periodic, therefore this inventory problem is solved at each period and the stocks of the previous periods can be used for the next and so on. However, the mathematical formulation of multi-period problems under lead time uncertainty is more difficult. Orders may cross, for that reason they may not be received in the same sequence in which they were placed. In certain publications it is assumed that orders
do not crossover, and so a single-period problem is solved.

A multi-period model was proposed in Gurnani et al. [1996] for assembly systems with two types of components and the lead time probability distributions are limited to two periods. In this model, authors supposed that components either arrive in the current period with a given probability ($\alpha$) or in the next period with probability $(1 - \alpha)$. This two period lead time model gives the optimal quantity of each component to order from each supplier.


A state of the art on production planning models under uncertainties is presented by Dolgui and Prodhon [2007].

### 2. PROBLEM DESCRIPTION

We consider a one-level assembly system with $n$ different components from $n$ different suppliers. The uncertainty of lead times causes an important level of inventory of the components and a delay of the finished product. The assembly system capacity is supposed infinite. The demand of finished product is known and we consider one period. The lead times of components type $i = 1, \ldots, n$ are independent random variables. This problem is formulated by Louly and Dolgui [2012], Louly and Dolgui [2011] for one-level multi-period case and by Hnaien et al. [2010] for the multi-level one period discrete random lead times. The following notations are used in this paper:

<table>
<thead>
<tr>
<th>Table 1. Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameters</strong></td>
</tr>
<tr>
<td>$i$</td>
</tr>
<tr>
<td>$j$</td>
</tr>
<tr>
<td>$h_i$</td>
</tr>
<tr>
<td>$H = b + \sum_{i=1}^{n} h_i$</td>
</tr>
<tr>
<td>$L_i^j$</td>
</tr>
<tr>
<td>$u_i^j$</td>
</tr>
<tr>
<td>$w_i^j$</td>
</tr>
<tr>
<td>$i$</td>
</tr>
<tr>
<td>$j$</td>
</tr>
<tr>
<td>$\sum_{i=1}^{n}$</td>
</tr>
<tr>
<td>$\leq$</td>
</tr>
<tr>
<td>$\forall i$</td>
</tr>
<tr>
<td>$\forall j$</td>
</tr>
<tr>
<td>$\in \mathbb{N}$</td>
</tr>
<tr>
<td>$\in (0,1)$</td>
</tr>
</tbody>
</table>

The optimization model without taking into account prices consists in minimizing the average cost composed of the sum of holding cost of the components and the backlogging cost of the finished product.

$$
\min \text{EC}(X) = \sum_{i=1}^{n} h_i (x_i - E(L_i)) + H \sum_{k=0}^{\infty} \left( 1 - \prod_{i=1}^{n} F_i(x_i + k) \right)
$$

s.t.

$$
1 \leq x_i \leq u_i \quad \forall i = 1, \ldots, n
$$

$$
x_i \in \mathbb{N} \quad \forall i = 1, \ldots, n
$$

where:

$$
X = (x_1, x_2, \ldots, x_1, \ldots, x_n)
$$

To solve this non-linear problem we can use the B&B proposed by Louly et al. [2008] for the multi-period case. In this paper we treat the same problem differently. Our idea is to propose to modify purchasing cost depending on the risk of not respecting the due date. The new objective function is presented in the equation (4).
The model is expressed as follows:

$$
\begin{align*}
\min EC(X, Y) &= \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} APC_i^j \cdot y_i^j \\
+ \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} y_i^j \cdot h_i \left( x_i^j - E(L_i^j) \right) \\
+ H \cdot \sum_{k \geq 0} \left( 1 - \prod_{i=1}^{n} \left( \sum_{j=0}^{u_i^0-1} y_i^j \cdot F_i^j (x_i^j + k) \right) \right) \\
\text{s.t.} \\
\sum_{j=0}^{u_i^0-1} y_i^j &= 1 \quad \forall i = 1, ..., n \quad (5) \\
x_i^j &\leq (u_i^0 - j) \cdot y_i^j \quad \forall i = 1, ..., n, \forall j = 0, ..., u_i^0 - 1 \quad (6) \\
y_i^j &\in \{0, 1\} \quad \forall i = 1, ..., n, \forall j = 0, ..., u_i^0 - 1 \quad (7) \\
\end{align*}
$$

where for a given supplier \((i = 1, ..., n)\) we have the following relation for two purchasing policies \(j\) and \(j + 1\) \((j = 0, ..., u_i^0 - 1)\):

$$
\begin{align*}
p_{i}^{j+1}(k) &= \begin{cases} 
p_{i}^{j}(k) & \text{if } k < u_i^j - 1 \\
p_{i}^{j}(k) + p_{i}^{j}(u_i^j) & \text{if } k = u_i^j - 1 
\end{cases} \\
\end{align*}
$$

Or

$$
\begin{align*}
F_{i}^{j+1}(k) &= \begin{cases} 
F_{i}^{j}(k) & \text{if } k < u_i^j - 1 \\
F_{i}^{j}(k) + F_{i}^{j}(u_i^j) & \text{if } k = u_i^j - 1 
\end{cases} \\
\end{align*}
$$

The objective function \((4)\) represents the total cost composed of additional purchasing cost noted \(APC\), the holding component costs \((HC)\) and the backlogging cost \((BC)\):

$$
\begin{align*}
APC &= \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} APC_i^j \cdot y_i^j \\
HC &= \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} \left( y_i^j \cdot h_i \left( x_i^j - E(L_i^j) \right) \right) \\
&\quad + \sum_{i=1}^{n} \left( h_i \right) \cdot \sum_{k \geq 0} \left( 1 - \prod_{i=1}^{n} \left( \sum_{j=0}^{u_i^0-1} y_i^j \cdot F_i^j (x_i^j + k) \right) \right) \\
BC &= b \cdot \sum_{k \geq 0} \left( 1 - \prod_{i=1}^{n} \left( \sum_{j=0}^{u_i^0-1} y_i^j \cdot F_i^j (x_i^j + k) \right) \right)
\end{align*}
$$

The total cost \(EC(X, Y) = APC(Y) + HC(X, Y) + BC(X, Y)\).

The constraint \((5)\) expresses that only one purchasing policy \((j = 0, ..., u_i^0 - 1)\) is assigned for each supplier \((i = 1, ..., n)\). The constraint \((6)\) expresses that each planned lead time \(x_i^j\) is limited by the upper bound for each lead time, the upper bound for the purchasing policy \((j=0)\) is equal to \(u_i^0\).

The problem is to minimize the model given by \((4)-(8)\). This minimization is rather difficult because the function is not linear and the decision variables \(X = (x_i^j; i = 1, ..., n; j = 0, ..., u_i^0 - 1)\) are integer and \(Y = (y_i^j; i = 1, ..., n; j = 0, ..., u_i^0 - 1)\) are binary. The general optimization problem is difficult, in the following section the problem is solved under some assumptions.

### 3. Optimization Method

The problem is solved under the assumption that holding cost and additional purchase cost per period are the same, and the lead times \(L_i^j\) of the different components have the same distribution probability. Then, the costs \(h_i, i = 1, ..., n\), can be replaced by \(h\), the distribution \(F_i^j\) can be noted by \(F^j\) and the lead time \(L_i^j\) by \(L\). The model given by \((4)-(8)\) can be rewritten as follows:

$$
\begin{align*}
\min EC(Z) &= \sum_{j=0}^{u_0^0-1} \sum_{s=1}^{u_0} z_j^s \cdot APC^j \cdot n \\
+ \sum_{j=0}^{u_0^0-1} \sum_{s=1}^{u_0} z_j^s \cdot s \cdot n \cdot h \\
- \sum_{j=0}^{u_0^0-1} \sum_{s=1}^{u_0} n \cdot h \cdot z_j^s \cdot E(L^j) \\
+ H \sum_{k \geq 0} \left( 1 - \left( \sum_{j=0}^{u_0^0-1} \sum_{s=1}^{u_0} z_j^s \cdot F^j(s + k) \right)^n \right) \\
\text{s.t.} \\
\sum_{s=1}^{u_0} \sum_{j=0}^{u_0^0-1} z_j^s &= 1 \quad (10)
\end{align*}
$$

Under the assumptions that all components have the same characteristics the objective function \((4)\) is rewritten as follows:

$$
\begin{align*}
EC(X, Y) &= \sum_{j=0}^{u_0^0-1} \sum_{s=1}^{u_0} y_j^s \cdot APC^j \cdot y_j^s \\
+ H \sum_{k \geq 0} \left( 1 - \left( \sum_{j=0}^{u_0^0-1} \sum_{s=1}^{u_0} y_j^s \cdot F^j(x_j^s + k) \right)^n \right) \\
\text{We introduce the binary variable } z_j^k, \forall j = 0, ..., u_0^0 - 1: \\
&z_j^k = \begin{cases} 
1 & \text{if } x_j^s = k \\
0 & \text{otherwise} 
\end{cases} \\
\text{Thus we can rewrite } x_j^s \text{ and } y_j^s \text{ as follows:} \\
x_j^s &= \sum_{s=1}^{u_0} z_j^s \cdot s \quad \forall j = 0, ..., u_0^0 - 1 \\
y_j^s &= \sum_{s=1}^{u_0} z_j^s \quad \forall j = 0, ..., u_0^0 - 1
\end{align*}
$$

Where:

$$
\sum_{s=1}^{u_0} \sum_{j=0}^{u_0^0-1} z_j^s = 1
$$
Finally, the objective function can be rewritten as follows:

\[
\min EC(Z) = \sum_{j=0}^{u^0} \sum_{s=1}^{u_0} z_j^s \cdot APC^j \cdot n + \sum_{j=0}^{u^0} z_j^s \cdot s \cdot n \cdot h - \sum_{j=0}^{u^0} z_j^s \cdot E(L^j) + H \sum_{k=0}^{u_0-1} \left(1 - \left(\sum_{j=0}^{u^0} \sum_{s=1}^{u_0} z_j^s (F^j(s+k))^n\right)\right)
\]

s.t.

\[
\sum_{s=1}^{u_0} \sum_{j=0}^{u^0} z_j^s = 1
\]

3.1 Partial increments of cost functions

We will use the following partial increment functions (Louly and Dolgui [2012]):

\[
G_j^+(Z) = EC(z_j^1, ..., z_j^{s+1}, ..., z_j^{u_0+1-1}) - EC(z_j^0, ..., z_j^s, ..., z_j^{u_0+1-1}), \quad \forall j = 0, ..., u^0 - 1
\]

\[
G_j^-(Z) = EC(z_j^0, ..., z_j^s, ..., z_j^{u_0+1-1}), \quad \forall j = 0, ..., u^0 - 1
\]

\[
G_s^+(Z) = EC(z_1^1, ..., z_{s+1}, ..., z_{u_0}^{u_0-1}), \quad \forall s = 1, ..., u^0 - 1
\]

\[
G_s^-(Z) = EC(z_1^1, ..., z_{s-1}, ..., z_{u_0}^{u_0-1}), \quad \forall s = 1, ..., u^0 - 1
\]

These partial incremental functions represent the evolution of the objective function due to the incrementation or the decrementation of the variable \(z_j^s\). An optimal solution \(Z^*\) must satisfy the requirements (13)- (16):

\[
G_j^+(Z) \geq 0, \quad \forall j = 0, ..., u^0 - 1 \quad (13)
\]

\[
G_j^-(Z) \geq 0, \quad \forall j = 0, ..., u^0 - 1 \quad (14)
\]

\[
G_s^+(Z) \geq 0, \quad \forall s = 0, ..., u^0 \quad (15)
\]

\[
G_s^-(Z) \geq 0, \quad \forall s = 0, ..., u^0 \quad (16)
\]

**Proposition 1.** The optimal solution over \(s\) is equivalent to the well known newsvendor model:

\[
F^j(s-1) \leq \left(\frac{b}{b+nh}\right)^n \leq F^j(s),
\]

\[
\forall j = 1, ..., u^0 - 1
\]

**Proof.** First we compute \(G_j^{\pm}(Z)\) as follows:

\[
G_j^+(Z) = APC^j \cdot n + (s+1) \cdot n \cdot h - n \cdot h \cdot E(L^j) + H \sum_{k=0}^{u_0-1} (F^j(s+k))^n
\]

\[
G_j^-(Z) = n \cdot h + H \sum_{k=0}^{u_0-1} 1 - (F^j(s+k))^n
\]

After simplification, we get:

\[
G_j^+(Z) = n \cdot h + H \sum_{k=0}^{u_0-1} 1 - (F^j(s+k))^n
\]

\[
G_j^-(Z) = n \cdot h - H(1 - (F^j(s))^n)
\]

As the optimal solution should satisfy the requirements (13) we have:

\[
\frac{b}{nh + b} \leq (F^j(s))^n
\]
If we combine inequalities 17 and 18 we obtain:
\[ F^j(s-1)^n \leq \frac{b}{b + n \cdot h} \leq F^j(s)^n, \quad \forall j = 1, ..., u_0 - 1 \]
Which is equivalent to
\[ F^j(s-1) \leq \left( \frac{b}{b + n \cdot h} \right)^{\frac{1}{n}} \leq F^j(s), \quad \forall j = 1, ..., u_0 - 1 \quad (19) \]

Proposition 2. The optimal solution over \( j \) must satisfy the following inequalities (20): (21)
\[ \left( n \cdot \Delta APC^j + n \cdot h \cdot \Delta E^{j+1} + H \right) \leq F^j(u_0 - 1) \]
Where : \( \Delta APC^{j+1} = APC^j - APC^{j+1} \)
\( \Delta E^{j+1} = E(L^{j+1}) - E(L^j) \)
\[ F^{j-1}(u^j) \leq \left( \frac{n(\Delta APC^j + n \cdot h \cdot \Delta E^j + H)}{H} \right)^{\frac{1}{n}} \quad (21) \]
Where : \( \Delta APC^j = APC^j - APC^{j-1} \)
\( \Delta E^j = E(L^j) - E(L^{j-1}) \)

\[ G^*_j(Z) = n \cdot APC^{j+1} + s \cdot n \cdot h - n \cdot h \cdot E(L^{j+1}) \]
\[ G^*_j(Z) = n \cdot APC^{j+1} - APC^j - n \cdot h \cdot (E(L^{j+1}) - E(L^j)) \]
\[ + H \sum_{k \geq 0} \left(1 - (F^j + 1(s + k))^n \right) \]
\[ G^*_j(Z) = n \cdot APC^{j+1} - APC^j - n \cdot h \cdot (E(L^{j+1}) - E(L^j)) \]
\[ + H \left( \sum_{k \geq 0} (F^j(s + k))^n - (F^{j+1}(s + k))^n \right) \]
We know that:
\[ F^{j+1}(x) = F^j(x) \quad \forall x < u^j - 1 \quad \text{and} \quad F^{j+1}(u^j - 1) = 1 \]
The equation becomes:
\[ G^*_j(Z) = n \cdot APC^{j+1} - APC^j - n \cdot h \cdot (E(L^{j+1}) - E(L^j)) \]
\[ + H \left( \left( F^j(u^j - 1)^n \right) - 1 \right) \]
\[ G^*_j(Z) \geq 0 \Rightarrow n \cdot APC^{j+1} - APC^j - n \cdot h \cdot (E(L^{j+1}) - E(L^j)) \]
\[ + H \left( (F^j(u^j - 1)^n \right) - 1 \right) \geq 0 \]
We obtain:
\[ \left( \frac{n \cdot \Delta APC^j + n \cdot h \cdot \Delta E^{j+1} + H}{H} \right)^{\frac{1}{n}} \leq F^j(u^j - 1) \]
Where : \( \Delta APC^{j+1} = APC^j - APC^{j+1} \)
\( \Delta E^{j+1} = E(L^{j+1}) - E(L^j) \)

In the same way we can compute \( G^*_j(Z) \):

4. NUMERICAL EXAMPLE

We give an illustrative example with 5 types of components (\( n = 5 \)) with same characteristics. The lead time of each type of component is a discrete random variable, which takes values from 1 to 5 (\( u = 5 \)). The unit holding cost \( h = 15 \), the unit backlogging cost \( b = 100 \) and the unit purchasing cost is given in the Table 2.

<table>
<thead>
<tr>
<th>Policy ( j )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>APC^j</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>APC^1</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>APC^2</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>APC^3</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

The distribution probability of all lead times are given in Table 3.

<table>
<thead>
<tr>
<th>( s )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Pr(L^s = s) )</td>
<td>0.80</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>( Pr(L^s = s) )</td>
<td>0.80</td>
<td>0.05</td>
<td>0.05</td>
<td>0.10=0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>( Pr(L^s = s) )</td>
<td>0.80</td>
<td>0.05</td>
<td>0.15=0.1</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>( Pr(L^s = s) )</td>
<td>0.80</td>
<td>0.20=0.15</td>
<td>0.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Pr(L^s = s) )</td>
<td>1=0.80</td>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We know that the optimal purchasing policy should satisfy inequalities (22) and (23). In tables 4 and 5 we determine the policies which do not fulfill these conditions.

Inequality sets (22) and (23) eliminates the purchasing policies \( j = 0 \) and \( j = 4 \), respectively. For a given optimal policy \( j \), the optimal purchasing date should satisfy inequality (19). As \( \left( \frac{b}{b + n \cdot h} \right)^{\frac{1}{n}} = 0.89 \) for given parameters, we can deduce following values of \( s \) for purchasing policies \( j = 1, 2, 3 \) in table 6:
Table 4. Dominance rule for inequality (20)

<table>
<thead>
<tr>
<th>j</th>
<th>(F^1(u^j))</th>
<th>(F^1(u^j - 1) - F^1(u^j))</th>
<th>(\frac{F^1(u^j - 1) - F^1(u^j)}{F^1(u^j)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.964743</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.826520</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.739685</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.564147</td>
<td>0.80</td>
<td></td>
</tr>
</tbody>
</table>

Table 5. Dominance rule for inequality (21)

<table>
<thead>
<tr>
<th>j</th>
<th>(F^{j-1}(u^j))</th>
<th>(\frac{n(APC^j - APC^{j+1}) + n(h(E(L^{j+1}) - E(L^j)) + H_j)}{H_j})</th>
<th>(\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.95</td>
<td>0.982213</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.90</td>
<td>0.989131</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.85</td>
<td>0.986949</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.80</td>
<td>0.751097</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Best values of \(s\) by inequalities (19)

<table>
<thead>
<tr>
<th>j</th>
<th>(F^j(s - 1))</th>
<th>(\left(\frac{\sum s_j}{n}\right))</th>
<th>(F^j(s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.85</td>
<td>0.89</td>
<td>0.88</td>
</tr>
<tr>
<td>2</td>
<td>0.85</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>3</td>
<td>0.80</td>
<td>0.89</td>
<td>0.89</td>
</tr>
</tbody>
</table>

To find the optimal purchasing policy as long as the purchasing date, it is enough to compare the values of \(EC(z_j^1), EC(z_j^2)\) and \(EC(z_j^3)\):

Table 7. Optimal purchasing policy and date

<table>
<thead>
<tr>
<th>(j)</th>
<th>(z_j^1)</th>
<th>(z_j^2)</th>
<th>(z_j^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>212.91</td>
<td>217.91</td>
<td>217.91</td>
</tr>
<tr>
<td>2</td>
<td>258.75</td>
<td>258.75</td>
<td>258.75</td>
</tr>
<tr>
<td>3</td>
<td>310.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can see clearly in table 7 that the optimal purchasing policy is policy 1 and the best purchasing date is 3.

If we compute the cost of all purchasing policy and date combinations, we obtain the following values in table 8:

Table 8. Cost of purchasing policy and date combinations

<table>
<thead>
<tr>
<th>(s)</th>
<th>(j)</th>
<th>(APC)</th>
<th>(APC)</th>
<th>(APC)</th>
<th>(APC)</th>
<th>(APC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>226.76</td>
<td>226.76</td>
<td>226.76</td>
<td>226.76</td>
<td>226.76</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>277.92</td>
<td>277.92</td>
<td>277.92</td>
<td>277.92</td>
<td>277.92</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>313.76</td>
<td>313.76</td>
<td>313.76</td>
<td>313.76</td>
<td>313.76</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>352.66</td>
<td>310.00</td>
<td>310.00</td>
<td>310.00</td>
<td>310.00</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>400.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

The paper deals with the problem of the reducing of lead times for one level assembly systems. It is focused on searching the optimal values of the planned lead times by the integration of additional purchase cost. The general optimization model is presented. Under the assumption that the components has the same lead time distributions, holding and additional purchasing costs, the optimal solution can be obtained by the well known Newsboy model.

A numerical example is proposed for the particular case of assembly system. However, further study is required to develop an efficient optimization algorithms for the general case when the lead time distributions, holding and additional purchasing cost are different.

REFERENCES


