Integration of additional purchase cost to reduce the lead time uncertainty for one level assembly system

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Abstract: We consider the supply planning for one level assembly system under uncertainty of lead times. The finished product need several components for the assembly operation. The lead times can be uncertain for different reasons such as machine breakdowns, transport delay, strike, etc. In literature, this type of problem was studied in the case of random lead times that follow continuous distributions as well as in the case of any discrete laws with known distribution. We consider the case of discrete distributions to be closer to the industrial reality because often in real life situations the lead time is expressed as the number of periods. Typically, this is an inventory control problem where the objective is to minimize the component holding and backlog costs for the finished product due to the uncertainty of lead times. Indeed, as the finished product is assembled by using several types of components, the assembly is stopped even if a single type of component is delayed. The other components are stored between their arrival and the arrival of the latest component. To reduce stock and backlog costs, we propose to pay the supplier more, if he/she agrees to decrease his lead time uncertainty. A model describing this choice is suggested.

Keywords: Assembly systems, Random lead time, Replenishment, Pricing, Purchase cost.

1. INTRODUCTION AND RELATED WORKS

The additional difficulty of the assembly systems is the dependence among the necessary components for the assembly operation (Dolgui and Proth [2010]). In the following, the assembly systems with stochastic lead times are reviewed. Liberatore [1979] tried to extend directly the EOQ model so as to treat stochastic lead time, but no closed-form solution was given for optimal order size. Yano [1987] considered the problem of two-level assembly systems with only two types of components at level 2 and one type of components at level 1. The lead times of the three components are stochastic Poisson and negative binomial distribution. The problem was to find planned lead times minimize the sum of inventory holding costs and tardiness costs. An algorithm was developed which exploits properties of the objective function to find optimal solutions. Computational results indicate that optimal solutions often have negative safety times, there are two situations. First, where the component holding and storage cost are both relatively high. Second case, the situation in which one component lead time is much longer than the other. Negative safety times do not have any practical signification. But, in our opinion since that safety time is the difference between planned and expected lead times, then this difference can be negative if the planned lead time is shorter than the expected lead time. Evidently, this result depends also in the values of holding and tardiness cost. Kumar [1987] presented a generic study of inventory control in an assembly system of several different components where the component procurement lead times are stochastic and the assembly date and quantity are fixed. The problem consists in determining the timing of each component's order so that the total cost, composed of the component holding and the tardiness of the assembly costs is minimized. Many of her results are based on exact analysis which is only possible to carry out for special types of distributions (exponential, uniform, and normal).

Another interesting single-period model of this type was developed in Chu et al. [1993]. Their model deals with a fixed demand for one finished product. To assembly this product several types of components are needed. The lead times of components are random variables. It is necessary to determine the order date for each type of component. The criterion considered is the mathematical expectation of the sum of the holding cost for the components and the backlogging cost for the finished product. The authors prove the convexity of the expected average cost and propose an iterative algorithm to minimize it.

In manufacturing systems, the demand is periodic, therefore this inventory problem is solved at each period and the stocks of the previous periods can be used for the next and so on. However, the mathematical formulation of multi-period problems under lead time uncertainty is more difficult. Orders may cross, for that reason they may not be received in the same sequence in which they were placed. In certain publications it is assumed that orders

do not crossover, and so a single-period problem is solved Graves [1993]. A multi-period model was proposed in Gurnani et al. [1996] for assembly systems with two types of components and the lead time probability distributions are limited to two periods. In this model authors supposed that components either arrive in the current period with a given probability (α) or in the next period with probability $(1-\alpha)$. This two period lead time model gives the optimal quantity of each component to order from each supplier.

Fujiwara and Sedarage [1997] studied a (Q,r)-type model for a simple assembly system with stochastic component procurement lead times. Assembly is instantaneous and takes place intermittently in batches but cannot start until all the components are available. The author used the following general assumptions: one finished product and several types of components, constant and known demand rate, and infinite capacity of the assembly system. They considered the inventory holding cost for the components and the assembled product, shortage cost for the assembled product and setup cost. In Bookbinder and Cakanyildirim [1999] Bookbinder considered inventories for (Q,r) models with constant demand and stochastic lead times. The authors developed two probabilistic models. For each model, the convexity of the expected cost is proven and the minimum is obtained. The author's motivation was to help an inventory manager of a JIT system who could invest in decreasing the lead time in a stochastic-order sense. They used (Q,r) approach and gave comparison of their model with the classical (Q,r) model (stochastic demand, fixed lead time) and the EOQ (deterministic demand). In Dolgui and Ould-Louly [2002], and Louly and Dolgui [2004] treated the same type of assembly systems as in Chu et al. [1993] providing some generalizations. Their model is a multi-period model with random lead times and integer decision variables. The finished product demand is periodic and constant (the same for all periods). The criterion considered is the sum of the average holding cost for the components and the average backlogging cost for the finished product. This model gives the optimal values of the safety stocks when the component lead times are i.i.d. random variables and the unit holding costs are the same for all types of components.

Tang and Grubbström [2005] considered a two component assembly system problem with stochastic lead times for components and deterministic demand for the finished item . Their study is similar to the work of Yano [1987]. However, the process time of item at level 1 is assumed to be stochastic, the due date is known and the optimal planned lead time are smaller than the due date. The objective is to minimize the total stockout cost and inventory holding costs. A Laplace transformation procedure is used to capture the stochastic properties of lead times. The optimal safety lead time, which is the difference between planned and expected lead time is presented. In Axsater [2005] a multi-level assembly network was considered with independent stochastic operation times. The objective was to choose starting times for different operations in order to minimize the total expected costs composed of holding costs of components and delay cost of end items. An approximate decomposition technique based on repeat application of the solution of a single-stage problem was suggested.

A state of the art on production planning models under uncertainties is presented by Dolgui and Prodhon [2007].

2. PROBLEM DESCRIPTION

We consider a one-level assembly system with n different components from n different suppliers. The uncertainty of lead times causes an important level of inventory of the components and a delay of the finished product. The assembly system capacity is supposed infinite. The demand of finished product is known and we consider one period. The lead times of components type $i = 1, ..., n$ are independent random variables. This problem is formulated by Louly and Dolgui [2012], Louly and Dolgui [2011] for one-level multi-period case and by Hnaien et al. [2010] for the multi-level one period discrete random lead times. The following notations are used in this paper:

Table 1. Notation

Parameters	
i	supplier index
j	purchasing price index
h_i	unit stock cost of the product purchased from the supplier i
$H = b + \sum_{i=1}^{n} h_i$	global holding and backlog costs for one finished product
L_i^j	random variable lead time representing the supplier i under the purchasing price j
u_i^j	upper value of lead time i under the purchasing price j; i.e., $1 \leq L_i^j \leq u_i^j$
Variables	
x_i^j	planned lead time for the supplier i with the purchasing price j
y_i^j	binary variable taking the value 1 if the purchasing price j is used for the supplier i
Functions	
APC_i^j	additional purchasing cost under the purchasing price j for the supplier i
$E(L_i^j)$	lead time expected value of supplier i under the purchasing price j
$F_i^j(x_i^j)$	cumulative distribution function of lead time of the supplier i under the purchasing price j
$p_i^j(k)$	probability of having lead time equal to k for the supplier i under the purchasing price j

The optimization model without tacking into account prices consists in minimizing the average cost composed of the sum of holding cost of the components and the backlogging cost of the finished product.

$$\min EC(X) = \sum_{i=1}^{n} h_i \left(x_i - E(L_i) \right)$$
$$+ H \sum_{k \geq 0} \left(1 - \prod_{i=1}^{n} F_i(x_i + k) \right) \quad (1)$$

s.t.

$$1 \leq x_i \leq u_i \quad \forall i = 1, ..., n$$
$$x_i \in \mathbb{N} \quad \forall i = 1, ..., n \quad (2)$$

where:

$$X = (x_1, x_2, ..., x_i, ..., x_n) \quad (3)$$

To solve this non-linear problem we can use the $B\&B$ proposed by Louly et al. [2008] for the multi-period case. In this paper we trait the same problem differently. Our idea is to propose to modify purchasing cost depending on the risk of not respecting the due date. The new objective function is presented in the equation (4).

The model is expressed as follows:

$$\min EC(X,Y) = \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} APC_i^j \cdot y_i^j$$

$$+ \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} y_i^j \cdot h_i \left(x_i^j - E(L_i^j) \right) \qquad (4)$$

$$+ H \cdot \sum_{k \geq 0} 1 - \prod_{i=1}^{n} \left(\sum_{j=0}^{u_i^0-1} y_i^j \cdot F_i^j(x_i^j + k) \right)$$

s.t.

$$\sum_{j=0}^{u_i^0-1} y_i^j = 1 \qquad \forall i = 1, ..., n \quad (5)$$

$$x_i^j \leq (u_i^0 - j) \cdot y_i^j \qquad \forall i = 1, ..., n, \forall j = 0, ..., u_i^0 - 1 \quad (6)$$

$$x_i^j \in \mathbb{N} \qquad \forall i = 1, ..., n \quad \forall j = 0, ..., u_i^0 - 1 \quad (7)$$

$$y_i^j \in \{0,1\} \qquad \forall i = 1, ..., n \quad \forall j = 0, ..., u_i^0 - 1 \quad (8)$$

where for a given supplier $(i = 1, ..., n)$ we have the following relation for two purchasing policies j and $j + 1$ $(j = 0, ..., u_i^0 - 1)$:

$$p_i^{j+1}(k) = \begin{cases} p_i^j(k) & \text{if } k < u_i^j - 1 \\ p_i^j(k) + p_i^j(u_i^j) & \text{if } k = u_i^j - 1 \end{cases}$$

Or

$$F_i^{j+1}(k) = \begin{cases} F_i^j(k) & \text{if } k < u_i^j - 1 \\ F_i^j(k) + p_i^j(u_i^j) & \text{if } k = u_i^j - 1 \end{cases}$$

The objective function (4) represents the total cost composed of additional purchasing cost noted APC, the holding component costs (HC) and the backlogging cost (BC):

$$APC = \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} APC_i^j \cdot y_i^j$$

$$HC = \sum_{i=1}^{n} \sum_{j=0}^{u_i^0-1} \left(y_i^j \cdot h_i \left(x_i^j - E(L_i^j) \right) \right) +$$

$$\sum_{i=1}^{n} (h_i) \cdot \sum_{k \geq 0} \left(1 - \prod_{i=1}^{n} \left(\sum_{j=0}^{u_i^0-1} y_i^j \cdot F_i^j(x_i^j + k) \right) \right)$$

$$BC = b \cdot \sum_{k \geq 0} \left(1 - \prod_{i=1}^{n} \left(\sum_{j=0}^{u_i^0-1} y_i^j \cdot F_i^j(x_i^j + k) \right) \right)$$

The total cost $EC(X,Y) = APC(Y) + HC(X,Y) + BC(X,Y)$.

The constraint (5) expresses that only one purchasing policy $(j = 0, ..., u_i^0 - 1)$ is assigned for each supplier $(i = 1, ..., n)$. The constraint (6) expresses that each planned lead time x_i^j is limited by the upper bound for each lead time, the upper bound for the purchasing policy $(j=0)$ is equal to u_i^0.

The problem is to minimize the model given by (4)--(8). This minimization is rather difficult because the function is not linear and the decision variables $X = (x_i^j; i = 1, ..., n; j = 0, ..., u_i^0 - 1)$ are integer and $Y = (y_i^j; i = 1, ..., n; j = 0, ..., u_i^0 - 1)$ are binary. The general optimization problem is difficult, in the following section the problem is solved under some assumptions.

3. OPTIMIZATION METHOD

The problem is solved under the assumption that holding cost and additional purchase cost per period are the same, and the lead times L_i^j of the different components have the same distribution probability. Then, the costs $h_i, i = 1, ..., n$, can be replaced by h, the distribution F_i^j can be noted by F^j and the lead time L_i^j by L^j. The model given by (4)--(8) can be rewritten as follows:

$$\min EC(Z) = \sum_{j=0}^{u^0-1} \sum_{s=1}^{u_0} z_j^s \cdot APC^j \cdot n$$

$$+ \sum_{j=0}^{u^0-1} \sum_{s=1}^{u_0} z_j^s \cdot s \cdot n \cdot h$$

$$- \sum_{j=0}^{u_0-1} \sum_{s=1}^{u_0} n \cdot h \cdot z_j^s \cdot E(L^j) \qquad (9)$$

$$+ H \sum_{k \geq 0} 1 - \left(\sum_{j=0}^{u_0-1} \sum_{s=1}^{u_0} z_j^s \left(F^j(s+k) \right)^n \right)$$

s.t.

$$\sum_{s=1}^{u_0} \sum_{j=0}^{u_0-1} z_j^s = 1 \qquad (10)$$

Under the assumptions that all components have the same characteristics the objective function (4) is rewritten as follows:

$$EC(X,Y) = \sum_{j=0}^{u^0-1} n \cdot APC^j \cdot y^j + \sum_{j=0}^{u^0-1} y^j \cdot n \cdot h \left(x^j - E(L^j) \right)$$

$$+ H \sum_{k \geq 0} \left(1 - \left(\sum_{j=0}^{u^0-1} y^j F^j(x^j + k) \right)^n \right)$$

We introduce the binary variable $z_j^k, \forall j = 0, ..., u^0 - 1$:

$$z_j^k = \begin{cases} 1 & \text{if } x^j = k \\ 0 & \text{otherwise} \end{cases}$$

Thus we can rewrite x^j and y_j as follows:

$$x^j = \sum_{s=1}^{u_0} z_j^s \cdot s \qquad \forall j = 0, ..., u^0 - 1$$

$$y^j = \sum_{s=1}^{u_0} z_j^s \qquad \forall j = 0, ..., u^0 - 1$$

Where:

$$\sum_{s=1}^{u_0} \sum_{j=0}^{u_0-1} z_j^s = 1$$

Which gives us the new distribution function for a given k:

$$F^j(x^j + k) = \sum_{s=1}^{u_0} z_j^s \cdot F^j(s+k)$$

$$\left(\sum_{j=0}^{u_0-1} y^j \cdot \left(F^j(x^j+k)\right)\right)^n = \sum_{j=0}^{u_0-1} \sum_{s=1}^{u_0} z_j^s \left(F^j(s+k)\right)^n$$

Finally, the objective function can be rewritten as follows:

$$\min EC(Z) = \sum_{j=0}^{u^0-1} \sum_{s=1}^{u_0} z_j^s \cdot APC^j \cdot n$$
$$+ \sum_{j=0}^{u^0-1} \sum_{s=1}^{u_0} z_j^s \cdot s \cdot n \cdot h$$
$$- \sum_{j=0}^{u_0-1} \sum_{s=1}^{u_0} n \cdot h \cdot z_j^s \cdot E(L^j) \qquad (11)$$
$$+ H \sum_{k \geq 0} \left(1 - \left(\sum_{j=0}^{u_0-1} \sum_{s=1}^{u_0} z_j^s \left(F^j(s+k)\right)^n\right)\right)$$

s.t.

$$\sum_{s=1}^{u_0} \sum_{j=0}^{u_0-1} z_j^s = 1 \qquad (12)$$

3.1 Partial increments of cost functions

We will use the following partial increment functions (Louly and Dolgui [2012]):

$$G_j^{s+}(Z) = EC(z_0^1, ..., z_j^{s+1}..., z_{u_0-1}^{u_0})$$
$$- EC(z_0^1, ..., z_j^s..., z_{u_0-1}^{u_0}), \quad \forall j = 0, ..., u^0 - 1$$
$$G_j^{s-}(Z) = EC(z_0^1, ..., z_j^{s-1}..., z_{u_0-1}^{u_0})$$
$$- EC(z_0^1, ..., z_j^s..., z_{u_0-1}^{u_0}), \quad \forall j = 0, ..., u^0 - 1$$
$$G_s^{j+}(Z) = EC(z_0^1, ..., z_{j+1}^s, ..., z_{u_0-1}^{u_0})$$
$$- EC(z_0^1, ..., z_j^s..., z_{u_0-1}^{u_0}), \quad \forall s = 1, ..., u^0$$
$$G_s^{j-}(Z) = EC(z_0^1, ..., z_{j-1}^s..., z_{u_0-1}^{u_0})$$
$$- EC(z_0^1, ..., z_j^s..., z_{u_0-1}^{u_0}), \quad \forall s = 1, ..., u^0$$

These partial incremental functions represent the evolution of the objective function due to the incrementation or the decrementation of the variable z_j^s. An optimal solution Z^* must satisfy the requirements (13)- -(16):

$$G_j^{s+}(Z) \geq 0 \quad \forall j = 0, ..., u^0 - 1 \qquad (13)$$
$$G_j^{s-}(Z) \geq 0 \quad \forall j = 0, ..., u^0 - 1 \qquad (14)$$
$$G_{j+}^s(Z) \geq 0 \quad \forall s = 0, ..., u^0 \qquad (15)$$
$$G_{j-}^s(Z) \geq 0 \quad \forall s = 0, ..., u^0 \qquad (16)$$

Proposition 1. The optimal solution over s is equivalent to the well known newsboy model :

$$F^j(s-1) \leq \left(\frac{b}{b+nh}\right)^{\frac{1}{n}} \leq F^j(s),$$
$$\forall j = 1, ..., u^0 - 1$$

Proof. First we compute $G_j^{s+}(Z)$ as follows :

$$G_j^{s+}(Z) = APC^j \cdot n + (s+1) \cdot n \cdot h - n \cdot h \cdot E(L^j)$$
$$+ H \sum_{k \geq 0} 1 - \left(F^j(s+k+1)\right)^n$$
$$- APC^j \cdot n - s \cdot n \cdot h + n \cdot h \cdot E(L^j)$$
$$- H \sum_{k \geq 0} 1 - \left(F^j(s+k)\right)^n$$

After simplification, we get :

$$G_j^{s+}(Z) = n \cdot h + H \sum_{k \geq 0} 1 - (F^j(s+k+1))^n$$
$$- H \sum_{k \geq 0} 1 - (F^j(s+k))^n$$
$$G_j^{s+}(Z) = n \cdot h$$
$$+ H \left(\sum_{k \geq 0} 1 - (F^j(s+k+1))^n - \sum_{k \geq 1} 1 - (F^j(s+k))^n\right)$$
$$- H(1 - (F^j(s))^n)$$
$$G_j^{s+}(Z) = n \cdot h$$
$$+ H(\sum_{k \geq 0} 1 - (F^j(s+k+1))^n$$
$$- \sum_{k \geq 0} 1 - (F^j(s+k+1))^n)$$
$$- H(1 - (F^j(s))^n)$$
$$G_j^{s+}(Z) = n \cdot h - H(1 - (F^j(s))^n)$$

As the optimal solution should satisfy the requirements (13) we have:

$$\frac{b}{nh+b} \leq (F^j(s))^n \qquad (17)$$

In the same way, we compute $G_j^{s-}(Z)$:

$$G_j^{s-}(Z) = APC^j \cdot n + (s-1) \cdot n \cdot h - n \cdot h \cdot E(L^j)$$
$$+ H \sum_{k \geq 0} 1 - \left(F^j(s+k-1)\right)^n$$
$$- APC^j \cdot n - s \cdot n \cdot h + n \cdot h \cdot E(L^j)$$
$$- H \sum_{k \geq 0} 1 - \left(F^j(s+k)\right)^n$$
$$G_j^{s-}(Z) = -n \cdot h + H \sum_{k \geq 0} 1 - (F^j(s+k-1))^n$$
$$- H \left(\sum_{k \geq 0} 1 - (F^j(s+k))^n\right)$$
$$G_j^{s-}(Z) = -n \cdot h + H \cdot (1 - F^j(s-1)^n)$$
$$+ H \left(\sum_{k \geq 1} 1 - (F^j(s+k-1))^n - \sum_{k \geq 0} 1 - (F^j(s+k))^n\right)$$
$$G_j^{s-}(Z) = -n \cdot h + H \cdot (1 - F^j(s-1)^n)$$
$$+ H(\sum_{k \geq 1} 1 - (F^j(s+k-1))^n$$
$$- \sum_{k \geq 1} 1 - (F^j(s+k-1))^n)$$
$$G_j^{s-}(Z) = -n \cdot h + H \cdot (1 - F^j(s-1)^n)$$

As the optimal solution should satisfy the requirements (14) we have:

$$(F^j(s-1))^n \leq \frac{b}{nh+b} \qquad (18)$$

If we combine inequalities 17 and 18 we obtain:

$$F^j(s-1))^n \leq \frac{b}{b+nh} \leq (F^j(s))^n, \quad \forall j=1,...,u^0-1$$

Which is equivalent to

$$F^j(s-1) \leq \left(\frac{b}{b+nh}\right)^{\frac{1}{n}} \leq F^j(s), \quad \forall j=1,...,u^0-1 \quad (19)$$

Proposition 2. The optimal solution over j must satisfy the following inequalities (20)- (21)

$$\left(\frac{n \cdot \Delta APC^j + n \cdot h \cdot \Delta E^{j+1} + H}{H}\right)^{\frac{1}{n}} \leq F^j(u^j-1) \quad (20)$$

Where : $\Delta APC^{j+1} = APC^j - APC^{j+1}$
$\Delta E^{j+1} = E(L^{j+1}) - E(L^j)$

$$F^{j-1}(u^j) \leq \left(\frac{n(\Delta APC^j + n \cdot h \cdot \Delta E^j + H)}{H}\right)^{\frac{1}{n}} \quad (21)$$

Where : $\Delta APC^j = APC^{j-1} - APC^j$
$\Delta E^j = E(L^j) - E(L^{j-1})$

Proof.

$$G^s_{j+}(Z) = n \cdot APC^{j+1} + s \cdot n \cdot h - n \cdot h \cdot E(L^{j+1})$$
$$+H \sum_{k \geq 0} 1 - (F^{j+1}(s+k))^n$$
$$-n \cdot APC^j - s \cdot n \cdot h + n \cdot h \cdot E(L^j)$$
$$-H \sum_{k \geq 0} 1 - (F^j(s+k))^n$$

$$G^s_{j+}(Z) = n(APC^{j+1} - APC^j) - n \cdot h \left(E(L^{j+1}) - E(L^j)\right)$$
$$+H \left(\sum_{k \geq 0} 1 - (F^{j+1}(s+k))^n - \sum_{k \geq 0} 1 - (F^j(s+k))^n\right)$$
$$G^s_{j+}(Z) = n(APC^{j+1} - APC^j) - n \cdot h \left(E(L^{j+1}) - E(L^j)\right)$$
$$+H \left(\sum_{k \geq 0} (F^j(s+k))^n - (F^{j+1}(s+k))^n\right)$$

We know that:

$$F^{j+1}(x) = F^j(x) \quad \forall x < u^j-1 \quad \text{and} \quad F^{j+1}(u^j-1) = 1$$
The equation becomes:
$$G^s_{j+}(Z) = n(APC^{j+1} - APC^j) - n \cdot h \left(E(L^{j+1}) - E(L^j)\right)$$
$$+H \left((F^j(u^j-1))^n - 1\right)$$
$$G^s_{j+}(Z) \geq 0 \Rightarrow n(APC^{j+1} - APC^j) - n \cdot h \left(E(L^{j+1}) - E(L^j)\right)$$
$$+H \left((F^j(u^j-1))^n - 1\right) \geq 0$$

We obtain:

$$\left(\frac{n \cdot \Delta APC^j + n \cdot h \cdot \Delta E^{j+1} + H}{H}\right)^{\frac{1}{n}} \leq F^j(u^j-1) \quad (22)$$

Where : $\Delta APC^{j+1} = APC^j - APC^{j+1}$
$\Delta E^{j+1} = E(L^{j+1}) - E(L^j)$

In the same way we can compute $G^s_{j-}(Z)$:

$$G^s_{j-}(Z) = n \cdot APC^{j-1} + s \cdot n \cdot h - n \cdot h \cdot E(L^{j-1})$$
$$+H \sum_{k \geq 0} 1 - (F^{j-1}(s+k))^n -$$
$$n \cdot APC^j - s \cdot n \cdot h + n \cdot h \cdot E(L^j)$$
$$-H \sum_{k \geq 0} 1 - (F^j(s+k))^n$$
$$G^s_{j-}(Z) = n(APC^{j-1} - APC^j)$$
$$-n \cdot h \left(E(L^{j-1}) - E(L^j)\right)$$
$$+H \left(\sum_{k \geq 0} (F^j(s+k))^n - (F^{j-1}(s+k))^n\right)$$
$$G^s_{j-}(Z) = n(APC^{j-1} - APC^j)$$
$$-n \cdot h \left(E(L^{j-1}) - E(L^j)\right)$$
$$+H \left(1 - (F^{j-1}(u^j))^n\right)$$
$$G^s_{j-}(Z) \geq 0 \Rightarrow n(APC^{j-1} - APC^j)$$
$$-n \cdot h \left(E(L^{j-1}) - E(L^j)\right) + H \left(1 - (F^{j-1}(u^j))^n\right) \geq 0$$

So we have

$$F^{j-1}(u^j) \leq \left(\frac{n(\Delta APC^j + n \cdot h \left(\Delta E^j\right) + H)}{H}\right)^{\frac{1}{n}} \quad (23)$$

Where : $\Delta APC^j = APC^{j-1} - APC^j$
$\Delta E^j = E(L^j) - E(L^{j-1})$

4. NUMERICAL EXAMPLE

We give an illustrative example with 5 types of components ($n=5$) with same characteristics. The lead time of each type of component is a discrete random variable, which takes vales from 1 to 5 ($u = 5$). The unit holding cost $h = 15$, the unit backlogging cost $b = 100$ and the unit purchasing cost is given in the Table 2.

Table 2. Unit purchasing cost

policy j	0	1	2	3	4
APC^j	0	5	25	50	80

The distribution probability of all lead times are given in Table 3.

Table 3. Probability distributions of the lead time of each policy ($j = 0,...,u-1$)

s	1	2	3	4	5
$Pr(L^0 = s)$	0.80	0.05	0.05	0.05	0.05
$Pr(L^1 = s)$	0.80	0.05	0.05	0.10=0.05 +0.05	
$Pr(L^2 = s)$	0.80	0.05	0.15=0.1 +0.05		
$Pr(L^3 = s)$	0.80	0.20=0.15 +0.05			
$Pr(L^4 = s)$	1=0.80 +0.20				

We know that the optimal purchasing policy should satisfy inequalities (22) and (23). In tables 4 and 5 we determine the policies which do not fulfil these conditions.

Inequality sets (22) and (23) eliminates the purchasing policies ($j = 0$) and ($j = 4$), respectively. For a given optimal policy j, the optimal purchasing date should satisfy inequality (19). As $\left(\frac{b}{b+nh}\right)^{\frac{1}{n}} = 0.89$ for given parameters, we can deduce following values of s for purchasing policies $j = 1, 2, 3$ in table 6:

5

Table 4. Dominance rule for inequality (20)

j	$\left(\frac{n(APC^j - APC^{j+1}) + n \cdot h(E(L^{j+1}) - E(L^j)) + H}{H}\right)^{\frac{1}{n}}$	$F^j(u^j - 1)$
0	0.964743	0.95
1	0.826520	0.90
2	0.739685	0.85
3	0.564147	0.80

Table 5. Dominance rule for inequality (21)

j	$F^{j-1}(u^j)$	$\left(\frac{n(APC^{j-1} - APC^j) + n \cdot h(E(L^j) - E(L^{j-1})) + H}{H}\right)^{\frac{1}{n}}$
1	0.95	0.982213
2	0.90	0.909131
3	0.85	0.860049
4	0.80	0.751097

Table 6. Best values of s by inequalities (19)

j	$F^j(s-1)$	$\left(\frac{b}{b+nh}\right)^{\frac{1}{n}}$	$F^j(s)$
1	$F^1(2) = 0.85$	0.89	$F^1(3) = 0.90$
2	$F^2(2) = 0.85$	0.89	$F^2(3) = 1.00$
3	$F^3(1) = 0.80$	0.89	$F^3(2) = 1.00$

To find the optimal purchasing policy as long as the purchasing date, it is enough to compare the values of $EC(z_1^3), EC(z_2^3)$ and $EC(z_3^2)$):

Table 7. Optimal purchasing policy and date

z_j^s	$EC(z_j^s)$
$j=1, \ s=3$	212.91
$j=2, \ s=3$	248.75
$j=3, \ s=2$	310.00

We can see clearly in table 7 that the optimal purchasing policy is policy 1 and the best purchasing date is 3.

If we compute the cost of all purchasing policy and date combinations, we obtain the following values in table 8:

Table 8. Cost of purchasing policy and date combinations

	$s=1$	$s=2$	$s=3$	$s=4$	$s=5$
$j=0$	288.76	246.10	223.75	227.09	262.50
$j=1$	277.92	235.27	**212.91**	216.25	
$j=2$	313.76	271.10	**248.75**		
$j=3$	352.66	**310.00**			
$j=4$	400.00				

5. CONCLUSIONS

The paper deals with the problem of the reducing of lead times for one level assembly systems. It is focused on searching the optimal values of the planned lead times by the integration of additional purchase cost. The general optimization model is presented. Under the assumption that the components has the same lead time distributions, holding and additional purchasing costs, the optimal solution can be obtained by the well known Newsboy model. A numerical example is proposed for the particular case of assembly system. However, further study is required to develop an efficient optimization algorithms for the general case when the lead time distributions, holding and additional purchasing cost are different.

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