An exact solution approach for disassembly line balancing problem under uncertainty of the task processing times

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http://dx.doi.org/10.1080/00207543.2014.961212

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The purpose of this work is to efficiently design disassembly lines taking into account the uncertainty of task processing times. The main contribution of the paper is the development of a decision tool that allows decision-makers to choose the best disassembly alternative (process), for an End of Life product (EOL), and assign the corresponding disassembly tasks to the workstations of the line under precedence and cycle time constraints. Task times are assumed to be random variables with known normal probability distributions. The case of presence of hazardous parts is studied and cycle time constraints are to be jointly satisfied with at least a certain probability level, or service level, fixed by the decision-maker. An AND/OR graph is used to model the precedence relationships among tasks. The objective is to minimise the line cost composed of the workstation operation costs and additional costs of workstations handling hazardous parts of the EOL product. To deal with task time uncertainties, lower and upper-bounding schemes using second-order cone programming and approximations with convex piecewise linear functions are developed. The applicability of the proposed solution approach is shown by solving to optimality a set of disassembly problem instances (EOL industrial products) from the literature.

Keywords: sustainable manufacturing; disassembly process planning; line design; chance constraints; uncertainty

1. Introduction

Disassembly lines as disassembly systems, play a crucial role in End of Life (EOL) product recovery (Güngör and Gupta 2002; Ilgin and Gupta 2010). This paper addresses the design of disassembly lines under uncertainty of task times of the disassembly process. Such a line consists of an ordered sequence of workstations connected by a material handling system which allows the transportation of work pieces from one workstation to another (Güngör and Gupta 2002; Meacham, Uzsoy, and Venkatadri 1999). At each workstation, an EOL product or one or more of its subassemblies are separated into their components and subassemblies for recycling, remanufacturing and reuse. Certain parts or subassemblies may be hazardous and require a particular treatment incurring a supplementary cost.

The studied optimisation problem aims to assign a given set of disassembly tasks (a set I), which models all possible disassembly processes of an EOL product, to an ordered sequence of workstations (a set J), to be determined, while respecting precedence and cycle time constraints. Task times are assumed to be random variables with known normal probability distributions. Therefore, cycle time constraints are to be jointly satisfied with at least a certain probability level $1-\alpha$ fixed by the decision-maker. In industrial terms, the probability level $1-\alpha$ reflects the level of the EOL product (components and subassemblies) demand satisfaction; hence, it defines the level of the customers' satisfaction. Indeed, the joint satisfaction of cycle time constraints with the probability level $1-\alpha$ means that the line to be designed would be a paced line in $[(1-\alpha)\times 100]\%$ of its total operating time.

The deterministic version of the disassembly line design problem is commonly known as Disassembly Line Balancing Problem (DLBP) introduced by Güngör and Gupta (1999) and has been proven to be NP-complete in Mcgovern and Gupta (2007).

In the literature, only few studies dealing with the uncertainty of disassembly task times have appeared. Task times, in fact, may vary during the disassembly process because of multiple factors, such as: operator skill, motivation and fatigue, the structure and quality of EOL products, changes in material composition of product items, workstation characteristics, etc. (Battaïa and Dolgui 2013).

A collaborative ant colony metaheuristic for stochastic mixed-model U-shaped DLBP was developed by Agrawal and Tiwari (2008). Task times were assumed to be uncertain with known independent normal probability distributions

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and the objective was to minimise the probability of line stoppage. A genetic algorithm to solve a non-linear binary bi-objective programme was developed in Aydemir-Karadag and Turkbey (2013) for disassembly line design and balancing under uncertainty of the task times. In (Bentaha, Battaïa, and Dolgui 2014a, 2014b, 2014c; Bentaha et al. 2014), mathematical models for designing this line under uncertainty of task processing times were developed. In (Bentaha, Battaïa, and Dolgui 2014a), uncertainty was modelled using the notion of recourse cost and a sample average approximation method was developed to solve the studied optimisation problem. In (Bentaha et al. 2014), uncertainty was modelled using workstation expectation times instead of direct station times. In (Bentaha, Battaïa, and Dolgui 2014b), the joint problem of DLB and sequencing was studied. In (Bentaha, Battaïa, and Dolgui 2014c), a lagrangian relaxation was proposed to maximise the disassembly line profit.

Other aspects of uncertainty in DLBP were also considered in the literature. A fuzzy optimisation model was proposed in Tripathi et al. (2009) with the objective to maximise the net revenue of the disassembly process under uncertainty of the quality of EOL products. A 'self-guided ants' metaheuristic was proposed as a solution method. To deal with the imprecise or fuzzy nature of decision-makers' targeted goals in DLBP and help to find the criteria priority in multi-objective DLBP, fuzzy multi-objective programming and fuzzy AHP approaches were, respectively, proposed in Paksoy et al. (2013) and Avikal, Mishra, and Jain (2014). Tuncel, Zeid, and Kamarthi (2012) used a Monte Carlo-based reinforcement learning technique to solve the multi-objective DLBP under demand variations of the EOL products. A MIP based predictive-reactive approach to deal with task failures was also developed in Altekin and Akkan (2011) for DLBP aiming to maximise the profit generated by a disassembly line. Güngör and Gupta (2001) proposed a heuristic to deal with task failures caused by defective parts of the EOL product. The objective was to minimise the cost of defective parts.

In assembly line balancing problems, to take into account the variability of task processing times, several models were proposed. Usually, task times were assumed to be independent normally distributed random variables with known parameters. Binary linear programmes with disjoint probabilistic constraints for both straight and U-line balancing problems were proposed in Agpak and Gokcen (2007). A goal programming approach for minimising the mean station time and variance differences between stations was presented. Various methods were developed to solve stochastic U-type assembly line balancing problem: genetic algorithm (Baykasoğlu and Özbakır 2007), a two-phase hybrid heuristic (Chiang and Urban 2006), a beam search-based method (Erel, Sabuncuoglu, and Sekerci 2005), and an exact solution approach based on a piecewise linear integer programme with disjoint probabilistic constraints (Urban and Chiang 2006).

From the review above, it can be concluded that the study of processing time variability in both assembly and disassembly line balancing is limited to heuristic or metaheuristic approaches or to exact solution methods based on integer linear programmes with disjoint probabilistic constraints. In the case of disjoint probabilistic constraints, the decision-maker seeks an assignment of tasks to workstations where, for each station taken separately, the probability that the station time be smaller than cycle time should remain greater than a predetermined value. An important and more challenging issue is to study the case where the cycle time constraints are satisfied jointly; i.e. the probability that all station times be smaller than cycle time should remain greater than a fixed value. This case is considered in this paper which is organised as follows: a description and formulation of the studied stochastic DLBP are given in Section 2. The proposed solution method using convex piecewise linear approximation is presented in Section 3. The numerical experiments are given in Section 4 and Section 5 concludes the paper with future research directions.

2. Problem description and formulation

A single type of an EOL product is to be completely disassembled on a straight paced line. All received EOL products contain all their parts with no addition or removing of components. The optimisation problem deals with the assignment of the disassembly tasks I to an ordered sequence of workstations J satisfying the precedence relations among tasks and cycle time constraints under uncertainty of the task times. A disassembly task can be performed by any but only one workstation. Disassembly task processing times $\tilde{t}_i, i \in I$, are assumed to be independent random variables with known normal probability distributions, i.e. $\tilde{t}_i \sim \mathcal{N}(\mu_i, \sigma_i)$, $t_i > 0$, $\forall i \in I$; means μ_i, $\forall i \in I$ and standard deviations σ_i, $\forall i \in I$, are known values. Let $\tilde{t}_i = t_i(\tilde{\xi}), i \in I$, where $\tilde{\xi} = (\tilde{t}_1, \ldots, \tilde{t}_{|I|}) \in \Xi \subset \mathbb{R}_+^{|I|}$, is a random vector of the task times and Ξ is a set of a given probability space (Ξ, F, P) introduced by ξ.

Only a subset I^* of the set I is selected which constitutes tasks of the best disassembly alternative. The subset I^* and a number of workstations $|J^*| \leq |J|$ forming the line to be designed are known after optimisation; $|J|$ represents an upper bound on the number of stations of the line. The objective is the minimisation of the line cost including the opened workstation operation costs and additional costs of workstations handling hazardous parts of the EOL product. A task $i \in H \subset I$ is called hazardous if it generates a hazardous part; H represents the set of all hazardous tasks.

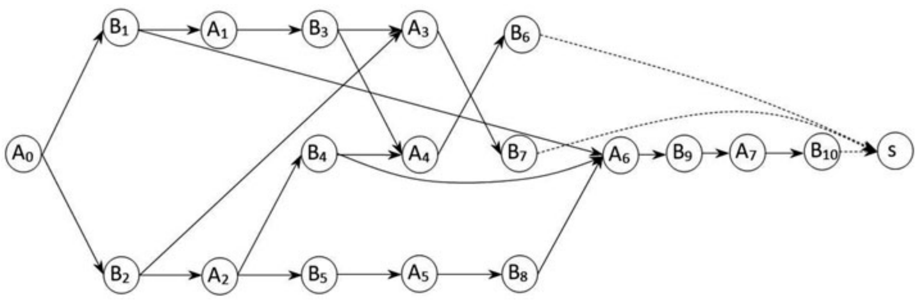

Figure 1. AND/OR graph of a hand light adapted from Özceylan, Paksoy, and Bektaş (2014).

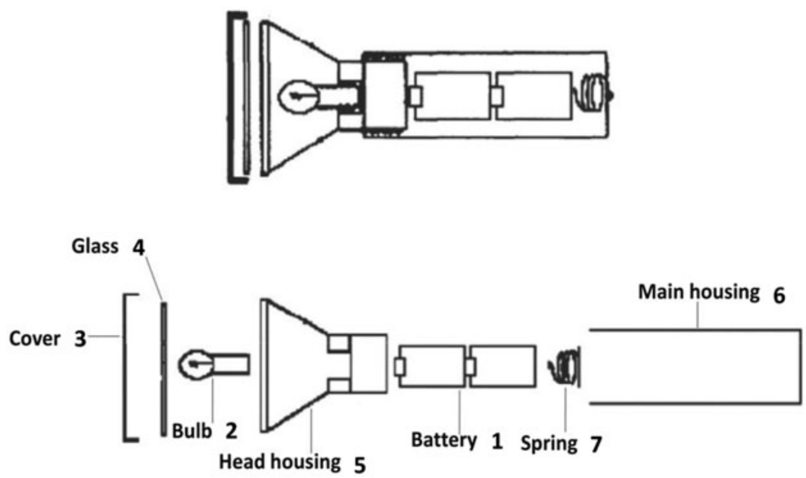

Figure 2. A hand light and its corresponding subassemblies (Tang et al. 2002).

An AND/OR graph is used to model the precedence relationships among tasks. Such a graph represents explicitly all the possible disassembly alternatives (Wang and Johnson 2000) of an EOL product and the precedence relationships among tasks and subassemblies. An example of such a graph is given in Figure 1. The corresponding EOL product (a hand light as an industrial product (Paksoy et al. 2013)) is illustrated in Figure 2.

The AND/OR graph of Figure 1 is constructed from the EOL product in Figure 2 as follows: each subassembly is modelled by a node labelled $A_k, k \in K$, and each node labelled $B_i, i \in I$, represents a disassembly task. For simplicity, subassemblies with one component are not represented. Two types of arcs define the precedence relations among subassemblies and disassembly tasks: AND and OR. If a disassembly task generates two subassemblies, or more, then, it is related to these subassemblies by AND-type arcs. If several concurrent tasks may be performed on a subassembly, this latter is related to these tasks by OR-type arcs. Table 1 below gives for each disassembly task the generated

Table 1. The hand light associated disassembly tasks and the corresponding generated subassemblies and/or components.

Task	Subassemblies	Components
1	3425/671	–
2	25,671/34	–
3	34/25	–
4	671/25	–
5	5671	2
6	–	2, 5
7	–	3, 4
8	671	5
9	67	1
10	–	6, 7

subassemblies and/or components. For instance, if task B_1 is performed, then two subassemblies are generated: subassembly '3425' represented by node A_1 in the AND/OR graph of Figure 1 and subassembly '671' represented by node A_6.

A sink node 's' is introduced and linked with dashed (dummy) arcs to all disassembly tasks with no successor. The dummy task s is used in the optimisation model to define the number of opened workstations $|J^*|$ of the disassembly line to be designed.

The developed mathematical model for this optimisation problem is given below.

Parameters

F_c fixed cost per operating a time unit of a workstation;
C_h additional fixed cost per a time unit of stations handling hazardous tasks;
C cycle time;
P_k set of indices for predecessors of $A_k, k \in K$;
S_k set of indices for successors of $A_k, k \in K$;
I_s index set of tasks preceding the dummy task s, i.e. $I_s = \{i | B_i \text{ precedes } s\}$

Decision variables

$$x_{ij} = \begin{cases} 1 & \text{if task } B_i \text{ is assigned to station } j, \\ 0 & \text{otherwise.} \end{cases}$$

$$x_{sj} = \begin{cases} 1 & \text{if the dummy task } s \text{ is assigned to station } j, \\ 0 & \text{otherwise.} \end{cases}$$

$$h_j = \begin{cases} 1 & \text{if a hazardous task is assigned to station } j, \\ 0 & \text{otherwise.} \end{cases}$$

Chance constrained binary programme

$$\min\left\{ C \times F_c \sum_{j \in J} j\, x_{sj} + C \times C_h \sum_{j \in J} h_j \right\} \quad \text{(CCBP)}$$

s.t.

$$\sum_{i \in S_0} \sum_{j \in J} x_{ij} = 1 \tag{1}$$

$$\sum_{j \in J} x_{ij} \le 1, \quad \forall i \in I \tag{2}$$

$$\sum_{i \in S_k} \sum_{j \in J} x_{ij} = \sum_{i \in P_k} \sum_{j \in J} x_{ij}, \quad \forall k \in K \backslash \{0\} \tag{3}$$

$$\sum_{i \in S_k} x_{iv} \le \sum_{i \in P_k} \sum_{j=1}^{v} x_{ij}, \quad \forall k \in K \backslash \{0\}, \forall v \in J \tag{4}$$

$$\sum_{j \in J} x_{sj} = 1 \tag{5}$$

$$\sum_{j \in J} j\, x_{ij} \le \sum_{j \in J} j\, x_{sj}, \quad \forall i \in I_s \tag{6}$$

$$h_j \geq x_{ij}, \quad \forall j \in J, \quad \forall i \in H \tag{7}$$

$$P\left(\sum_{i \in I} t_i(\tilde{\xi}) x_{ij} \leq C, \quad \forall j \in J\right) \geq 1 - \alpha \tag{8}$$

$$x_{sj}, x_{ij}, h_j \in \{0, 1\}, \quad \forall i \in I, \forall j \in J \tag{9}$$

The terms of the objective function represent the cost of operating opened workstations and the additional cost of handling hazardous parts. If the dummy task s is assigned to a workstation j, then j defines the number of opened workstations; i.e. $j = |J^*|$.

Constraint (1) imposes the selection of only one disassembly task to begin the disassembly process. Constraint set (2) indicates that a task is to be assigned to at most one workstation. Constraints (3) ensure that only one OR-successor is selected for each subassembly $A_k, k \in K$. Constraint set (4) defines the precedence relations among tasks: the selected successor is assigned to upper-indexed station (or the same) than the one to which the selected predecessor is assigned. Constraint (5) imposes the assignment of the dummy task s to one workstation. Constraints (6) ensure that all the disassembly tasks preceding s are assigned to lower or equal-indexed workstations than the one to which s is assigned. The constraints (7) ensure the value of h_j to be 1 if at least one hazardous task is assigned to a workstation j; $\sum_{j \in J} h_j$ defines the number of hazardous stations. The constraint (8) enforces the station operating time to remain within the cycle time, for all opened workstations, with a probability at least $(1 - \alpha)$ determined by the decision-maker. Finally, set (9) represents constraints on all possible values of the decision variables.

3. Solution method

In this section, lower and upper-bounding schemes for (CCBP) are proposed. The purpose of these bounds (one lower bound and two upper bounds) is to approximately solve the studied problem (CCBP). In such a case, for a given instance, if the lower bound value is equal to the upper one, then, an exact solution of (CCBP) is found; otherwise, the quality of a solution generated by the lower bound or the upper bound can be computed. Therefore, this developed decision tool is of a critical importance for a decision-maker since it permits him to, either compute an exact solution, or evaluate the quality of an approximate feasible solution. The development of these schemes is based on convex piecewise linear approximation and second-order cone programming. The convex piecewise linear approximation is used in order to approximate, linearly, non-linear functions and integrate them in a linear programme. Second-order programming is used to efficiently model and then solve the studied problem.

Approximation of (CCBP)

Sine disassembly task times $\tilde{t}_i, i \in I$ are assumed to be independent random variables with known normal probability distribution, then using the results of Cheng and Lisser (2012), we have:

$$P\left(\sum_{i \in I} t_i(\tilde{\xi}) x_{ij} \leq C, \quad \forall j \in J\right) \geq (1 - \alpha) \Leftrightarrow \begin{cases} P\left(\sum_{i \in I} t_i(\tilde{\xi}) x_{ij} \leq C\right) \geq (1 - \alpha)^{q_j}, \quad \forall j \in J \\ \sum_{j \in J} q_j = 1 \\ q_j \geq 0, \quad \forall j \in J \end{cases}$$

and

$$P\left(\sum_{i \in I} t_i(\tilde{\xi}) x_{ij} \leq C\right) \geq (1 - \alpha)^{q_j}, \quad \forall j \in J \Leftrightarrow \sum_{i \in I} \mu_i x_{ij} + \Phi^{-1}(\beta_j) \sqrt{\sum_{i \in I} \sigma_i^2 x_{ij}} \leq C, \quad \forall j \in J \tag{10}$$

where $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function $\Phi(\cdot)$ and $\beta_j = (1 - \alpha)^{q_j}, \quad \forall j \in J$.

Let $\boldsymbol{x}$ be a vector of the decision variables x_{sj}, x_{ij}, h_j and $X = \{\boldsymbol{x} | \text{constraints}(1) - (7\ ) \text{ and } (9) \text{ are satisfied}\}$. From (10), the programme (CCBP′) given below represents an equivalent version of problem (CCBP):

$$\min\left\{C\times F_c\sum_{j\in J} j\,x_{sj}+C\times C_h\sum_{j\in J} h_j\right\}\quad (\mathrm{CCBP}')$$

$$\text{s.t. } \boldsymbol{x}\in X$$

$$\sum_{i\in I}\mu_i x_{ij}+\Phi^{-1}\left((1-\alpha)^{q_j}\right)\sqrt{\sum_{i\in I}\sigma_i^2 x_{ij}}\le C,\quad \forall j\in J \tag{11}$$

$$\sum_{j\in J} q_j=1$$

$$q_j\ge 0,\quad \forall j\in J$$

Let $(s,\boldsymbol{r})\in\mathbb{R}\times\mathbb{R}^{\ell-1}$. The unit second-order convex cone of dimension ℓ is defined as

$$\mathcal{Q}_\ell=\left\{\begin{pmatrix}\boldsymbol{r}\\ s\end{pmatrix}\middle| s\ge||\boldsymbol{r}||\right\}$$

where $\|\cdot\|$ refers to the standard Euclidean norm. Constraint (11) is equivalently written as a second-order cone constraint of dimension $\ell=|I|+1$ as follows:

$$\sum_{i\in I}\mu_i x_{ij}+\Phi^{-1}\left((1-\alpha)^{q_j}\right)\sqrt{\sum_{i\in I}\sigma_i^2 x_{ij}}\le C,\quad \forall j\in J$$

$$\Leftrightarrow \boldsymbol{\mu}^T\,\boldsymbol{x}_j+\Phi^{-1}\left((1-\alpha)^{q_j}\right)\|\Sigma^{1/2}\boldsymbol{x}_j\|\le C,\quad \forall j\in J$$

Since $\Phi^{-1}\left((1-\alpha)^{q_j}\right)>0$ and $x_{ij}\in\{0,1\}\Leftrightarrow x_{ij}^2\in\{0,1\}$ (x_{ij} is a binary variable and α < 50%; α represents a risk and in general α ≤ 10%), then:

$$\boldsymbol{\mu}^T\,\boldsymbol{x}_j+\Phi^{-1}\left((1-\alpha)^{q_j}\right)\|\Sigma^{1/2}\boldsymbol{x}_j\|\le C,\quad \forall j\in J$$

$$\Leftrightarrow \|\Sigma^{1/2}\boldsymbol{x}_j\|\le\frac{1}{\Phi^{-1}\left((1-\alpha)^{q_j}\right)}\left(C-\boldsymbol{\mu}^T\,\boldsymbol{x}_j\right),\quad \forall j\in J$$

$$\Leftrightarrow\begin{pmatrix}\Sigma^{1/2}\\ \dfrac{-\boldsymbol{\mu}^T}{\Phi^{-1}\left((1-\alpha)^{q_j}\right)}\end{pmatrix}\boldsymbol{x}_j+\begin{pmatrix}\boldsymbol{0}\\ \dfrac{C}{\Phi^{-1}\left((1-\alpha)^{q_j}\right)}\end{pmatrix}\in\mathcal{Q}_{|I|+1}$$

where $\boldsymbol{\mu}=\left(\mu_1,\ldots,\mu_{|I|}\right)$, $\Sigma^{1/2}=\begin{pmatrix}\sigma_1 & & 0\\ & \ddots & \\ 0 & & \sigma_{|I|}\end{pmatrix}$ is a diagonal matrix and $\boldsymbol{x}_j=\left(x_{1j},\ldots,x_{|I|j}\right), j\in J$.

The Second-Order Cone mixed Binary Programme (SOCBP) given below represents an equivalent version of problem (CCBP').

$$\min\left\{C\times F_c\sum_{j\in J} j\,x_{sj}+C\times C_h\sum_{j\in J} h_j\right\}\quad (\mathrm{SOCBP})$$

$$\text{s.t. } \boldsymbol{x}\in X$$

$$s_j\le\frac{1}{\Phi^{-1}\left((1-\alpha)^{q_j}\right)}\left(C-\boldsymbol{\mu}^T\boldsymbol{x}_j\right),\quad \forall j\in J$$

$$r_{ij}\ge\sigma_i x_{ij},\quad \forall i\in I,\forall j\in J$$

$$s_j\ge\|\boldsymbol{r}_j\|,\quad \forall j\in J$$

$$\sum_{j\in J} q_j = 1$$

$$q_j \geq 0, \quad \forall j \in J$$

$$s_j, r_{ij} \geq 0, \quad \forall i \in I, \forall j \in J$$

where $s_j, r_{ij} \geq 0, \quad \forall i \in I, \forall j \in J$ are intermediate variables; $\boldsymbol{r}_j = \left(r_{1j}, \ldots, r_{|I|j}\right)^T, \ \forall j \in J$.

(a) Lower bounding scheme

A special case of Linear Programmes with joint Probabilistic Constraints (LPPC) has been studied in Cheng and Lisser (2012). The coefficients of the matrix of the probabilistic constraints were assumed to be normally distributed and the vector rows to be independent. In our case, coefficients are the task times and each row vector is the task time of a workstation. It is clear that task times of workstations are independent of each other since task times are mutually independent. Although decision variables of (LPPC) studied in Cheng and Lisser (2012) are positive continuous, the main results remain valid for (CCBP) with discrete decision variables.

Convex piecewise tangent approximation of $\Phi^{-1}((1-\alpha)^q)$

Let $1-\alpha = \overline{\alpha}$. The function $\Phi^{-1}(\overline{\alpha}^q), q \in]0,1]$ is approximated using the first-order Taylor series and the input data $\left(q_j, \Phi^{-1}(\overline{\alpha}^{q_j})\right), j = 1, \ldots, m$; $\left(q_j, \Phi^{-1}(\overline{\alpha}^{q_j})\right)$ is a tangent point of the curve of the function $\Phi^{-1}(\overline{\alpha}^q), q \in]0,1]$. Assume without loss of generality that $q_1 < q_2 < \ldots < q_m$. A piecewise tangent linear approximation of $\Phi^{-1}(\overline{\alpha}^q), q \in]0,1]$, is given as follows:

$$g(q) = \max_{j=1,\ldots,m} \left\{a_j + b_j q\right\}, \quad q \in]0,1] \tag{12}$$

$$b_j = \left(\Phi^{-1}\right)^{(1)}(\overline{\alpha}^{q_j}) \cdot \overline{\alpha}^{q_j} \ln(\overline{\alpha}), \quad j = 1, \ldots, m$$

$$a_j = \Phi^{-1}(\overline{\alpha}^{q_j}) - b_j \cdot q_j, \quad j = 1, \ldots, m$$

$$\left(\Phi^{-1}\right)^{(1)}(\overline{\alpha}^{q_j}) = \frac{1}{f\left(\Phi^{-1}(\overline{\alpha}^{q_j})\right)}, \quad j = 1, \ldots, m$$

where f represents here the standard normal probability density function. Using approximation (12), programme (SOCLB) below is an approximation of (CCBP).

$$\min \left\{ C \times F_c \sum_{j\in J} j\, x_{sj} + C \times C_h \sum_{j\in J} h_j \right\} \quad \text{(SOCLB)}$$

$$\text{s.t. } \boldsymbol{x} \in X$$

$$s_j \leq C - \boldsymbol{\mu}^T \boldsymbol{x}_j, \quad \forall j \in J$$

$$r_{ij} \geq \sigma_i x_{ij}, \quad \forall i \in I, \forall j \in J$$

$$s_j \geq \|\boldsymbol{r}_j\|, \quad \forall j \in J$$

$$z_{ij} \geq a_k x_{ij} + b_k y_{ij}, \quad \forall i \in I, \forall j \in J, k = 1, \ldots, m$$

$$\sum_{j\in J} y_{ij} = \sum_{j\in J} o_{ij}, \quad \forall i \in I$$

$$o_{ij} \leq x_{ij}, \quad \forall i \in I, \forall j \in J$$

$$o_{ij} \leq q_j, \quad \forall i \in I, \forall j \in J$$

$$q_j + x_{ij} \leq 1 + o_{ij}, \quad \forall i \in I, \forall j \in J$$

$$\sum_{j\in J} q_j = 1$$

$$s_j, q_j, o_{ij}, r_{ij}, y_{ij}, z_{ij} \geq 0, \quad \forall i \in I, \forall j \in J$$

In addition, the optimal value of (SOCLB) is a lower bound of (CCBP); this approximation is based on the lower bound given in Cheng and Lisser (2012) for continuous decision variables.

(b) Upper bounding schemes

In this subsection, two approximations of (CCBP) will be developed, the value of each approximation represents an upper bound. These two approximations are based on Bonferroni's inequality (Galambos 1997) and convex piecewise linear approximation of $\Phi^{-1}(\overline{\alpha}^q), q \in]0,1]$.

Convex piecewise linear approximation of $\Phi^{-1}(\overline{\alpha}^q)$

Since $\Phi^{-1}(\overline{\alpha}^q), q \in]0,1]$ is a convex function, then for the input data $\left(q_j, \Phi^{-1}(\overline{\alpha}^{q_j})\right), j = 1, \ldots, m$, where $\left(q_j, \Phi^{-1}(\overline{\alpha}^{q_j})\right)$ is an interpolation point, a convex piecewise linear function g of $\Phi^{-1}(\overline{\alpha}^q)$ is defined by:

$$g(q) = \max_{j=1,\ldots,m-1} \left\{a_j + b_j q\right\}, \quad q \in]0,1] \tag{13}$$

$$a_j = \frac{q_{j+1}\Phi^{-1}(\overline{\alpha}^{q_j}) - q_j \Phi^{-1}(\overline{\alpha}^{q_{j+1}})}{u_{j+1} - u_j}, \quad j = 1, \ldots, m-1$$

$$b_j = \frac{\Phi^{-1}(\overline{\alpha}^{q_{j+1}}) - \Phi^{-1}(\overline{\alpha}^{q_j})}{q_{j+1} - q_j}, \quad j = 1, \ldots, m-1$$

$$q_1 < q_2 < \ldots < q_m, q_j \in]0,1], \quad j = 1, \ldots, m$$

The first upper bound approximation (SOCUB1) of (CCBP) is defined by replacing (a_k, b_k) values in (SOCLB) by their values defined in (13). This approximation is based on the one given in Cheng and Lisser (2012) for continuous decision variables. It defines an upper bound value of (CCBP) if

$$\gamma = \prod_{j\in J} \Phi\left(\frac{C - \boldsymbol{\mu}^T \boldsymbol{x}_j}{\|\Sigma^{1/2} \boldsymbol{x}_j\|}\right) \geq 1 - \alpha$$

The second upper bound approximation (SOCUB2) of the addressed problem is known for problems with joint probabilistic constraints and is based on Bonferroni's inequality:

$$\min\left\{C \times F_c \sum_{j\in J} j x_{sj} + C \times C_h \sum_{j\in J} h_j\right\} \quad \text{(SOCUB2)}$$

$$\text{s.t. } \boldsymbol{x} \in X$$

$$s_j \leq \frac{1}{\Phi^{-1}(1-\alpha_j)}\left(C - \boldsymbol{\mu}^T \boldsymbol{x}_j\right), \quad \forall j \in J$$

$$r_{ij} \geq \sigma_i x_{ij}, \quad \forall i \in I, \forall j \in J$$

$$s_j \geq \|\boldsymbol{r_j}\|, \quad \forall \boldsymbol{j} \in \boldsymbol{J}$$

$$s_j, r_{ij} \geq 0, \quad \forall i \in I, \forall j \in J$$

Note that α_j, $j \in J$ are not decision variables but parameters verifying $\sum_{j\in J} \alpha_j = \alpha$.

4. Numerical experiments

The developed lower and upper bounding schemes were implemented in MS VC++ 2008 and Cplex 12.5 was used to solve seven instances on a PC with Pentium(R) Dual-Core CPU T4500, 2.30 GHz and 3 GB RAM. These used instances available in the literature contain process alternatives for disassembly of different EOL products. The names of these problem instances are respectively composed of the first letters of authors' names and the year of publication, i.e. BBD13a represents a compass (Bentaha, Battaïa, and Dolgui 2013a), BBD13b is a piston and connecting rod (Bentaha, Battaia, and Dolgui 2013b), KSE09 is a sample product created by the authors Koc, Sabuncuoglu, and Erel (2009), L99a and L99b are, respectively, a radio set and a ball-point pen (Lamberta 1999), MJKL11 from (Ma et al. 2011) is an automatic pencil and TZC02 from (Tang et al. 2002) is a hand light. Instance TZC02 corresponds to the graph of Figure 1. The input data for each problem instance is given in Table 2.

The columns 'AND-relations' report the number of disassembly tasks with no successor in subcolumn '0', with one AND-type arc in subcolumn '1' and with two AND-type arcs in subcolumn '2'. The column 'arcs' gives the total number of AND- and OR-type arcs.

Table 3 reports the optimisation results of the studied instances using the proposed lower and upper bounds. The number of points for convex piecewise linear approximation was fixed at 15, α = 5, 25% of the disassembly tasks were assumed to be hazardous and the first point of input data for piecewise approximation was 0.0001; all sampled points were equidistant. The remaining parameters were randomly generated. Columns 'LB', 'UB_1' and 'UB_2' report, respectively, the lower (SOCLB) and upper (SOCUB1), (SOCUB2) bound values. Column 'Gap' reports the optimality gap value $\frac{UB-LB}{LB}$, columns '$|I^*|$', 'h-stat.' and 'CPU time' report, respectively, the number of selected tasks of the selected alternative, the number of hazardous workstations with the corresponding rank, in the line, for each hazardous station and the resolution time in seconds. The second upper bound UB_2 was computed for $\alpha_j = \frac{\alpha}{|J|},\ \forall j \in J$.

The results of Table 3 show that, for each solved instance, the upper and lower bound values are equal. As mentioned earlier, this means that all instances are solved to optimality. In Table 3(a), the values of parameter γ are greater than 95%. Since α = 5%, then each value of UB_1 gives an upper bound for (CCBP). The CPU time of UB_2 for each processed instance in Table 3(b) is better than the CPU time of UB_1 of the same instance in Table 3(a). The conclusion is that, in our case, UB_2 is preferred to UB_1. Note that the returned best solution of instance L99b with UB_1 is different from the returned optimal solution of this same instance with UB_2.

Table 4 below aims to analyse the impact on the objective function value of the number of points 'Pts_{nbr}' or segments of the piecewise linear functions used to approximate the non-linear ones of the two problems (SOCLB) and (SOCUB1). The number of segments corresponds to the number of points minus one in the case of piecewise linear approximation and to the number of points in the case of tangent piecewise linear approximation.

As shown in Table 4 for the processed instances with the defined parameters, there is no impact of the number of segments of the approximate piecewise linear functions on the optimal values of the objective functions. The optimal values of the objective functions of all instances were reached with approximate piecewise linear functions composed of four segments.

Figure 3 bellow details and illustrates the returned optimal solution of the hand light product, instance TZC02, with (SOCLB). The selected disassembly alternative highlighted with bold arcs is composed of six tasks. These tasks define the optimal disassembly process for the considered EOL hand light. The selected tasks are assigned to three workstations which constitute the stations of the disassembly line, see Figure 3.

Table 2. Problem instances.

	$\|I\|$	$\|K\|$	arcs	AND relations			$\|J\|$	C
				0	1	2		
BBD13a	10	5	18	3	6	1	3	0.61
BBD13b	25	11	49	4	18	3	4	120
KSE09	23	13	47	4	14	5	6	20
L99a	30	18	60	2	26	2	9	50
L99b	20	13	41	5	9	6	9	10
MJKL11	37	22	76	4	27	6	10	40
TZC02	10	7	21	3	3	4	6	90

Table 3. Obtained results: (a) the lower bound and the first upper bound; (b) the lower bound and the second upper bound.

	LB	$\|I^*\|$	$\|J^*\|$	h-stat.	CPU time	UB_1	$\|I^*\|$	$\|J^*\|$	h-stat.	CPU time	γ%	Gap%
(a)												
BBD13a	8.54	3	2	–	0.14	8.54	3	2	–	0.23	99.63	0
BBD13b	1680	4	2	–	26.38	1680	4	2	–	29.73	99.08	0
KSE09	1160	6	3	(1, 3)	2.70	1160	6	3	(1, 2)	2.48	99.23	0
L99a	850	9	3	(1, 3)	3.56	850	9	3	(1, 1)	38.52	99.96	0
L99b	150	9	3	–	2.26	150	9	3	–	1.51	99.50	0
MJKL11	720	7	3	(1, 2)	6.97	720	7	3	(1, 2)	7.18	99.98	0
TZC02	990	6	3	(1, 2)	0.70	990	6	3	(1, 1)	0.78	99.13	0
(b)												
	LB	$\|I^*\|$	$\|J^*\|$	h-stat.	CPU time	UB_2	$\|I^*\|$	$\|J^*\|$	h-stat.	CPU time	Gap%	
BBD13a	8.54	3	2	–	0.14	8.54	3	2	–	0.05	0	
BBD13b	1680	4	2	–	26.38	1680	4	2	–	0.09	0	
KSE09	1160	6	3	(1, 3)	2.70	1160	6	3	(1, 3)	0.17	0	
L99a	850	9	3	(1, 3)	3.56	850	9	3	(1, 2)	0.66	0	
L99b	150	9	3	–	2.26	150	7	3	–	0.67	0	
MJKL11	720	7	3	(1, 2)	6.97	720	7	3	(1, 2)	2.54	0	
TZC02	990	6	3	(1, 2)	0.70	990	6	3	(1, 1)	0.09	0	

Table 4. Obtained results for main upper and lower bounds: changing the accuracy of the convex piecewise linear approximation.

	Pts_{nbr}	LB	CPU time	UB_1	CPU time	γ%	Gap%
BBD13a	5	8.54	0.08	8.54	0.14	99.63	0
	10	8.54	0.11	8.54	0.09	99.63	0
	20	8.54	0.11	8.54	0.17	99.63	0
BBD13b	5	1680	1.62	1680	11.95	99.08	0
	10	1680	27.69	1680	18.67	99.08	0
	20	1680	42.56	1680	42.28	99.08	0
KSE09	5	1160	1.31	1160	1.87	99.23	0
	10	1160	1.78	1160	1.59	97.25	0
	20	1160	4.13	1160	3.62	97.25	0
L99a	5	850	1.84	850	2.39	96.92	0
	10	850	2.56	850	3.82	99.96	0
	20	850	4.19	850	54.23	99.66	0
L99b	5	150	0.48	150	0.55	98.87	0
	10	150	0.89	150	0.73	97.59	0
	20	150	0.89	150	0.94	98.39	0
MJKL11	5	720	6.46	720	6.89	99.98	0
	10	720	8.88	720	4.23	99.98	0
	20	720	16.65	720	169.85	99.78	0
TZC02	5	990	0.39	990	0.36	99.13	0
	10	990	0.37	990	0.55	98.81	0
	20	990	0.67	990	0.76	98.81	0

The disassembly task B_7 is hazardous, i.e. task B_7 generates hazardous parts of the EOL product after its execution. Task B_7 is assigned to the second workstation (a hazardous station with rank 2) of the designed line. Note that the objective function of the studied problem enforces the assignment of the hazardous tasks (under constraints) to the first workstation or, if not possible, to the closest following workstation. Thus, the negative impact of hazardous components or material on operators, disassembly tools or machines and handling systems would be reduced and additional costs would be avoided.

Although the modelling process was defined using a simple hand light as EOL product, the developed methodology can be easily adapted for real life cases like End of Life Vehicles (ELV) or Waste Electrical and Electronic Equipment (WEEE). In addition, the proposed solution method can be applied efficiently.

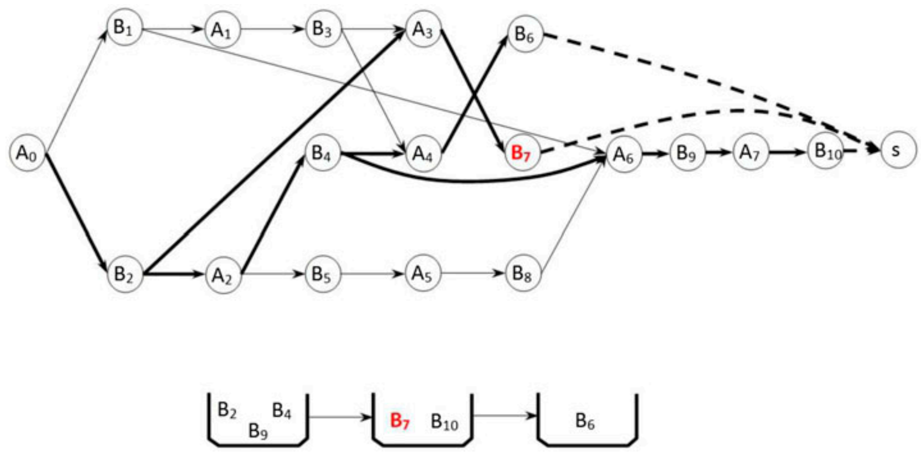

Figure 3. The selected (optimal) disassembly alternative and the corresponding task assignment to the determined workstations of the instance TZC02.

5. Conclusion

In this paper, a cost-oriented disassembly line design problem was studied under uncertainty. Task processing times were assumed to be random variables with known normal probability distributions. The case of presence of hazardous parts was integrated. Cycle time constraints were to be jointly respected with at least a certain probability level fixed by the decision-maker. To solve the addressed problem with an assessment of the solution quality, a mixed binary mathematical programme with joint probabilistic constraints along with one lower bound and two upper bounds were proposed. The developed lower and upper bounding schemes were based on second-order cone programming and convex piecewise linear approximation. These schemes define a basis for a decision aiding tool of critical importance for decision-makers. In fact, with these schemes, a decision-maker can compute a lower bound value of the line operation cost to be designed and an upper one, which allows him to choose a best disassembly process for an EOL product. Moreover, through its ability to assess the disassembly cost for a product at the end of life, such a tool may help to take the decisions not only for EOL options (landfill, combustion, recycle, refurbish, reuse, etc.) but even at the product design stage.

The developed models were evaluated using a set of instances (EOL products) from the literature. All instances were solved to optimality. The numerical results have shown that the upper bound based on Bonferroni's inequality solves the problem instances faster than the upper bound based on approximation using convex piecewise linear functions.

The presented modelling process can be easily adapted for real industrial cases like ELV or WEEE. In order to consider such cases in the developed solution method, a cutting-plane approach will be investigated and compared to the default solution method of the Cplex solver.

References

Agpak, K., and H. Gokcen. 2007. "A Chance-constrained Approach to Stochastic Line Balancing Problem." *European Journal of Operational Research* 180: 1098–1115.

Agrawal, S., and M. K. Tiwari. 2008. "A Collaborative Ant Colony Algorithm to Stochastic Mixed-model U-shaped Disassembly Line Balancing and Sequencing Problem." *International Journal of Production Research* 46 (6): 1405–1429.

Altekin, F. T., and C. Akkan. 2011. "Task-failure-driven Rebalancing of Disassembly Lines." *International Journal of Production Research* 50: 4955–4976.

Avikal, S., P. K. Mishra, and R. Jain. 2014. "A Fuzzy AHP and PROMETHEE Method-based Heuristic for Disassembly Line Balancing Problems." *International Journal of Production Research* 52: 1306–1317.

Aydemir-Karadag, A., and O. Turkbey. 2013. "Multi-objective Optimization of Stochastic Disassembly Line Balancing with Station Paralleling." *Computers & Industrial Engineering* 65 (3): 413–425.

Battaïa, O., and A. Dolgui. 2013. "A Taxonomy of Line Balancing Problems and Their Solutionapproaches." *International Journal of Production Economics* 142 (2): 259–277.

Baykasoğlu, A., and L. Özbakır. 2007. "Stochastic U-line Balancing Using Genetic Algorithms." *The International Journal of Advanced Manufacturing Technology* 32: 139–147.

Bentaha, M. L., O. Battaïa, and A. Dolgui. 2013a. "A Stochastic Formulation of the Disassembly Line Balancing Problem." In *Advances in Production Management Systems Competitive Manufacturing for Innovative Products and Services*, edited by C. Emmanouilidis, M. Taisch, and D. Kiritsis, 397–404. Rhodes: Springer.

Bentaha, M. L., O. Battaia, and A. Dolgui. 2013b. "Chance Constrained Programming Model for Stochastic Profit-Oriented Disassembly Line Balancing in the Presence of Hazardous Parts." In *Advances in Production Management Systems Sustainable Production and Service Supply Chains*, edited by V. Prabhu, M. Taisch, and D. Kiritsis, 103–110. State College, PA: Springer.

Bentaha, M. L., O. Battaïa, and A. Dolgui. 2014a. "A Sample Average Approximation Method for Disassembly Line Balancing Problem under Uncertainty." *Computers & Operations Research* 51: 111–122.

Bentaha, M. L., O. Battaïa, and A. Dolgui. 2014b. "Disassembly Line Balancing and Sequencing under Uncertainty." *Procedia CIRP* 15: 239–244.

Bentaha, M. L., O. Battaïa, and A. Dolgui. 2014c. "Lagrangian Relaxation for Stochastic Disassembly Line Balancing Problem." *Procedia CIRP* 17: 56–60.

Bentaha, M. L., O. Battaïa, A. Dolgui, and S. J. Hu. 2014. "Dealing with Uncertainty in Disassembly Line Design." *CIRP Annals – Manufacturing Technology* 63 (1): 21–24.

Cheng, J., and A. Lisser. 2012. "A Second-order Cone Programming Approach for Linear Programs with Joint Probabilistic Constraints." *Operations Research Letters* 40 (5): 325–328.

Chiang, W.-C., and T. L. Urban. 2006. "The Stochastic U-line Balancing Problem: A Heuristic Procedure." *European Journal of Operational Research* 175: 1767–1781.

Erel, E., I. Sabuncuoglu, and H. Sekerci. 2005. "Stochastic Assembly Line Balancing Using Beam Search." *International Journal of Production Research* 43 (7): 1411–1426.

Galambos, J. 1997. "Bonferroni Inequalities." *The Annals of Probability* 5 (4): 577–581.

Güngör, A., and S. M. Gupta. 1999. "Disassembly Line Balancing." *Proceedings of the Annual Meeting of the Northeast Decision Sciences Institute*, 193–195. Newport, RI.

Güngör, A., and S. M. Gupta. 2001. "A Solution Approach to the Disassembly Line Balancing Problem in the Presence of Task Failures." *International Journal of Production Research* 39 (7): 1427–1467.

Güngör, A., and S. M. Gupta. 2002. "Disassembly Line in Product Recovery." *International Journal of Production Research* 40 (11): 2569–2589.

Ilgin, M. A., and S. M. Gupta. 2010. "Environmentally Conscious Manufacturing and Product Recovery (ECMPRO): a Review of the State of the Art." *Journal of Environmental Management* 91 (3): 563–591.

Koc, A., I. Sabuncuoglu, and E. Erel. 2009. "Two Exact Formulations for Disassembly Line Balancing Problems with Task Precedence Diagram Construction Using an aND/oR Graph." *IIE Transactions* 41 (10): 866–881.

Lamberta, J. D. 1999. "Linear Programming in Disassembly/Clustering Sequence Generation." *Computers & Industrial Engineering* 36 (4): 723–738.

Ma, Y.-S., H.-B. Jun, H.-W. Kim, and D.-H. Lee. 2011. "Disassembly Process Planning Algorithms for End-of-Life Product Recovery and Environmentally Conscious Disposal." *International Journal of Production Research* 49 (23): 7007–7027.

McGovern, S., and S. A. Gupta. 2007. "Balancing Method and Genetic Algorithm for Disassembly Line Balancing." *European Journal of Operational Research* 179 (3): 692–708.

Meacham, A., R. Uzsoy, and U. Venkatadri. 1999. "Optimal Disassembly Configurations for Single and Multiple Products." *Journal of Manufacturing Systems* 18 (5): 311–322.

Özceylan, E., T. Paksoy, and T. Bektaş. 2014. "Modeling and Optimizing the Integrated Problem of Closed-loop Supply Chain Network Design and Disassembly Line Balancing." *Transportation Research Part E: Logistics and Transportation Review* 61: 142–164.

Paksoy, T., A. Güngör, E. Özceylan, and A. Hancilar. 2013. "Mixed Model Disassembly Line Balancing Problem with Fuzzy Goals." *International Journal of Production Research* 51: 6082–6096.

Tang, Y., M. Zhou, E. Zussman, and R. Caudill. 2002. "Disassembly Modeling, Planning, and Application." *Journal of Manufacturing Systems* 21 (3): 200–217.

Tripathi, M., S. Agrawal, M. K. Pandey, R. Shankar, and M. K. Tiwari. 2009. "Real World Disassembly Modeling and Sequencing Problem: Optimization by Algorithm of Self-Guided Ants (ASGA)." *Robotics and Computer-Integrated Manufacturing* 25 (3): 483–496.

Tuncel, E., A. Zeid, and S. Kamarthi. 2012. "Solving Large Scale Disassembly Line Balancing Problem with Uncertainty Using Reinforcement Learning." *Journal of Intelligent Manufacturing* 2: 1–13.

Urban, T. L., and W.-C. Chiang. 2006. "An Optimal Piecewise-linear Program for the U-line Balancing Problem with Stochastic Task times." *European Journal of Operational Research* 168: 771–782.

Wang, M. H., and M. R. Johnson. 2000. "Disassembly Sequence Generation Using a Neural Network Approach." *Journal of Manufacturing Systems* 19 (2): 73–82.