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Numerical modeling of local capillary effects in porous media as a pressure discontinuity acting on the interface of a transient bi-fluid flow

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Abstract

Transient flows through porous media can be controlled by local capillary forces. In an attempt to ease the representation of these complex multi-scale flows, this article presents a new numerical approach to account for these local forces, viewed as a global pressure discontinuity acting in bi-fluid flows through smeared-out porous media. A finite element discretization of the Darcy's equations is considered and a pressure enriched space is locally introduced at the fluid interface in order to capture the pressure discontinuity. Then, a Variational Multiscale Stabilization (*VMS*) method is selected to take into account the subgrid effects on the finite element solution and hence ensure the consistency of the finite element formulation. The fluid front is represented by a level set function, convected with the fluid velocity thanks to a finite element scheme stabilized with a Streamline-Upwind/Petrov-Galerkin (*SUPG*) method. Both convergence and implementation are first validated with the Method of Manufactured Solution (*MMS*) and the model shows a good convergence. Second, a comparison with experimental measurements in the case of capillary wicking of water into carbon reinforcements shows a very good correlation between experimental and numerical results.

Keywords: Capillary stress, Darcy's equations, stabilized finite element

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method, discontinuous pressure, capillary wicking

1. Introduction

Capillary effects define the ability of a liquid to maintain contact with a solid. They are localized and play a key role in the description of liquid flows in porous media. Their best visualization is the spontaneous wicking mechanisms, where the liquid flows without the assistance of any external force. Such phenomena are related in one hand to the surface tension between the liquid and the surrounding media such as air, and on the other hand, to the liquid-solid and air-solid surface energies.

During flows in a porous medium, micro- and macro-voids may develop following the competition between viscous and capillary effects. This work focuses on the capillary effects occurring in porous media such as fibrous reinforcements during infusion process, especially Liquid Resin Infusion (LRI) process. [Indeed, capillary effects are traditionally neglected in the flow simulation during high pressure composite manufacturing processes.](#) Meanwhile, experimental studies have shown that the [capillary stress](#) resulting from the interaction of carbon reinforcements and liquid, such as water or epoxy resin, can reach a value of 0.3 – 0.4 bar [in quasi-UD fabrics \[62, 63\], at the fibre scale \[76\] or in carbon woven fabrics \[5\]](#). This value represents approximately one third of the 1 bar driving force available in the LRI process, which is too significant to be neglected and even permits to manufacture composite parts that could not be without its contribution. Consequently, the aim of this paper is to introduce these local capillary effects, in order to assess their influence on the filling stage scenarios at the scale of composite parts [46, 70].

Indeed, following the multi-scale nature of high performance composites, the study can be conducted at three different scales as shown in many studies [15, 22, 36, 60]: at the fiber or microscopic scale ($\sim 10^{-6}\text{m}$), at the tow or mesoscopic scale ($\sim 10^{-3}\text{m}$) and at the process or macroscopic scale ($\sim 10^{-1}\text{m}$). At microscopic scale, numerical modeling of the capillary rise is well-documented [11, 13, 52, 71]. Capillary effects are accounted for into Navier-Stokes or Stokes equations by the mean of the surface tensions between the three phases (solid-liquid-air) [4, 20, 24]. At mesoscopic scale, capillary effects are taken into account through the micro-diffusion within fiber tows. Generally, unsaturated flow models are adopted to describe this

35 problem. A sink term which depends on the capillary number is incorporated
36 in the governing equations to model the capillary effects [65, 67, 72], **but**
37 **standing only for isotropic representations**. In many research areas such as
38 ground water infiltration or oil recovery, the capillary pressure is related to
39 the saturation according to different analytical parameterizations such as the
40 ones proposed by Van Genuchten, Stauffer, Kalaydjian, Hassanizadeh, Gray
41 and Bareblatt [38, 47, 50].

42 This approach has been extended to composite manufacturing fields in
43 order to simulate the filling stage and to assess void formation [17, 54, 59].
44 **Capillary effects are usually represented through the introduction a single**
45 **capillary pressure, although capillary effects do come from local mechanisms**
46 **related to both orthotropic micro(meso)-structure architecture and surface**
47 **tensions. Dedicated studies concentrate on these local phenomena [24] which**
48 **are, for the moment, out of reach in tractable models at the structure scale.**
49 **An alternative way of introducing these 3D effects at an upper scale is to**
50 **consider their effect as a capillary stress tensor (3D representation) acting**
51 **on the fluid-gas interface in a slug-flow approach, *i.e.* no saturation zone is**
52 **considered at this scale. Accounting properly for the 3D pressure disconti-**
53 **nuity in a numerical approach will allow to complete full models of infusion**
54 **processes at the structure scale [15] including coupling with the wet/dry pre-**
55 **form mechanical response, as well as to model dual-scale flows at the tow**
56 **scale [75] provided an equivalent homogeneous medium can be used to rep-**
57 **resent populations of fibres. Notwithstanding any local effects related either**
58 **to velocity or pressure fields which are of utmost interest to represent lo-**
59 **cal physical changes, such as for shear-rate dependant fluids, void creations**
60 **issues, or fluid pressure acting in wet preforms for coupling issues [22, 15].**

61 In this work, an innovative macroscopic approach is adopted. The cap-
62 illary action is described by a capillary stress tensor acting on the liquid-air
63 interface. The subsequent jump of the pressure field at the flow front is taken
64 into account numerically in the weak formulation of Darcy's equations. Those
65 equations, established in a velocity-pressure mixed form, are solved using a
66 Finite Element Method (FEM). Both velocity and pressure are approximated
67 by continuous and linear fields. According to the Brezzi - Babuška theory,
68 such an approximation is not stable. This issue is overcome by stabilizing the
69 Finite Element (FE) formulation thanks to the Variational MultiScale (VMS)
70 framework introduced by Hughes [39, 40] and extensively used and studied
71 by Badia & Codina to stabilize Stokes', Darcy's and Maxwell's equations in
72 a unified setting [7, 8, 9, 10]. However, a special attention is mandatory to

73 accurately capture the pressure discontinuity across the flow front, which is
74 described here with a level-set method [57, 58, 69].

75 The literature provides several techniques to capture this local phenomenon.
76 For instance, the Extended-Finite Element Method (X-FEM) [21, 32, 45] con-
77 sists in enriching the pressure approximation space by discontinuous func-
78 tions. This enrichment, which is not localized in the mesh elements, pro-
79 vides additional degrees of freedom, resulting in some computational issues
80 when the discontinuity is moving (the mesh, and consequently the global
81 "stiffness" matrix, need to be updated). A discontinuous Galerkin formu-
82 lation [1, 12, 48, 56] can be another way of dealing with singular forces at
83 the interface. As the continuity between elements is weakly imposed, it al-
84 lows the solution to be discontinuous: each element has its own degrees of
85 freedom and is connected to its neighboring by numerical fluxes. However,
86 in our simulations, the interface does not necessarily correspond to edges of
87 elements, but can cut these elements. In this work, the jump of pressure
88 field is captured using the technique developed by Ausas *et al.* [6], which
89 consists in a local enrichment of the pressure space by discontinuous func-
90 tions. Unlike the X-FEM approach, the corresponding additional degrees of
91 freedom are local to an interface element, and can therefore be eliminated at
92 the elementary level before the final assembly.

93 The rest of this article is divided into four parts. Section 2 focuses on
94 the mathematical description of the fluid flow problem, and the finite ele-
95 ment strategy implemented. Section 3 describes the level-set method used
96 to capture the fluid front, *i.e.* the interface across which pressure is discon-
97 tinuous. An error analysis is given in Section 4 to assess the accuracy of the
98 numerical developments. Finally, Section 5 compares simulation and exper-
99 imental results for water capillary wicking in carbon reinforcements. [Also,](#)
100 [a 3D simulation of the resin flow through an orthotropic stiffener is carried](#)
101 [out.](#)

102 2. Fluid flow problem

103 2.1. Physical and mathematical description

104 Let Ω be a region of \mathbb{R}^d (with $d = 2, 3$ the spatial dimension) bounded
105 by $\partial\Omega$ (see Fig. 1). Ω represents a porous medium, the fibrous preform in
106 our context, considered as an equivalent homogeneous orthotropic medium
107 characterized by a porosity ϕ and [a saturated permeability \$\mathbf{K}\$ independent](#)
108 [on the fluid.](#) The permeability is a measure indicating the capacity of the

2.1 Physical and mathematical description 2 FLUID FLOW PROBLEM

109 material to allow fluids to pass through it. In a realistic description, the fi-
 110 brous reinforcement is anisotropic and \mathbf{K} is a symmetric tensor. The domain
 111 Ω is filled with two immiscible, Newtonian and incompressible fluids: a liquid
 112 of viscosity μ_l , occupying the subdomain Ω_l and a surrounding medium (for
 113 instance, the air) of viscosity $\mu_a \ll \mu_l$, occupying Ω_a . Hence: $\Omega = \Omega_l \cup \Omega_a$.
 114 The interface is denoted $\Gamma_{l/a}$: $\Gamma_{l/a} = \partial\Omega_l \cap \partial\Omega_a$.

115 The domain boundary $\partial\Omega$ is divided into two types of boundary: $\partial\Omega_D$
 116 and $\partial\Omega_N$ such that $\partial\Omega = \partial\Omega_D \cup \partial\Omega_N$ and $\partial\Omega_D \cap \partial\Omega_N = \emptyset$, where respectively
 117 Dirichlet and Neumann boundary conditions are prescribed.

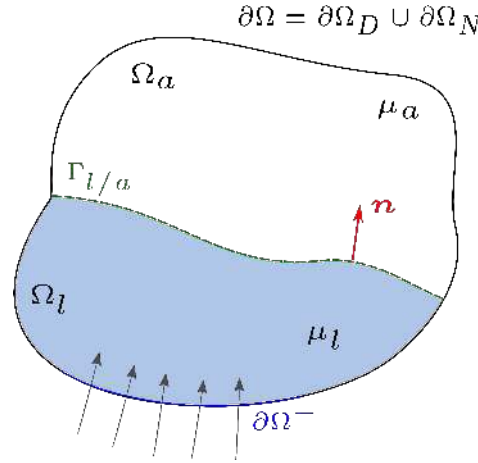


Figure 1: 2D-description of the domain Ω .

118 Darcy's equations [28] express the flow velocity \mathbf{v} at the scale of the
 119 homogeneous equivalent medium, *i.e.* the superficial fluid velocity, with
 120 respect to the pressure gradient ∇p and two parameters: the fluid viscosity
 121 and the permeability (Eq. 1). The mass conservation is introduced through
 122 the divergence of the velocity \mathbf{v} (Eq. 2). Hence, the governing equations are

$$\mu \mathbf{K}^{-1} \mathbf{v} + \nabla p = \mathbf{f}, \quad \mu = \begin{cases} \mu_l & \text{in } \Omega_l \\ \mu_a & \text{in } \Omega_a \end{cases} \quad (1)$$

$$\nabla \cdot \mathbf{v} = h \quad (2)$$

123 with \mathbf{f} the external forces and h a source/sink term, equal to zero when
 124 the fluids are assumed to be incompressible. Assuming, for a while, that the
 125 axes of the orthonormal coordinate system $\{x, y, z\}$ coincide with the normals
 126 to the three symmetry planes of the orthotropic material, the permeability

127 tensor writes in this eigen-system:

$$\mathbf{K} = \mathbf{K}_{LOC} = \begin{pmatrix} K_x & 0 & 0 \\ 0 & K_y & 0 \\ 0 & 0 & K_z \end{pmatrix} \quad (3)$$

128 with $K_{\{x,y,z\}}$ the values of the permeability in the x -, y - and z -directions,
129 the index LOC refers to the local or material coordinate system.

130 The capillary effects are described at the macroscopic scale by a capil-
131 lary stress tensor, $\boldsymbol{\sigma}_{cap}$, having the same eigen-directions as the permeability
132 tensor, since they correspond to the symmetries of the orthotropic porous
133 medium. Hence,

$$\boldsymbol{\sigma}_{cap} = \boldsymbol{\sigma}_{cap}^{LOC} = \begin{pmatrix} \sigma_{cap}^x & 0 & 0 \\ 0 & \sigma_{cap}^y & 0 \\ 0 & 0 & \sigma_{cap}^z \end{pmatrix} \quad (4)$$

134 with $\sigma_{cap}^{\{x,y,z\}}$ the components of the equivalent capillary stress at the interface
135 $\Gamma_{l/a}$ in the x -, y - and z -directions. In general situations, described in
136 section 5.3, the eigen-directions of the previous tensors vary from point to
137 point, and consequently, do not match with the axes of the global coordinate
138 system. In this case, tensor \mathbf{K} is expressed in the global system by: $\mathbf{K} =$
139 $\mathbf{Q}\mathbf{K}_{LOC}\mathbf{Q}^T$, where \mathbf{Q} is the orthogonal tensor expressing the passage from
140 the local to the global bases. Similarly, $\boldsymbol{\sigma}_{cap} = \mathbf{Q}\boldsymbol{\sigma}_{cap}^{LOC}\mathbf{Q}^T$.

141 Capillary effects give rise to a jump of pressure across $\Gamma_{l/a}$. This jump,
142 denoted $[p]$, is expressed as

$$[p] = \mathbf{n} \cdot \boldsymbol{\sigma}_{cap} \cdot \mathbf{n} \quad \text{on} \quad \Gamma_{l/a} \quad (5)$$

143 where \mathbf{n} is the normal vector to the interface.

144 Finally, system (Eq. 1), (Eq. 2), completed by (Eq. 5), is closed by
145 prescribing a normal velocity v_0 on $\partial\Omega_D$ and a pressure p_0 on $\partial\Omega_N$:

$$\mathbf{v} \cdot \mathbf{n} = v_0 \quad \text{in} \quad \partial\Omega_D \quad (6)$$

$$p = p_0 \quad \text{in} \quad \partial\Omega_N \quad (7)$$

$$[\mathbf{v} \cdot \mathbf{n}] = 0 \quad \text{on} \quad \Gamma_{l/a} \quad (8)$$

146 where the last condition (Eq. 8), the continuity of the normal velocity, ex-
147 presses the mass conservation across the interface $\Gamma_{l/a}$.

148 2.2. Weak formulation

149 In order to solve the previous Darcy's system with a finite element method,
 150 the weak formulation of these equations has first to be established. Two ap-
 151 proaches exist to express the weak formulation. First, the Darcy's problem
 152 can be formulated in pressure only, and the velocity post-calculated apart.
 153 However, mass conservation issues can appear when considering a jump of
 154 material properties such as a jump of permeability [29]. Second, and this
 155 is the strategy adopted here, a full velocity/pressure mixed weak formula-
 156 tion can be formulated, ensuring the mass conservation. Moreover, what is
 157 called the dual formulation of Darcy's equations [8, 34] is chosen, in order to
 158 naturally enforce the pressure discontinuity.

159 The dual variational formulation is obtained by multiplying the strong
 160 equations (Eq. 1) and (Eq. 2) respectively by any admissible and smooth
 161 enough velocity test function \mathbf{w} and pressure test function q , and then by
 162 integrating by part the term $\mathbf{w} \cdot \nabla p$. The natural enforcement of the [capillary](#)
 163 [stress](#) results from this integration by parts:

$$\begin{aligned} \langle \mathbf{w}, \nabla p \rangle_{\Omega} &= \langle \mathbf{w}, \nabla p \rangle_{\Omega_l} + \langle \mathbf{w}, \nabla p \rangle_{\Omega_a} \\ &= - \langle \nabla \cdot \mathbf{w}, p \rangle_{\Omega} + \langle \mathbf{w} \cdot \mathbf{n}, [p] \rangle_{\Gamma_{l/a}} + \langle \mathbf{w} \cdot \mathbf{n}, p \rangle_{\partial\Omega_N} \\ &= - \langle \nabla \cdot \mathbf{w}, p \rangle_{\Omega} + \langle \mathbf{w} \cdot \mathbf{n}, \mathbf{n} \cdot \boldsymbol{\sigma}_{cap} \cdot \mathbf{n} \rangle_{\Gamma_{l/a}} + \langle \mathbf{w} \cdot \mathbf{n}, p_0 \rangle_{\partial\Omega_N} \end{aligned}$$

164 where, for a bounded region R , the bilinear form $\langle \cdot, \cdot \rangle_R$ denotes the
 165 $L^2(R)^n$ inner-product ($n = 1$ if a and b are scalars, $n = d$ if they are vectors):
 166 $\langle a, b \rangle_R = \int_R a \cdot b \, dR$, for a and b in $L^2(R)^n$ the classical Lebesgues functional
 167 space,

$$L^2(R) = \{q : R \rightarrow \mathbb{R} \mid \int_R q^2 \, dR < \infty\}$$

168 In order to complete the functional setting associated with the weak
 169 Darcy's equations, the Sobolev space $H(\nabla \cdot, \Omega)$ is also introduced:

$$H(\nabla \cdot, \Omega) = \{\mathbf{u} \in L^2(\Omega)^d \mid \nabla \cdot \mathbf{u} \in L^2(\Omega)\}$$

170 Finally, the dual formulation of the mixed Darcy system (Eq. 1) - (Eq.
 171 2) - (Eq. 5) - (Eq. 6) reads: Find $(\mathbf{v}, p) \in H(\nabla \cdot, \Omega) \times L^2(\Omega)$, with $\mathbf{v} \cdot \mathbf{n} = v_0$
 172 on $\partial\Omega_D$, such that

$$\begin{aligned} \langle \mu \mathbf{K}^{-1} \mathbf{v}, \mathbf{w} \rangle_{\Omega} - \langle \nabla \cdot \mathbf{w}, p \rangle_{\Omega} &= \langle \mathbf{f}, \mathbf{w} \rangle_{\Omega} + \langle \mathbf{w} \cdot \mathbf{n}, p_0 \rangle_{\partial\Omega_N} \\ &\quad + \langle \mathbf{w} \cdot \mathbf{n}, \mathbf{n} \cdot \boldsymbol{\sigma}_{cap} \cdot \mathbf{n} \rangle_{\Gamma_{l/a}} \end{aligned} \quad (9)$$

$$\langle \nabla \cdot \mathbf{v}, q \rangle_{\Omega} = \langle h, q \rangle_{\Omega} \quad (10)$$

173 $\forall (\mathbf{w}, q) \in H(\nabla \cdot, \Omega) \times L^2(\Omega)$, with $\mathbf{w} \cdot \mathbf{n} = 0$ on $\partial\Omega_D$, and $\mu = \mu_i$ in Ω_i .

174 2.3. Stabilized FE formulation

175 The computational domain Ω is discretized by using a mesh made up of
 176 triangles in 2D or tetrahedrons in 3D. Let Ω_h be this discretized domain. The
 177 velocity \mathbf{v} and the pressure p are approximated by \mathbf{v}_h and p_h , which are both
 178 continuous and piecewise linear functions ($P1/P1$ approximation). However,
 179 such an approximation is not stable [8, 3, 29] according to Ladyenskaya-
 180 Brezzi-Babuška theory. In this work, this difficulty is overcome by using a
 181 Variational Multi-Scale (VMS) technique [8, 40] consisting in adding some
 182 stabilization terms to the Galerkin formulation. More precisely, the velocity
 183 and pressure functional spaces, $\mathcal{V} \equiv H(\nabla \cdot, \Omega)$ and $\mathcal{P} \equiv L^2(\Omega)$ are split as

$$\mathcal{V} = \mathcal{V}_h \otimes \mathcal{V}' \quad \text{and} \quad \mathcal{P} = \mathcal{P}_h \otimes \mathcal{P}'$$

184 where \mathcal{V}_h and \mathcal{P}_h are the velocity and pressure finite element spaces and
 185 \mathcal{V}' and \mathcal{P}' are the so-called subgrid or unresolvable scale spaces of velocity
 186 and pressure. Following this approach, the solution (\mathbf{v}, p) of the variational
 187 problem (Eq. 9)-(Eq. 10), as well as the test functions (\mathbf{w}, q) are divided as

$$\begin{aligned} \mathbf{v} &= \mathbf{v}_h + \mathbf{v}', & p &= p_h + p' \\ \mathbf{w} &= \mathbf{w}_h + \mathbf{w}', & q &= q_h + q' \end{aligned}$$

188 Subsequently, the variational problem is broken down into a problem at
 189 the resolvable scale, the finite element problem, and a subgrid scale problem,
 190 which cannot be explicitly solved. Consequently, the strategy of VMS meth-
 191 ods consists in approximating the effects of the subgrid scale onto the finite
 192 element scale, leading to additional terms in the finite element formulation.
 193 In this work, the Algebraic SubGrid Scale (ASGS) technique is used, a sub-
 194 type of VMS method developed by Badia and Codina in [8, 9, 25, 37]. The
 195 subgrid terms are expressed as a function of the finite element residual such
 196 that, on a mesh element e

$$\mathbf{v}'|_e \approx -\tau_u^e (\mu \mathbf{K}^{-1} \mathbf{v}_h + \nabla p_h - \mathbf{f})|_e \quad (11)$$

$$p'|_e \approx -\tau_p^e (\nabla \cdot \mathbf{v}_h - h)|_e \quad (12)$$

197 where τ_u^e and τ_p^e are stabilization parameters (Eq. 13) on the element e .
 198 They depend on the mesh size h_e , the geometry (through L_0 , a characteristic
 199 length of the domain Ω), the fluid viscosity, the porous medium permeability
 200 and the stabilization coefficients c_u and c_p (in this work, $c_u = c_p = 1$). As we

201 are using $P1/P1$ approximation, these two parameters are expressed as [2, 8]:

$$\boldsymbol{\tau}_u^e = \frac{h_e \mathbf{K}|_e}{c_u L_0 \mu|_e}, \quad \tau_p^e = \frac{\mu|_e c_p L_0 h_e}{K_m|_e} \quad (13)$$

with K_m^e an equivalent permeability, chosen as [15]

$$K_m^e = \frac{1}{d} \text{trace}(\mathbf{K}|_e)$$

202 Including the subgrid scale effects, the discrete FE system reads: Find
203 $(\mathbf{v}_h, p_h) \in \mathcal{V}_h \times \mathcal{P}_h$, with $\mathbf{v}_h \cdot \mathbf{n} = v_0$ on $\partial\Omega_{hD}$, such that

$$\begin{aligned} & \langle \mu \mathbf{K}^{-1} \mathbf{v}_h, \mathbf{w}_h \rangle_{\Omega_h} - \langle \nabla \cdot \mathbf{w}_h, p_h \rangle_{\Omega_h} - \langle \nabla \cdot \mathbf{v}_h, q_h \rangle_{\Omega_h} \\ & + \sum_e \tau_p^e \langle \nabla \cdot \mathbf{v}_h, \nabla \cdot \mathbf{w}_h \rangle_e + \sum_e \boldsymbol{\tau}_u^e \langle -\mu \mathbf{K}^{-1} \mathbf{v}_h - \nabla p_h, \mu \mathbf{K}^{-1} \mathbf{w}_h + \nabla q_h \rangle_e \\ & = \langle \mathbf{w}_h \cdot \mathbf{n}, \mathbf{n} \cdot \boldsymbol{\sigma}_{cap} \cdot \mathbf{n} \rangle_{\Gamma_{hl/a}} + \langle \mathbf{w}_h \cdot \mathbf{n}, p_0 \rangle_{\partial\Omega_{hN}} + \langle \mathbf{f}, \mathbf{w}_h \rangle_{\Omega_h} + \langle h, q_h \rangle_{\Omega_h} \\ & + \sum_e \boldsymbol{\tau}_u^e \langle \mathbf{f}, \mu \mathbf{K}^{-1} \mathbf{w}_h + \nabla q_h \rangle_e + \sum_e \tau_p^e \langle h, -\nabla \cdot \mathbf{w}_h \rangle_e \end{aligned} \quad (14)$$

204 $\forall (\mathbf{w}_h, q_h) \in \mathcal{V}_h \times \mathcal{P}_h$ with $\mathbf{w}_h \cdot \mathbf{n} = 0$ on $\partial\Omega_{hD}$. In this formulation, \sum_e
205 stands for the summation over all the mesh elements e .

206 Note that when the porous medium is assumed to be isotropic, previous
207 formulation (Eq. 14) can be slightly simplified, since the permeability \mathbf{K} ,
208 the stabilization parameter $\boldsymbol{\tau}_u$ and the [capillary stress](#) become scalar.

209 2.4. Pressure discontinuity

210 The [capillary stress](#) generates a pressure discontinuity at the liquid/air
211 interface. Moreover, for two different liquids the jump of viscosity across
212 this same interface leads to a discontinuity of the pressure gradient. In a FE
213 framework, these two kinds of discontinuities represent a numerical difficulty
214 to be dealt with. A first approach found in the literature consists in circum-
215 venting the discontinuity by considering a smooth transition area around
216 the interface [16]. The performance of this method depends strongly on the
217 smoothing function, on the transition region thickness, and consequently on
218 the local mesh size [27]. Since no mesh adaptation strategy is used in this
219 work, the liquid-air interface will be identified by a continuous set of seg-
220 ments (2D) or triangles (3D) crossing the mesh elements [15, 61] and built
221 locally thanks to the level-set front-capturing method described in Section 3.

222 This approach allows to integrate the capillary term into (Eq. 14) directly
 223 on a segment or triangle, using one integration point if the [capillary stress](#)
 224 is piecewise constant. Moreover, additional integration points are considered
 225 in the elements e crossed by the interface, in order to evaluate accurately the
 226 term $\langle \mu \mathbf{K}^{-1} \mathbf{v}_h, \mathbf{w}_h \rangle_e$. Thus, in the 2D configuration (Fig. 2), assuming
 227 that both viscosities of the liquid and air are constant, 3 integrations points
 228 are used in each sub-element deriving from the element split.

229 However, such a split is not sufficient to ensure the accurate capture of
 230 the pressure and pressure gradient discontinuities. Especially, continuous
 231 and piecewise linear approximation of the pressure, piecewise linear approx-
 232 imation of the interface, give rise to the parasitic current phenomenon (even
 233 if the curvature is not involved in the equations), which consists in spu-
 234 rious oscillations of the velocity, possibly deteriorating the interface [33].
 235 Here again, several options are available in the literature to reduce these
 236 oscillations [23, 30, 55]. In particular, an enrichment of the pressure space
 237 [6, 20, 26, 42, 43, 44] can be set up, locally in the elements crossed by the
 238 fluid front. This work considers the pressure enrichment developed by R.
 239 Ausas *et al.* [6]. Originally introduced to deal with discontinuities involved
 240 in Navier-Stokes equations, this technique is applied here to Darcy's equa-
 241 tions. This consists in adding, in the elements crossed by the interface, the
 242 two discontinuous shape functions M_1 and M_2 described in Fig. 2 and derived
 243 as following:

$$M_1(\mathbf{x}) = (1 - S(\mathbf{x}))\chi^l(\mathbf{x}) \quad (15)$$

$$M_2(\mathbf{x}) = S(\mathbf{x})\chi^a(\mathbf{x}) \quad (16)$$

244 with

$$S(\mathbf{x}) = \sum_{J \in \mathcal{J}^a} N_J(\mathbf{x}) \quad (17)$$

245 where N_J is the usual linear shape function associated with node J , χ^l is
 246 equal to 1 in the liquid region, to 0 elsewhere, and $\chi^a = 1 - \chi^l$. The set \mathcal{J}^a
 247 corresponds to the element nodes being in Ω_a .

248 In such elements, the pressure field p_h is expressed as

$$p_h(\mathbf{x}) = \sum_J P_J N_J(\mathbf{x}) + C_1 M_1(\mathbf{x}) + C_2 M_2(\mathbf{x}) \quad (18)$$

249 where P_J are the degree of freedom associated with the element vertices J ,
 250 while C_1 and C_2 are those associated with the discontinuous shape functions

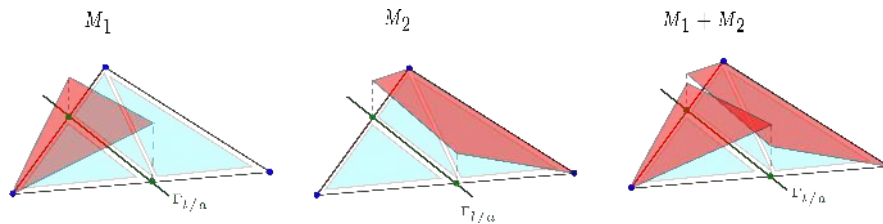


Figure 2: 2D local pressure enrichment and surface reconstruction (inspired of [6]).

251 M_1 and M_2 . However, since C_1 and C_2 are defined per element, they can
 252 be eliminated by static condensation, at the elementary level, prior to the
 253 final assembly. Therefore, the main advantage of combining the interface
 254 reconstruction and local pressure enrichment is that the discontinuity gener-
 255 ated by the [capillary stress](#) is treated without increasing the number of final
 256 degrees of freedom and affecting the computation time.

257 3. Fluid front capturing: level-set method

258 The moving flow front is captured by a level-set method [57, 69], con-
 259 sisting in choosing a continuous function, the so-called level-set function,
 260 $\psi(\mathbf{x}, t) : \Omega \times \mathbb{R}^+ \rightarrow \mathbb{R}$, negative in Ω_l and positive in Ω_a . Consequently, the
 261 interface $\Gamma_{l/a}$ is implicitly described as the zero-isovalue of function ψ :

$$\Gamma_{l/a}(t) = \{\mathbf{x} \in \Omega \mid \psi(\mathbf{x}, t) = 0\} \quad (19)$$

262 where t denotes the time variable. Note that the gradient of ψ allows the
 263 computation of the normal vector normal at the interface.

264 Assuming the flow velocity \mathbf{v} , defined both in Ω_l and Ω_a , known at each
 265 instant $t \in [0, T]$ (T is the final time of the simulation), the level-set function
 266 is then convected according to the hyperbolic equation (Eq. 20):

$$\frac{\partial \psi}{\partial t} + \mathbf{v} \cdot \nabla \psi = 0 \quad \forall (\mathbf{x}, t) \in \Omega \times [0, T] \quad (20)$$

$$\psi(\mathbf{x}, t = 0) = \psi_0 \quad \forall \mathbf{x} \in \Omega \quad (21)$$

$$\psi(\mathbf{x}, t) = g(\mathbf{x}, t) \quad \forall (\mathbf{x}, t) \in \partial\Omega^- \times [0, T] \quad (22)$$

267 where $g(\mathbf{x}, t)$ (Eq. 22) corresponds to the value of ψ to be imposed on the
 268 incoming boundary $\partial\Omega^-$ (Fig. 1)

$$\partial\Omega^-(t) = \{\mathbf{x} \in \partial\Omega \mid \mathbf{v}(\mathbf{x}, t) \cdot \mathbf{n} < 0\},$$

269 while (Eq. 21) states for the initial condition at $t = 0$.

270 *3.1. SUPG formulation*

271 Transport equation (Eq. 20) is solved by a FE technique, using the same
 272 mesh as for Darcy’s equations. The variational formulation is first obtained
 273 by multiplying (Eq. 20) by any admissible and smooth enough test function
 274 and integrating the product over Ω . The time interval is discretized by a set
 275 of points $0 = t_0 < t_1 < \dots < t_n < t_{n+1} < \dots < t_N$, and a finite difference
 276 scheme in time, the implicit Crank-Nicholson scheme, is then applied to
 277 (Eq. 20). It results that at each instant t_n , $\psi(\cdot, t_n)$ is approximated by
 278 $\psi_h(\cdot, t_n)$ a continuous piecewise linear function. However, the usual Galerkin
 279 approach is known to be not stable for hyperbolic equations. This issue can
 280 be avoided by considering a Streamline Upwind Petrov-Galerkin (SUPG)
 281 method, introduced by Hughes in [19], and consisting in taking the test
 282 functions in a space different of the shape functions. More precisely, shape
 283 functions are still the nodal functions N_J already introduced, but the test
 284 functions, denoted N_J^* , are now defined on a mesh element e , as

$$N_J^* = N_J + \tau^e \mathbf{v} \cdot \nabla N_J$$

285 where the stabilization parameter τ^e is chosen as

$$\tau^e = \frac{1}{2} \frac{h_e}{v_e}$$

286 with h_e the size of element e and v_e the norm of the average velocity in e .

287 This modification of the test functions adds, in a consistent way, an up-
 288 wind artificial diffusion term stabilizing the FE formulation, at least as long
 289 as the convective term remains under control.

290 *3.2. Filtered level-set*

291 For the level-set procedure described below to be effective, the level-set
 292 function ψ has to be initialized with a specific expression. Let $d_0(\mathbf{x})$ denote
 293 the signed distance function from point \mathbf{x} to the initial liquid-air interface
 294 $\Gamma_{la}(0)$. The initial expression of the level-set function, involved in the initial
 295 condition (Eq. 21) is then chosen as

$$\psi_0(\mathbf{x}) = \varepsilon \tanh \left(\frac{d_0(\mathbf{x})}{\varepsilon} \right) \tag{23}$$

296 where ε can be viewed as the thickness of the interface. In practice: $\varepsilon = 3h_e$.

297 Outside a narrow band around the interface, ψ_0 quickly tends towards
 298 the constant values $\pm\varepsilon$. Therefore, condition (Eq. 22) to be prescribed on
 299 the inflow boundary can easily be enforced. Additionally, within this tiny
 300 band close to the interface, ψ_0 is equal, in the first order, to the distance func-
 301 tion d_0 . A distance function have, by definition, a unit gradient: $\|\nabla d_0\| = 1$.
 302 This property ensures the “control” of the convection term in transport equa-
 303 tion (Eq. 20) and thus the efficiency of the SUPG stabilization. However,
 304 the initial ”tanh-like” shape (Eq. 23) is not preserved under the transport
 305 of ψ with the Darcy’s velocity field \mathbf{v} . That is why, as this velocity varies
 306 abruptly (but continuously) through the liquid-air interface, steep gradients
 307 of level-set function will develop in its vicinity, and the SUPG stabilization
 308 will fail. This problem is avoided by periodically reinitializing the level-set
 309 function: the zero-isovalue is preserved, while the tanh property is applied
 310 elsewhere. Based on the relation $d \tanh(x)/dx = 1 - \tanh^2(x)$, function ψ is
 311 of the form (Eq. 23) if

$$\|\nabla \psi\| = \left| 1 - \left(\frac{\psi}{\varepsilon} \right)^2 \right| \quad (24)$$

312 At a given time t_n , the reinitialization step consists in solving iteratively
 313 the Hamilton-Jacobi equation

$$\frac{\partial \tilde{\psi}}{\partial \tau} + \text{sgn}(\tilde{\psi}) \left(\|\nabla \tilde{\psi}\| - \left| 1 - \left(\frac{\tilde{\psi}}{\varepsilon} \right)^2 \right| \right) = 0 \quad (25)$$

$$\tilde{\psi}(x, \tau = 0) = \psi(x, t_n) \quad (26)$$

314 until reaching the steady state, *i.e.* $\partial \tilde{\psi} / \partial \tau = 0$, corresponding consequently
 315 to the property (Eq. 24). This state gives the reinitialized level-set func-
 316 tion. In practice, only a few increments (3 in our simulations) are necessary
 317 to recover the unit gradient property in the narrow band around the inter-
 318 face. In (Eq. 25), τ is a time-like variable, and sgn is the regularized sign
 319 function [57]

$$\text{sgn}(\psi) = \frac{\psi}{\sqrt{\psi^2 + \|\nabla \psi\|^2 h_e^2}} \quad (27)$$

320 Note that, classically, Hamilton-Jacobi equation (Eq. 25) can be consid-
 321 ered as a transport equation with a right-hand-side, and is then solved in the
 322 same way as the level-set convection equation (Eq. 20). The reinitialization

323 velocity is equal to $sgn(\psi) \frac{\nabla \tilde{\psi}}{\|\nabla \tilde{\psi}\|}$, while the non-linear terms are explicitly
 324 evaluated at the previous iteration.

325 3.3. Time-stepping strategy

326 The time-stepping strategy consists, for a given time increment, in solving
 327 Darcy's equations, then updating the flow front position by solving the level-
 328 set transport equation using the Darcy's velocity, and moving on to the next
 329 time increment. To sum up, the algorithm coupling Darcy's and level-set
 330 problems is as following:

Algorithm 1 Staggered algorithm for Darcy's and level-set problems

Require: $\psi(\mathbf{x}, t = 0) = \psi_0$ the initial value for the level set function
while $0 < t^{n+1} < T$ **do**
 1- **Fluid problem:**
 Find $(\mathbf{v}_h, p_h) \in \mathcal{V}_h \times \mathcal{P}_h$ by solving Darcy's equations (Eq. 14)
 2- **Flow front problem:**
 Find ψ_h by solving the level-set equations
 3- **Reinitialization problem:**
Repeat 3 times: Solving Hamilton-Jacobi's equations (Eq. 25)
end while

331 4. Convergence analysis

332 The FE model presented in the previous section has been implemented
 333 in the FE software Z-set [68]. The efficiency of the implementation, as well
 334 as the accuracy of the approach, are evaluated by an error analysis based
 335 on the Method of Manufactured Solutions (MMS) [64]. This consists in
 336 selecting velocity and pressure fields that satisfy Darcy's equations (Eq. 1)-
 337 (Eq. 2) and calculating the corresponding right-hand-side terms that are then
 338 prescribed in the FE problem. Performance of the implementation measures
 339 the capability of reproducing the initial fields.

340 The 2D-computational domain is the unit square $\Omega = [0, 1] \times [0, 1]$. The
 341 analytical pressure field is defined as

$$p(x, y) = \begin{cases} \sin(2\pi x) \sin(2\pi y) & \text{for } y < \frac{1}{2} \\ \sin(2\pi x) \sin(2\pi y) + \sigma_{cap} & \text{for } y > \frac{1}{2} \end{cases} \quad (28)$$

342 with σ_{cap} the scalar value of the capillary stress in the isotropic case.

343

344 Replacing (Eq. 28) inside Darcy's equation (Eq. 1) gives the components
345 of the velocity, v_x and v_y

$$\begin{aligned} v_x &= \frac{K}{\mu} 2\pi \cos(2\pi x) \sin(2\pi y) \\ v_y &= \frac{K}{\mu} 2\pi \sin(2\pi x) \cos(2\pi y) \end{aligned} \quad (29)$$

346 These fields (Eq. 28) and (Eq. 29) satisfy Darcy's system (Eq. 1)-(Eq.
347 2) with the term h taken as

$$h = \nabla \cdot \mathbf{v} = \frac{K}{\mu} 8\pi^2 \sin(2\pi x) \sin(2\pi y) \quad (30)$$

348 Only one type of boundary condition is considered here, the Dirichlet
349 one (Eq. 6): $\mathbf{v} \cdot \mathbf{n} = v_0 = 0$ on $\partial\Omega$. Thus, the so-called compatibility
350 condition is fulfilled, that is

$$\int_{\Omega} \nabla \cdot \mathbf{v} \, d\Omega = \int_{\partial\Omega} \mathbf{v} \cdot \mathbf{n} \, d\Gamma = \int_{\partial\Omega} v_0 \, d\Gamma = 0$$

351 In all the simulations shown in this section, the pressure jump across the
352 line $\{y = \frac{1}{2}\}$ is equal to 1, $[p] = \sigma_{cap} = 1$, while the ratio K/μ is also unit
353 (isotropic case). Pressure and velocity obtained by the FE strategy described
354 before are plotted in a 3D-representation in Fig. 3 using an unstructured mesh
355 of element size $h_e = 0.0125$. The pressure discontinuity is well-captured,
356 without apparent oscillations of pressure. This is qualitatively confirmed
357 in Fig. 4, where the computed pressure is satisfactorily compared to the
358 analytical one along two lines, $\{x = \frac{1}{4}\}$ and $\{x = \frac{1}{2}\}$.

359 Next, a quantitative analysis of the error made on velocity and pressure is
360 performed by considering 4 structured meshes of size, respectively, $h_e = 1/20$,
361 $1/40$, $1/80$ and $1/160$. On each of these meshes, pressure error is calculated
362 with the usual L^2 -norm denoted $\|\cdot\|_{L^2}$, while velocity error is estimated both
363 in L^2 -norm and in $H(\nabla\cdot)$ -norm denoted $\|\cdot\|_{H(\nabla\cdot)}$ (Eq. 31):

$$\|u\|_{L^2} = \left(\int_{\Omega} u^2 \, d\Omega \right)^{\frac{1}{2}}, \quad \|u\|_{H(\nabla\cdot)} = \left(\|u\|_{L^2}^2 + \|\nabla \cdot u\|_{L^2}^2 \right)^{\frac{1}{2}} \quad (31)$$

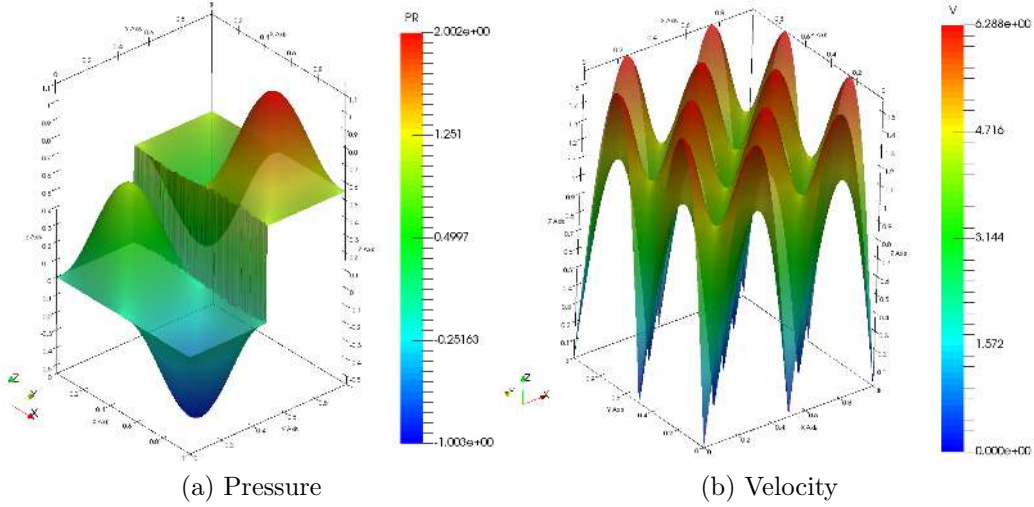


Figure 3: 3D-plot of pressure and velocity fields (Eq. 28)-(Eq. 29), obtained by the FE solution.

364 The theoretical convergence rates, without discontinuity of pressure, are
 365 2 in L^2 -norm both for the pressure and velocity, and 1 in the $H(\nabla\cdot)$ -norm
 366 for the velocity [3, 8, 53]. Therefore, three different cases are proposed here:
 367 a continuous case, corresponding to $\sigma_{cap} = 0$, in order to assess the Darcy's
 368 solver in a classical situation and have a reference situation; two discontinuous
 369 cases with $\sigma_{cap} = 1$ as mentioned above, but one without pressure enrichment
 370 of Section 2.4, and one with this technique. Results are summarized in
 371 Tables 1-2-3 and Fig. 5.

Mesh	h_e	$\ p - p_h\ _{L^2}$	$\ \mathbf{v} - \mathbf{v}_h\ _{L^2}$	$\ \mathbf{v} - \mathbf{v}_h\ _{H(\nabla\cdot)}$
20×20	0.05	0.0213803	0.0839136	6.76775
40×40	0.025	0.00389416	0.0188836	3.3258
80×80	0.0125	0.000714466	0.00416498	1.61289
160×160	$6.25 \cdot 10^{-3}$	0.000148175	0.00100372	0.812385

Table 1: Error in the L^2 -norm for the pressure, and both the L^2 -norm and $H(\nabla\cdot)$ -norm for the velocity. Case with a continuous pressure.

372 We observe that without pressure discontinuity, the convergence rate ob-
 373 tained is in agreement with the optimal one, since the rate is slightly higher

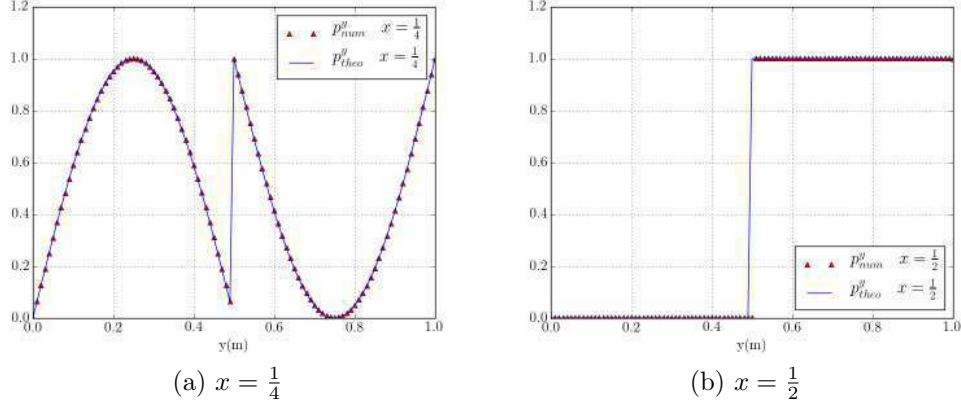


Figure 4: Comparison between analytical pressure field (Eq. 28) (continuous line) and results of simulation (dots), along the lines $x = \frac{1}{4}$ and $x = \frac{1}{2}$.

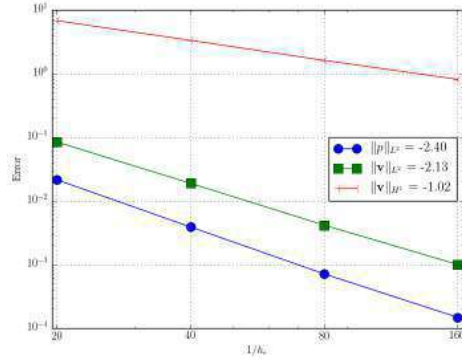
Mesh	h_e	$\ p - p_h\ _{L^2}$	$\ \mathbf{v} - \mathbf{v}_h\ _{L^2}$	$\ \mathbf{v} - \mathbf{v}_h\ _{H(\nabla \cdot)}$
20×20	0.05	0.0889837	0.109918	7.2842
40×40	0.025	0.0475362	0.0203123	3.33272
80×80	0.0125	0.0367567	0.00888546	2.1614
160×160	$6.25 \cdot 10^{-3}$	0.0231142	0.00189264	0.906225

Table 2: Error in the L^2 -norm for the pressure, and both the L^2 -norm and $H(\nabla \cdot)$ -norm for the velocity. Case with pressure discontinuity and no pressure enrichment.

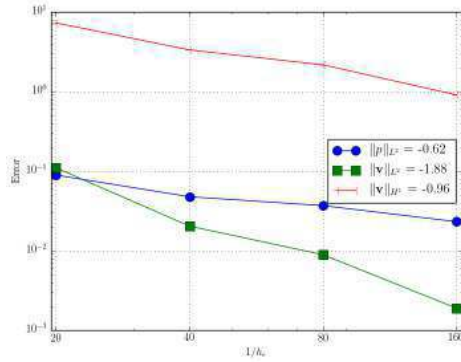
Mesh	h_e	$\ p - p_h\ _{L^2}$	$\ \mathbf{v} - \mathbf{v}_h\ _{L^2}$	$\ \mathbf{v} - \mathbf{v}_h\ _{H(\nabla \cdot)}$
20×20	0.05	0.0615517	0.0933167	6.94098
40×40	0.025	0.0211956	0.0247864	3.63211
80×80	0.0125	0.00571188	0.00801094	1.9443
160×160	$6.25 \cdot 10^{-3}$	0.00237119	0.00323276	1.55267

Table 3: Error in the L^2 -norm for the pressure, and both the L^2 -norm and $H(\nabla \cdot)$ -norm for the velocity. Case with pressure discontinuity and local pressure enrichment.

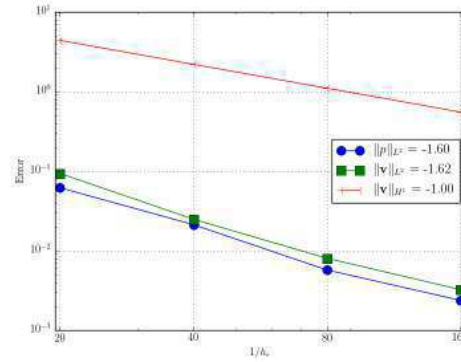
374 than 2 for both pressure and velocity in L^2 -norm, and equal to 1 for the veloc-
 375 ity in $H(\nabla \cdot)$ -norm. As expected, the two cases with pressure discontinuity let
 376 show lower convergence rates compared to the continuous case, especially for
 377 the pressure. Without enrichment strategy, a sub-optimal convergence rate



(a)



(b)



(c)

Figure 5: Error analysis: continuous case (a), discontinuous case without pressure enrichment (b) and discontinuous case with local pressure enrichment (c).

378 of 0.62 is obtained for pressure, corresponding to approximately 1/3 of the
 379 theoretical order predicted for the continuous case. However, when enriching
 380 locally the pressure space, this rate is greatly improved, since jumping up to
 381 80% of this same theoretical order, with the value of 1.6. Note that, with this
 382 same enrichment technique, but considered in the context of Navier-Stokes
 383 equations, Ausas and co-authors [6] obtained a pressure convergence rate
 384 equal to 75% of the one predicted with a continuous pressure. Hence, we can
 385 conclude that the numerical approach presented below allow us to describe
 386 with accuracy the pressure discontinuity when solving the Darcy's equations
 387 with a [capillary stress](#) applied on the moving flow front.

388 5. Numerical applications

389 This section assesses and demonstrates the performance of our numerical
 390 model in realistic contexts. First, numerical simulations of flows through
 391 porous media with a very low permeability are carried out, and the interest
 392 of local pressure enrichment is highlighted. Next, capillary wicking simula-
 393 tions are performed and the results are compared with experimental studies.
 394 Finally, a first approach of flows in 3D orthotropic materials is provided
 395 through the simulation of the filling stage of a T-stiffener during a LRI pro-
 396 cess. It is also used to demonstrate that simply shifting the pressure at a
 397 boundary condition by the capillary stress may hold for UD cases, but is not
 398 satisfactory in terms of filling scenario and results for general 3D cases.

399 5.1. Ascending capillary flow and realistic parameters

400 The numerical strategy is assessed by simulating a unidirectional flow
 401 in a porous medium with realistic properties, in terms of permeabilities,
 402 viscosities and capillary stresses, in the context of composite materials man-
 403 ufacturing. Thus, the computational domain Ω is a square of 1 meter side.
 404 The isotropic permeability K is equal to $3.0 \times 10^{-13} \text{m}^2$, while the isotropic
 405 capillary stress, applied on the interface $\Gamma_{l/a} \equiv \{y = h = 1/2\}$, is of 32×10^3
 406 Pa. Viscosities are $\mu_l = 10^{-3} \text{Pa.s}$ and $\mu_a = 10^{-5} \text{Pa.s}$. This pressure is the
 407 only driving force, since boundary conditions on both planes $\{y = 0\}$ and
 408 $\{y = 1\}$ are set to the atmospheric pressure. The remaining boundaries are
 409 considered as impervious walls, thus the $\mathbf{v} \cdot \mathbf{n} = 0$ condition is applied on the
 410 vertical edges of the domain, $\{x = 0\}$ and $\{x = 1\}$. All numerical values of
 411 material properties and boundary conditions are sum up in Fig. 6.

412 In the case of a unidirectional flow, the analytical solution of Darcy's
 413 equations is quite simple to determine. Indeed, the pressure is piecewise
 414 linear, while the velocity is constant. With the notations introduced in Fig. 6,
 415 the pressure and velocity fields can be written as

$$\begin{aligned}
 p(x, y) &= \mu_l \frac{p_1 - p_0 - \sigma_{cap}}{h\mu_l + (1-h)\mu_a} y + p_0 && \text{in } \Omega_l \\
 p(x, y) &= \mu_a \frac{p_1 - p_0 - \sigma_{cap}}{h\mu_l + (1-h)\mu_a} (y - 1) + p_1 && \text{in } \Omega_a \\
 v_x(x, y) &= 0 && \text{in } \Omega \\
 v_y(x, y) &= -K \frac{p_1 - p_0 - \sigma_{cap}}{h\mu_l + (1-h)\mu_a} && \text{in } \Omega
 \end{aligned} \tag{32}$$

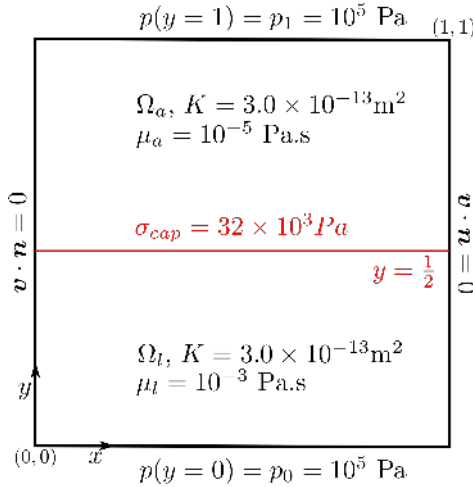


Figure 6: Material properties and boundary conditions used in the numerical simulation of unidirectional flow.

416 The velocity and pressure fields computed with a structured mesh of size
 417 $h_e = \frac{1}{50}$ (4800 triangular elements corresponding to 2499 nodes) and an
 418 interface crossing the elements, are given in Fig. 7 and 8. Two cases are
 419 considered: without and with the pressure enrichment introduced in section
 420 2.4. In the first case, the pressure jump is not well-captured at the interface
 421 (Fig. 7(b)), resulting in a spurious velocity around this interface (Fig. 7(a)).
 422 On the contrary, the discontinuity of the pressure field is accurately computed
 423 with the enrichment (Fig. 8(b)) leading to a uniform velocity field as expected
 424 by Equation (Eq. 32) (Fig. 8(a)). Numerical and analytical values of the
 425 velocity are identical, and equal to $1.901 \times 10^{-5} \text{ m.s}^{-1}$ in norm. This also
 426 proves the accuracy of the pressure description. To complete this analysis,
 427 analytical and numerical pressures have been plotted on the line $\{x = \frac{1}{2}\}$
 428 in Fig. 9, for different structured meshes. Again, no pressure oscillation is
 429 observed.

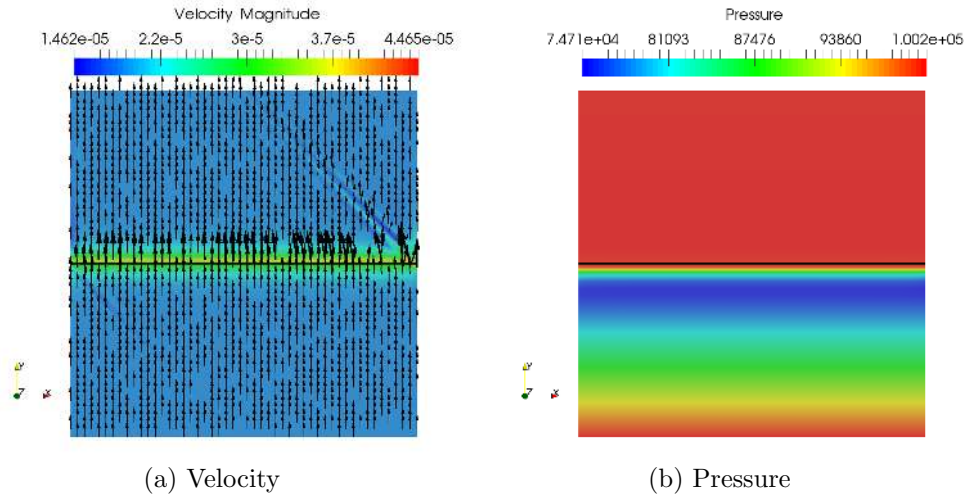


Figure 7: Velocity and pressure fields obtained without pressure enrichment

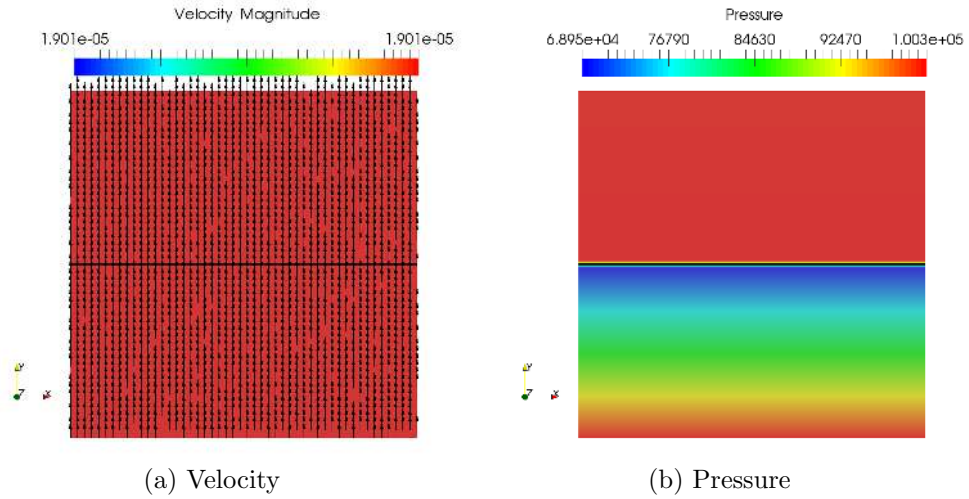


Figure 8: Velocity and pressure fields obtained with pressure enrichment

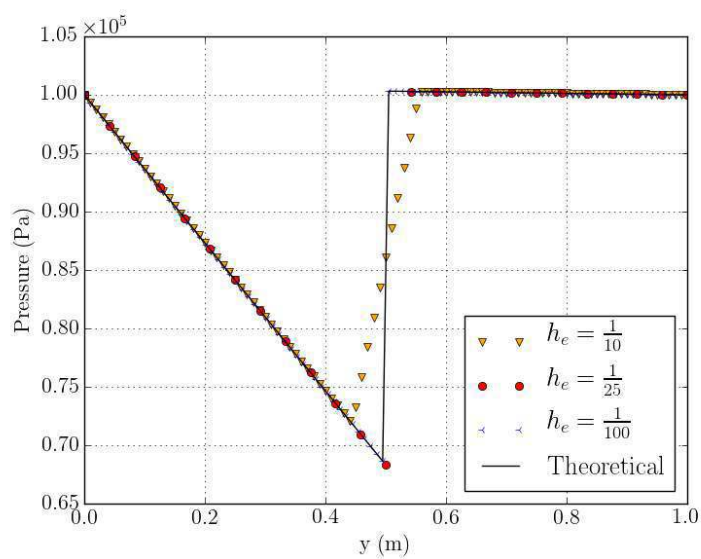


Figure 9: Comparison between analytical and numerical pressures, plotted along the line $\{x = \frac{1}{2}\}$, for different mesh sizes h_e .

430 5.2. Capillary wicking

431 In this section, simulation of wicking in carbon reinforcements is con-
 432 fronted to experimental data [62, 63]. The only driving force is thus due to
 433 capillary effects.

434 5.2.1. Experimental approach

435 Pucci *et al.* [62] proposed an experimental procedure to determine the
 436 scalar capillary stress σ_{cap} in the three main directions of a unidirectional
 437 (UD) carbon fabric (Fig. 11). On the one hand, for a given direction, the
 438 mass of water in the fabric $m(t)$ is recorded over time using a tensiometer.
 439 Wicking is commonly described by a modified Washburn equation [74] for
 440 porous media relating mass and time

$$m^2(t) = \left[\frac{(c\bar{r})\phi^2(\pi R^2)^2}{2} \right] \frac{\rho_l^2 \gamma \cos \theta_a t}{\mu_l} \quad (33)$$

441 where c is a constant accounting for the tortuous path of liquid in the equiv-
 442 alent capillary tube arrangement of mean radius \bar{r} . ϕ is the porosity and R
 443 the inner radius of the cylindrical sample holder. The first term in square
 444 brackets finally represents a geometric factor of the porous medium. ρ_l and
 445 μ_l are, respectively, the liquid (water) density and its viscosity. θ_a is the
 446 apparent mean advancing contact angle during the capillary rise and γ_l the
 447 liquid surface tension.

448 On the other hand, from Darcy's equation applied to a unidirectional
 449 flow (Eq. 32) following the assumption of spontaneous impregnation under
 450 the effect of capillary stress σ_{cap} (Fig. 10) the square of the water height $h^2(t)$
 451 (see Fig. 11) can be expressed as a function of time

$$h^2(t) = \frac{2K\sigma_{cap}t}{\mu_l\phi} \quad (34)$$

452 This expression is easily obtained from the last equation of the analytical
 453 model (Eq. 32), considering that $v_y = \phi \frac{dh}{dt}$, $p_1 = p_0$, $\mu_a = 0$ and integrating
 454 it with respect to time.

455 Taking into account the cylindrical shape of radius R , the mass gain can
 456 be related to the height by

$$m^2(t) = h^2(t)\phi^2\rho_l^2(\pi R^2)^2 \quad (35)$$

457 Considering the equivalence between Eq. 33 and Eq. 35, it is then possible
 458 to describe capillary stress σ_{cap} for a given permeability K [62].

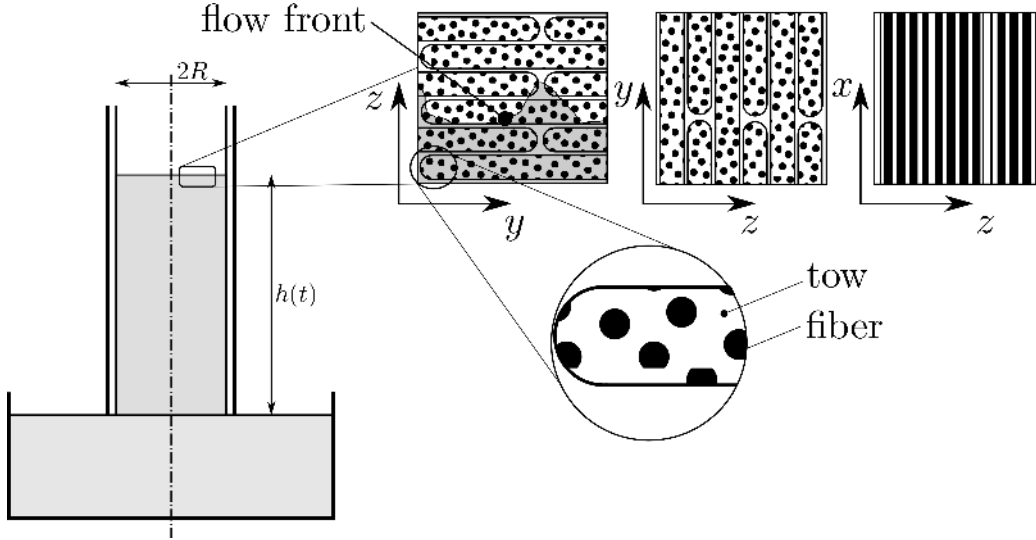


Figure 10: Capillary wicking in a cylindric quasi-UD carbon reinforcement according to Darcy law for the three principal directions of the medium.

459 5.2.2. Numerical simulations and results

460 Three 2D numerical simulations of wicking of water into a cylindrical
 461 quasi-UD carbon reinforcement have been carried out. For each simulation,
 462 the fabric is oriented in one of the directions x , y or z . Note that consequently,
 463 each of these simulations is reduced to an isotropic case. The computational
 464 domain, the $2R \times H$ rectangle described in Fig. 11, is discretized with a
 465 fixed mesh of 2,352 triangular elements and 1,250 nodes. The boundary
 466 conditions prescribed for this simulation are a zero normal velocity on the
 467 vertical sides and a pressure of 1 bar on the two other sides.

468 The definition of the orthotropic permeability tensor, orthotropic capil-
 469 lary stress tensor applied on the flow front and identified from experience,
 470 are given in Table 4, as well as the water and “air” viscosities, the water
 471 density, the porosity, and the dimensions $2R$ and H of the computational
 472 domain.

473 In order to have realistic simulations, the porosity has to be taken into ac-
 474 count in Darcy’s equations. This is achieved by substituting $\mathbf{v}\phi$ for \mathbf{v} Darcy’s
 475 equations. From the position of the water height $h(t)$ obtained by simula-
 476 tion, the corresponding water weight is calculated by Eq. 35 and compared
 477 (Fig. 12), with experimental data and the analytical expression given by Eq.

Permeabilities [49]	(m ²)
K_x	$3 \cdot 10^{-11}$
K_y	$1.5 \cdot 10^{-11}$
K_z	$3 \cdot 10^{-13}$
Capillary stress [62]	(kPa)
σ_{cap}^x	1.15 ± 0.30
σ_{cap}^y	0.51 ± 0.14
σ_{cap}^z	32.10 ± 11.60
Others	
μ_{water}	10^{-3} Pa.s
ρ_{water}	10^3 kg.m ⁻³
μ_{air}	10^{-5} Pa.s
ϕ	0.40
$2R$	12 mm
H	20 mm

Table 4: Capillary wicking parameters.

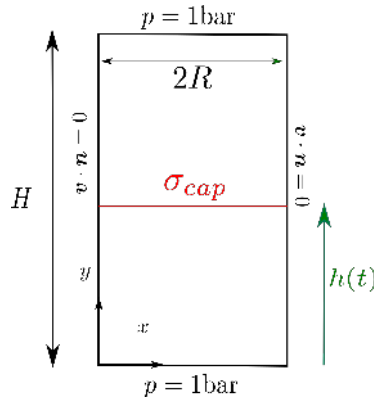


Figure 11: Geometrical parameters and boundary conditions of the capillary wicking.

478 34-35. It can be shown that numerical simulations and analytical expres-
 479 sion give comparable results. Therefore, the numerical simulations reflect
 480 the experimental wicking. It confirms that the numerical model is correct
 481 since wicking in each main directions were already correctly described by
 482 the analytical model. However, the key point is that the proposed numerical
 483 methodology simulates wicking, a transient phenomenon, here in an isotropic
 484 context, but with realistic values of parameters involved in the model.

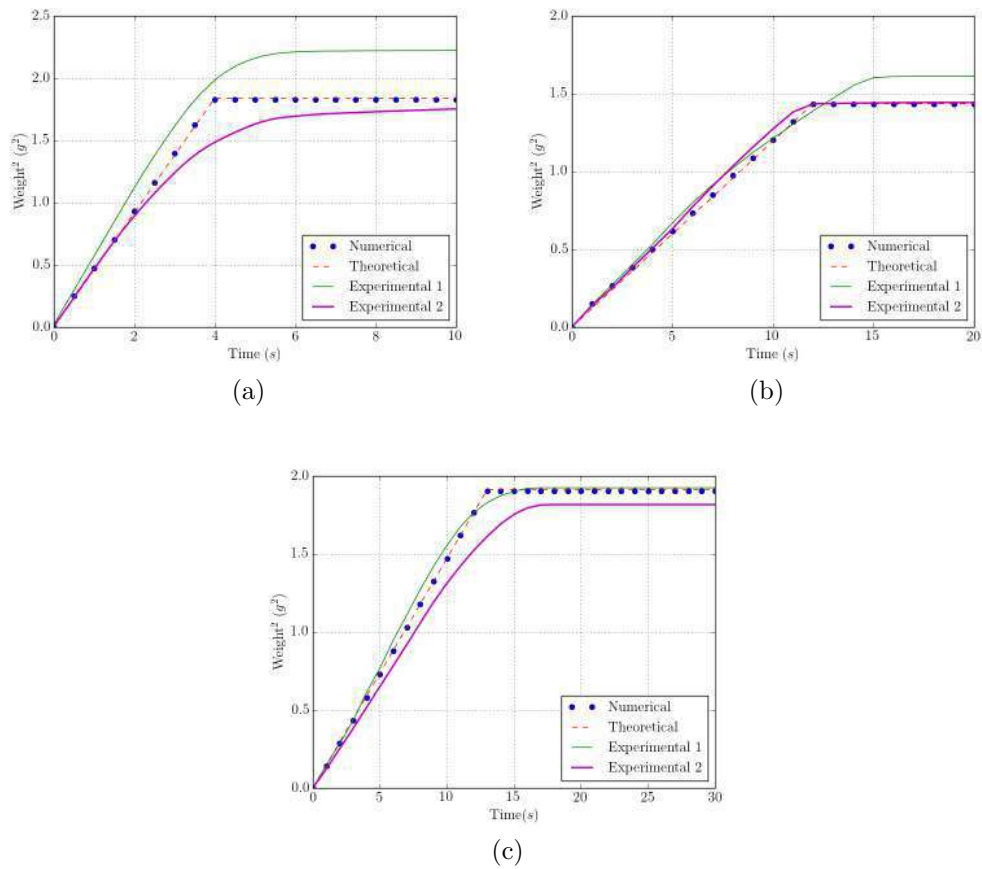


Figure 12: Comparison of the time dependent numerical and experimental weight in the x (a), y (b) and z (c) directions.

Parameters	Values	Units
K_t	10^{-14}	m^2
K_p	10^{-12}	m^2
σ_{cap}^t	0.3	bar
σ_{cap}^p	0.01	bar
μ_{resin}	0.1	Pa.s
μ_{air}	10^{-5}	Pa.s
ϕ	40%	
Δt	100	s

Table 5: Inputs - LRI simulation.

485 *5.3. Full 3D-simulations of LRI process filling stage*

486 This section investigates the 3D-simulation of the filling stage of a Liquid
487 Resin Infusion (LRI) process [15, 22]. More precisely, the objective is to
488 evaluate the influence of capillary effects on the resin impregnation. The part
489 to be infused is the aeronautic-like stiffener shown in Fig. 13. Furthermore,
490 the preform is assumed to have an additional symmetry: two eigen-values
491 of the permeability tensor, as well as two eigen-values of the capillary stress
492 tensor are equal. Hence, Table 5 gives the values of the in-plane permeability
493 K_p , which is a hundred times larger than the transverse permeability K_t . As
494 capillary forces are more significant in less permeable media, the **capillary**
495 **stress** σ_{cap}^t in the transverse direction is higher than the value in the plan
496 σ_{cap}^p . These values, completed by the resin viscosity, the air viscosity, the
497 porosity and the time step Δt are also provided in Table 5. Figure 13 shows
498 the corresponding materials eigen-directions y^t and x^p on a cutting plane.
499 That is the transverse direction and the plane orthogonal to this direction in
500 three different areas, allowing to compute numerical values of permeabilities
501 and **capillary stresses** at each integration point of finite elements. Moreover,
502 the boundary conditions both in velocity and pressure are given. The resin
503 flow front is initialized as the plane $\{y = 0.5\text{cm}\}$. The flow is driven by
504 the difference of pressure between the “inlet” (plane $\{y = 0\}$) and the “top”
505 (plane $\{y = 12\text{cm}\}$) boundaries, equal to 10^5 Pa, and additionally by the
506 capillary stress on $\Gamma_{l/a}$ when this is taken into account.

507 Regarding the solution for 1D cases, one may question about a simple
508 way of accounting for capillary effects by modifying the overall pressure gra-
509 dient. Although an extension to 3D is not straightforward, in order to assess

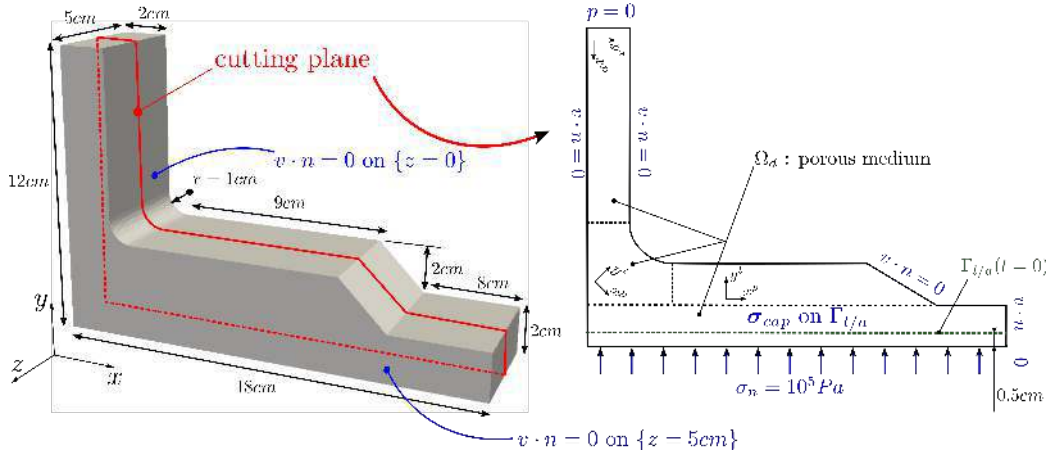


Figure 13: Geometrical dimensions and boundary conditions of the stiffener.

510 also such a basic approach, 3 simulations for the T-stiffener were considered:
 511 one carried out without capillary effects, one accounting for capillary effects
 512 by modifying the overall pressure gradient through changing the boundary
 513 condition in the plane $y = 0$: $\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} = 10^5 \text{ Pa} \rightarrow \mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} = 10^5 \text{ Pa} + \sigma_{cap}^t$ -
 514 *Modified BC*-, and finally integrating orthotropic capillary stresses with the
 515 proposed method - *Discontinuous Pressure*-. Figure 14 compares the flow
 516 front position during infusion for the corresponding three simulations. As
 517 expected, the part is filled more quickly when the capillary effects are taken
 518 into account: 1h58 min with the Modified BC approach using the highest
 519 capillary stress σ_{cap}^t , 2h 10min with our discontinuous pressure numerical
 520 model and 3 hours without any capillary effects. One can verify that capillary
 521 effects will help the filling of the preform. Besides, the pressure and
 522 fluid front kinetics resulting from the approaches integrating these effects
 523 differ largely.

524 More precisely, the pressure computed from the 3 methods are presented in
 525 Figure 15, for locations along a vertical line $\{0.01; y; 0.2\}$ as sketched in Fig-
 526 ure 14b. One can verify that the pressure profiles obtained without capillary
 527 effects and with the Discontinuous Pressure method are quite close, showing
 528 the ability of the latter method to capture properly the pressure field out of
 529 the interface region (pressure gradient) while integrating locally the capillary
 530 effects. As for the Modified BC, the obtained pressure profile is largely mod-
 531 ified. Also, it can be noticed that the Discontinuous Pressure approach yields

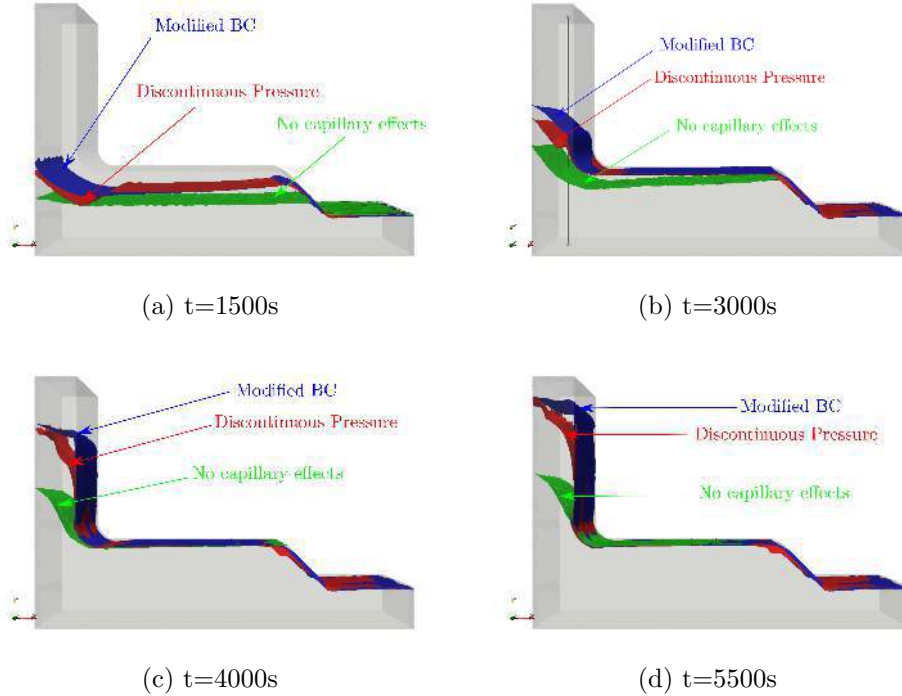


Figure 14: Numerical results - position of the fluid front during the filling of an orthotropic stiffener at different times

532 a pressure jump whose magnitude is a combination of both capillary stresses
 533 in the transverse and plane directions. This method is intrinsically able to
 534 account for the capillary stress orthotropy, and the corresponding flow front
 535 follows the preform principal directions.

536 It can be concluded that with the Modified BC method, first the or-
 537 thotropic character of the capillary effects will not be accounted for by the
 538 simple overall gradient correction, and second the pressure field will not be
 539 discontinuous, opposite to the physics of two-phase flows. Consequently, a
 540 finer analysis is not possible with this approach, and especially it will no
 541 longer hold for a more exhaustive modelling approach relying on velocity
 542 and fluid pressure fields. Conversely, the proposed approach with discontin-
 543 uous pressure will yield relevant pressure and velocity distributions which can
 544 then be incorporated in more exhaustive approaches of the filling stage in-
 545 cluding solid-fluid mechanics couplings through the fluid pressure. Of course,

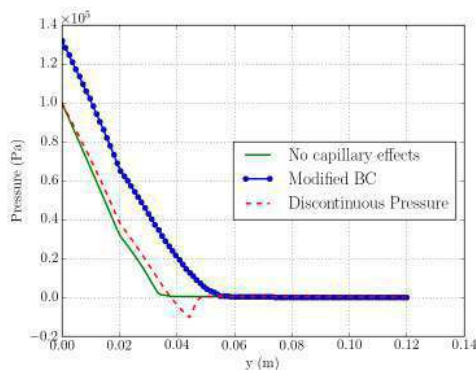


Figure 15: Comparison of the pressure fields with the *Modified BC - Discontinuous Pressure* methods and without capillary effects at $t=3000s$ along a vertical line $\{0.01; y; 0.2\}$ plotted in Fig. 14b.

546 further experimental studies are required to validate and calibrate the nu-
 547 merical model for industrial-like structures. However, these results show that
 548 capillary effects may have huge impact on the filling strategy for the out-of-
 549 autoclave processes targeted here.

550 6. Conclusion

551 In our macroscale configuration, the capillary action is represented by
 552 capillary stresses, acting at the liquid/air interface by the mean of the nor-
 553 mal vector. This stress is weakly enforced in Darcy's equations as an input
 554 parameter of the model and generates a pressure jump at the interface. These
 555 equations are discretized using a stabilized mixed FE method, linear in both
 556 velocity and pressure. The spurious velocities due to the pressure discontinu-
 557 ity are limited by using a local pressure enrichment technique. The numerical
 558 model gives the expected convergence rates, both for velocity and pressure.
 559 Besides, the 2D isotropic simulations of a capillary wicking of water inside
 560 a carbon fabric show a good correlation between the numerical results and
 561 the experimental data, as well as with the analytical model of Washburn's
 562 equation. Finally, a full 3D and orthotropic case has been investigated: the
 563 filling of an aeronautic part with a LRI process. This simulation enlightens
 564 the influence of the capillary effects on the progress of the filling stage, and
 565 demonstrates that the overall response is of highest importance, but also the

566 proper representation of the pressure discontinuity is mandatory for velocity
567 and pressure fields predictions to be used for solid-fluid mechanics couplings
568 for instance [15]. Further experimental studies are now required in order to
569 confirm this scenario.

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