Job-shop scheduling problem with energy consideration

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Abstract

These days, rising energy costs along with general concerns about major environmental issues (global warming, climate change), result in more and more strict production constraints for the industrial sector, which is known to be the first energy consumer and greenhouse gas emitter in the world. There is therefore a growing industrial need to address the problems of production systems related to energy aspects.
In this paper, a job-shop scheduling problem with energetic aspects is considered. The objective is to minimize production costs in terms of energy, while respecting a power peak limitation, along with more traditional production constraints. Two integer linear programming models are proposed for the addressed problem. In order to evaluate and compare the performance of these formulations, computational experiments are presented and numerical results are discussed and analysed.
Keywords: job-shop scheduling, power peak, energy prices, integer linear programming.

1. Introduction

As is known, most energy production systems generate greenhouse gas emissions. These cause the phenomenon of global warming responsible for rising temperatures, climate change, rising of sea levels and changing of the length of the seasons. Climate change also impacts our health and increases the risk of infectious diseases. In addition, most energies used are based on scarce and finite resources which tend to increase their price. This explains the growing general concern about energy aspects in our daily life.

According to Wang and Li [2013], the industrial sector is the first energy consumer and greenhouse gas emitter in the world. Due to increases in energy prices and environmental constraints, manufacturing companies are more and more required to control their production costs, energy consumption, waste and carbon emission.

In the last decades, the consideration of energy aspects in production have gained a lot of attention from long- to short-term planning. Strategic decision-making can concern the planning of investments related to for example the location of warehouses or production plants, or the purchase of machinery and equipment. The former aspect could be driven by factors such as the minimization of vehicle movements, and hence energy consumption. The latter can be affected by e.g. the choice of flexible, modular machines that allow to manage the energy consumption of the plant. At a tactical level, different problems can arise that concern energy aspects. For example, Rapine et al. [2018] tackles a single-item lot-sizing problem in a production system with identical, capacitated parallel machines, with a set of constraints limiting the energy consumption in each period. Similarly, Masmoudi et al. [2017] proposes a lot-sizing problem in a flow-shop system with energy consideration and Beck et al. [2018] addresses an extended economic lot scheduling problem. Finally, problems arise at operational level since electricity prices vary according to short periods and specific power limits are imposed by energy providers. In recent years, these aspects have inspired many research works in several fields, as shown for example in Zavanella et al. [2015] in which Queuing Theory is used to represent energy and power use in a production context, or in Fernandez et al. [2013], which uses buffers to allow an invariant system throughtput while reducing energy consumption during some periods by switching off a subset of machines. In the case of scheduling problems, the combination of energy cost and power limitations can lead to scheduling problems with non-trivial energy optimization criterion and/or constraints.

Concerning electricity profiles, two types of demand response programs are commonly considered (Goldman et al. [2010]). The first policy is the price-driven program where different rates of electricity are used during periods. This leads manufacturers to manage their production planning to produce during periods with a lower cost of energy (as examples Time Of Use (TOU), Critical Peak Pricing (CPP)). The second policy is the event-driven program. In this case, manufacturers are rewarded if they manage to reduce their energy consumption in order to react to specific triggering events such as weather conditions. In both cases, from the customers point of view, it is recommended to reduce the energy consumption during peak periods in order to reduce their overall electricity cost. However, there is a significant difference in the way these problems are dealt with: in the price-driven response program all the prices data are known in advance and thus we can consider deterministic scheduling problems, whereas in the event-driven response program we have to consider dynamic optimization scheduling problems. In this work, we will focus on the former.

Energy-related production costs add up with those related to production planning, which depend directly on the type of manufacturing system. Such situations occur, for example, in automotive industries, or industrial foundries, which are among the largest energy consumers. Therefore, as shown in the recent study of Wichmann et al. [2018], there is an industrial need to take into account the energy costs in production systems, and more specifically time-dependent energy prices in the planning phase, in order to yield an energy-aware scheduling of tasks.

Nowadays, factories need to be more agile in order to meet the challenges of mass customization. This explains the increasingly large number of industrial manufacturing systems organized to allow more flexibility than the previous mass production systems. This flexibility can come from more flexible or reconfigurable resources but also from some changes in the choice of resources used (e.g. with closed and open stations) or in the order of these resources (e.g. U-shaped flow lines). As a generalization of flow-shop, job-shop systems can take these new needs into account.

This work focuses on this type of issues and addresses the energy aspects of scheduling for job-shop production systems. Energy considerations are twofold.

On the one hand, we try to minimize energy costs related to the scheduling. On the other hand, we consider a power peak limitation that impacts the scheduling of operations as each machine is characterized by a nominal power. Hence, the contribution of this work consists of the definition of a job-shop scheduling problem that considers these aspects, which to the best of our knowledge has not yet been addressed. Two integer linear programming (ILP) models are proposed, namely a disjunctive model and a time-indexed model. In addition, a heuristic algorithm is developed in order to find good quality solutions in reasonable computational time.

The remainder of the paper is organized as follows. Section 2 presents the literature review. Section 3 describes the studied problem and introduces the main used notations. In Section 4, the two proposed mathematical formulations are presented. Computational experiments are described and analysed in Section 5. Finally, Section 6 presents conclusions and some perspectives.

2. Literature review

An increasing number of research works consider scheduling problems with energy-related criteria as well as constraints. In this section, a short review (summarized in Table 1) will be presented of some papers that deal with the limitation of power consumption all along the time horizon (power peak), the minimization of the overall energy required to process a set of operations (energy consumption) or the minimization of the economic cost of such energy (energy cost), so as to position the present work.

2.1. Power peak

Scheduling problems with a power peak aim at avoiding a given maximum amount of available power to be overused at any time. The works covered by our literature review that deal with a power peak resort to different solution methods. Kemmoe et al. [2017] and Fang et al. [2011] use mixed-integer linear programming (MILP) to minimize makespan in a job-shop system, and energy consumption, makespan and carbon footprint in a flow-shop system. Heuristic methods are proposed to minimize energy cost in a system with parallel machines (Artigues et al. [2013]) and total tardiness and makespan in a flexible flow-shop (Bruzzone et al. [2012]).

2.2. Energy consumption

In a general way, the minimization of energy consumption deals with either the state of machines (turning on, turning off, idle, processing) or their speed. In all the references that we studied, energy consumption is minimized along with other criteria. Among these latter we find e.g. the makespan (Yildirim and Mouzon [2012], Mansouri et al. [2016], Dai et al. [2013], May et al. [2015], He et al. [2005]) or the tardiness (Mouzon and Yildirim [2008], Fang and Lin [2013], Zhang and Chiong [2016]). A great variety of scheduling problem is dealt with, from systems with single or parallel machines, to job-shop, flow-shop, and flexible flow-shop systems. Heuristic approaches are developed in most of the works, and particularly in those dealing with large-sized instances (like Liu et al. [2016]), with some exception, like the exact approach of Fang et al. [2011].

2.3. Energy cost

Energy cost minimization amounts to take into account a fee schedule that associate different per-energy-unit prices to different slots of the planning horizon, and minimize the overall economic cost. Works in which energy cost is the only criterion, or in which is one of a multi-objective driver (along with e.g. holding costs or tardiness penalty) are found in almost equal number, the former ones dealing with single machine systems (Shrouf et al. [2014], Che et al. [2016]), parallel machine systems (Artigues et al. [2013]) and job-shop systems (Selmair et al. [2016]), the latter ones dealing with flow-shop (Fernandez et al. [2013], Wang and Li [2013]) and flexible flow-shop systems (Luo et al. [2013]). Heuristic algorithms are developed in most of the cases, except for Selmair et al. [2016] which proposes an Integer Linear Program.

Reference	Type of scheduling problem	Objective(s) to minimize	Additional constraint(s)	Optimization method(s)
Shrouf et al. [2014]	Single machine	**energy cost**		Heuristic
Yildirim and Mouzon [2012]		**energy consumption** makespan		Heuristic
Mouzon and Yildirim [2008]		**energy consumption** total tardiness		Heuristic
Che et al. [2016]		**energy cost**		Heuristic
Artigues et al. [2013]	Parallel machines	**energy cost**	**power peak**	Heuristic
Fang and Lin [2013]		**energy consumption** tardiness penalty		Heuristic
Mansouri et al. [2016]	Flow-shop	makespan **energy consumption**		Heuristic
Fang et al. [2011]		makespan **energy consumption** carbon footprint	**power peak**	Mixed integer programming (Disjunctive formulation)
Wang and Li [2013]		**energy cost** **energy consumption**		Heuristic
Dai et al. [2013]	Flexible flow-shop	makespan **energy consumption**		Heuristic
Bruzzone et al. [2012]		total tardiness makespan	**power peak**	Heuristic
Luo et al. [2013]		makespan **energy cost**		Heuristic
Zhang and Chiong [2016]	Job-shop	**energy consumption** total weighted tardiness		Heuristic
May et al. [2015]		**energy consumption** makespan		Heuristic
He et al. [2005]		**energy consumption** makespan		Heuristic
Kemmoe et al. [2017]		makespan	**power peak**	Heuristic
Selmair et al. [2016]		**energy cost**		Integer linear programming (Time-indexed formulation)
Liu et al. [2016]		**energy consumption** total weighted tardiness		Heuristic

Table 1: An overview of the literature review for production systems with energy consideration.

2.4. Conclusion

As we can observe from this review, and to the best of our knowledge, the joint consideration of energy cost minimization and power peak for a job-shop

system has not been studied yet. Moreover, most works propose heuristics, whereas few works consider exact approaches.
In this work, we study a job-shop scheduling problem without any case-specific constraints for the sake of generality, and we consider an energy cost driver and a power peak constraint. We propose for this problem two integer programming formulations inspired by classic formulations for the job-shop scheduling problem.

3. Problem introduction

This section is structured in two parts: in the first, the problem is formally stated, while in the second a detailed example is given to help the reader understand how the considered energy-related features (see Section 2.4) can impact the scheduling in a job-shop system.

3.1. Formal problem definition

In the classic job-shop scheduling problem, the jobs of a set J must be processed on a set M of machines. The processing of a job j on a machine m is called an operation (j, m): the sequence of operations of each job is a predefined, ordered subset $O_j \subseteq M$ of machines, and the machine that must process job j immediately after $m \in O_j$ is denoted with $s_j(m) \in O_j$. The processing time $q_{j,m}$ of each operation is also known and deterministic. The scheduling problem consists in determining the starting date of each operation. The goal is to optimize some given economic and/or production criteria while meeting some classic production constraints, for example preventing operations to be preempted, or any two operations to be executed at the same time on the same machine, or an upper bound $\overline{C}_{\max}$ on the last completion time (makespan), and possibly some additional problem-specific constraints.
In this study, some energy aspects of the scheduling are taken into account.
Each machine $m \in M$ has a nominal power ϕ_m, and each operation (j, m) has a power consumption equal to ϕ_m and constant over its duration. Moreover,

a limit $W_{\max}$ is imposed on the maximum overall power peak which cannot be exceeded at any time. Such a limit is often found in real life contexts, as electric power suppliers typically include a defined power limit in supply contracts for companies.

The energy consumption $\phi_m \cdot q_{j,m}$ of an operation (j, m) has a price, which evolves over time according to a given fee schedule. A set P of periods is defined, each $p \in P$ being associated with a length cap^p and a price per energy unit cp^p. We denote with $C^p = \sum_{p' \in P : p' < p} cap^{p'}$ the starting time of period p. Periods P are further defined so as to have $\sum\limits_{p \in P} cap^p = \overline{C}_{\max}$, i.e. the length of the time horizon.

Table 2 summarizes these notations.

symbol	meaning
J	set of jobs
M	set of machines
O_j	ordered subset of machines associated with job j
$s_j(m)$	machine that must process job j after machine m
$\overline{C}_{\max}$	length of the time horizon (upper bound on makespan)
$q_{j,m}$	processing time of operation (j, m)
ϕ_m	nominal power of machine m
$W_{\max}$	maximum overall power peak
P	set of periods of the fee schedule
cap^p	length of period p
cp^p	price per energy unit of period p
C^p	starting time of period p

Table 2: Main notations used to describe the problem parameters.

Figure 1 illustrates the cost associated with one job to schedule on one machine over a timespan of 11 hours. Let us consider a power consumption $\phi = 1$ KW for the machine and a processing time $q = 2$ hours for the operation. The energy prices are defined over three periods as $cp^1 = 3$, $cp^2 = 1$ and $cp^3 = 2$ €/KWh,

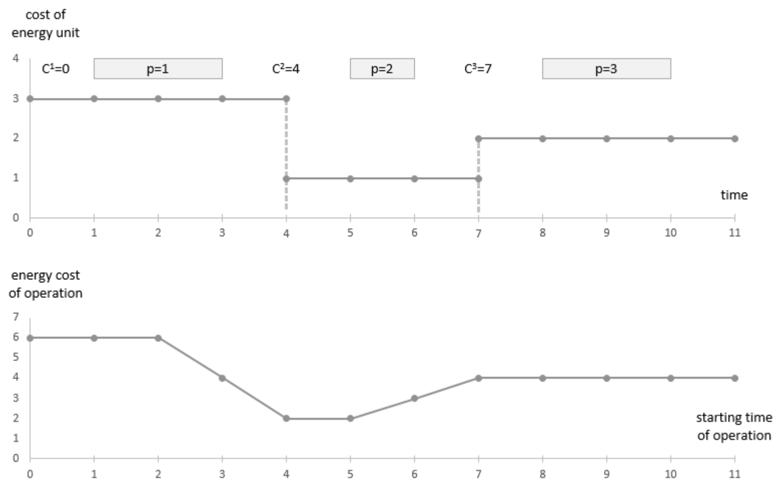

Figure 1: Price profile variation and generated energy costs.

and their durations are 4, 3 and 4 hours respectively, as shown in the upper part of the figure. The bottom part shows the generated energy cost depending on the starting time of the unique operation. If for instance the starting time is 2, then the operation starts and ends in the first period and its cost is $cp^1 \cdot q \cdot \phi = 6$ €, $q \cdot \phi = 2$ KWh being the energy consumption. If the starting time is 3, then the processing of the operation spans over periods $p = 1$ and $p = 2$ and its energy cost decreases to $cp^1 \cdot 1 \cdot \phi + cp^2 \cdot 1 \cdot \phi = 4$ €, as it is expected since the energy fee of period 2 is lower.

The problem proposed here consists in finding a schedule that complies with the classic constraints (precedence, non-preemption, non-overlap, makespan upper bound) and the maximal power peak while minimizing the overall energy cost. An example can help the reader to understand more in depth how scheduling can be affected by the energy aspects introduced before.

3.2. Impact of the energy aspects on the scheduling

This example deals with 3 jobs and 3 machines. The sets $O_1 = \{1, 3, 2\}$, $O_2 = \{2, 1, 3\}$ and $O_3 = \{2, 3, 1\}$ and the notations $s_1()$, $s_2()$, $s_3()$ (introduced at the beginning of Section 3.1) define how the operations of each job are sequenced, e.g. job $j = 2$ must be processed on machine $m = 2$ first, then on machine $m = 1$, i.e. $s_2(2) = 1$, and finally on machine $m = 3$, i.e. $s_2(1) = 3$.

Table 3 gives the processing times of operations in hours.

jobs	machines		
	1	2	3
1	2	1	3
2	0.5	1.5	1
3	1	1.5	2

Table 3: Processing time of each operation.

Moreover, let us suppose the following energy features:

▶ the nominal power values are $\phi_1 = 5$, $\phi_2 = 6$ and $\phi_3 = 8$ KW;

▶ the maximum authorized overall power consumption is 13 KW;

▶ the time horizon is divided in a sequence of ON/OFF-peak periods. Slots with odd p indices are ON-peak periods with $cap^p = 3$ hours and $cp^p = 0.159$ €/KWh, whereas OFF-peak periods have $cap^p = 4$ hours and $cp^p = 0.13$ €/KWh. These values are inspired by those proposed by the main French electricity provider, see also Section 5.1.

If every energy aspect is neglected and the objective function to minimize is the makespan, the optimal solution is the schedule represented by the Gantt chart of Figure 2. The optimal makespan is 7.5 hours.

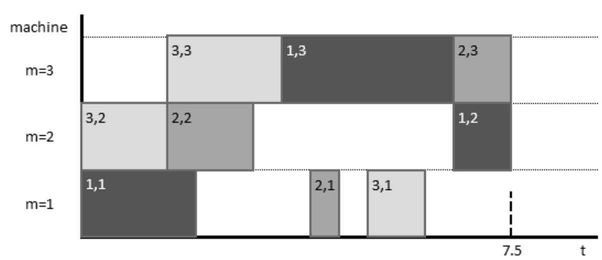

Figure 2: Gantt chart of the schedule that minimizes the makespan of the example, without taking into account any energy aspects. Each block represents an operation (j, m), operations of the same job have the same filling.

3.2.1. *Introduction of a power peak constraint*

The introduction of the maximum power limit alone affects considerably the scheduling. Figure 3 shows the solution schedule that minimizes the makespan while meeting the maximum power requirement. This latter not only prevents the three machines to be switched on at the same time, but even some subsets of them, like e.g. machines $m = 2$ and $m = 3$ in this case. Due to this, the schedule of some operations must be postponed as they must wait for power availability, as it is the case here for operation (2,2). The makespan of the optimal solution is then equal to 10 hours.

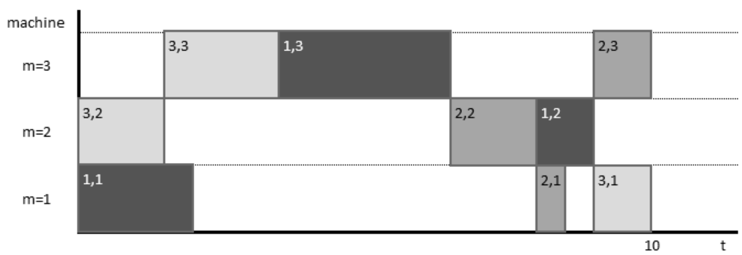

Figure 3: Gantt chart of the schedule that minimizes the makespan, while complying with the maximum power limitation. The optimal makespan increases of a considerable 33%.

3.2.2. *Energy cost minimization*

Changing the objective function to seek for the solution that minimizes the energy cost affects the production planning even more dramatically, as shown in Figure 4, which considers the alternate ON- and OFF-peak periods. We point out that the scheduling must not only comply with the power limitation: the solution schedule in Figure 4 is obtained with a timespan upper bound $\overline{C}_{\max}$ equal to 10 hours (i.e. the optimal value of makespan). Therefore such a solution is equivalent to that of Figure 3 w.r.t the makespan minimization criterion. However, as a result of the cost driver, as many operations as possible are planned during the only OFF-peak slot, thus changing the schedule significantly. The overall energy cost is 12.80 €, whereas the cost of the previous solution is 13.33 €.

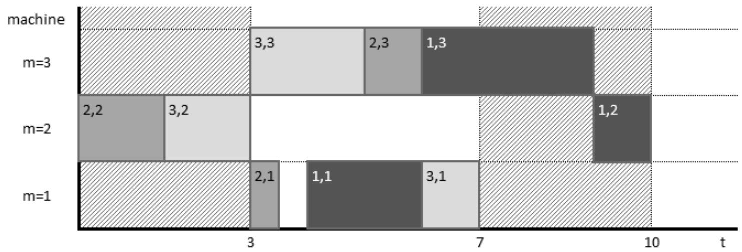

Figure 4: Production plan with minimum overall energy cost among those that have minimum makespan and comply with the maximum power limitation. Ruled areas represent ON-peak periods. Not surprisingly, operations tend to be scheduled as much as possible during OFF-peak periods.

In some cases the available timespan can be larger than the optimal makespan which allows to achieve further savings. In the case of Figure 5 we consider schedules with a makespan up to 12 hours. This allows to use one more OFF-peak period and some operations, e.g. (1,2), to be scheduled during it, which reduces the overall energy cost to 12.39 €.

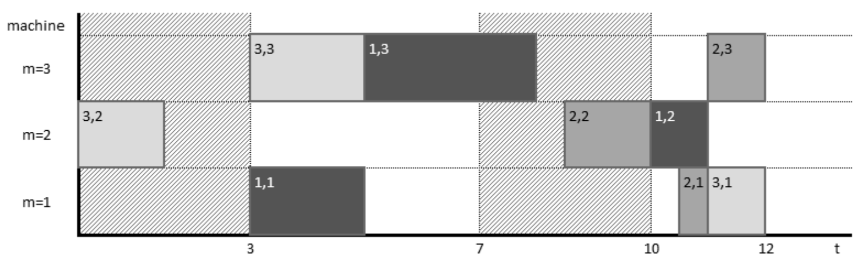

Figure 5: When priority is given to the energy cost and a higher makespan is tolerated, further economies can be achieved. In the example, a larger timespan allows a cost reduction of more than 3%.

4. Mathematical formulations and Heuristic algorithm IP2H$_n$

In this section, we present two integer programming models for the job-shop scheduling problem with energy aspects, along with a heuristic algorithm that can help find feasible solutions with a good ratio between solution quality and computational time.

4.1. Disjunctive formulation IP1

The disjunctive formulation for the job-shop scheduling problem is based on Manne [1960]. In spite of being known to have a weak linear relaxation (which can result in a reduced efficiency of solving algorithms, as we will discuss in Section 5.2), this formulation is among the most used in the literature. In this section, we extend the base model to take into account the power and energy cost features previously discussed (see Section 3.1). Moreover, in order to apply the disjunctive formulation, an additional assumption must be done, i.e. no operation can span over three or more periods in P. This can be expressed as $\max_{j,m} \{q_{j,m}\} \leq \min_{p} cap^p$, i.e. the longest operation must be shorter than the shortest period. We have also been inspired by Fang et al. [2011] concerning the disjunctive modeling of the usage of cumulative resources, in order to express the maximum power constraint.

4.1.1. Decision variables

The proposed disjunctive formulation for the job-shop scheduling problem with energy consideration is an Integer Linear Program (ILP) IP1 whose binary decision variables are the following:

- schedule variables $X^p_{j,m}$, $j \in J$, $m \in O_j$, $p \in P$, $X^p_{j,m} = 1 \Leftrightarrow$ operation (j, m) completes during period p;
- disjunction variables $y_{j,j',m}$, $j, j' \in J : j < j'$, $m \in O_j \cap O_{j'}$, $y_{j,j',m} = 1 \Leftrightarrow$ job j precedes job j' on machine m;
- interperiod variables $S_{j,m}$, $j \in J$, $m \in O_j$, $S_{j,m} = 1 \Leftrightarrow$ operation (j, m) starts and ends in same period, $S_{j,m} = 0 \Leftrightarrow$ operation (j, m) spans two consecutive periods;
- overlap variables: given jobs $j, j' \in J$ and machines $m \in O_j$ and $m' \in O_{j'}$:
  - $f_{j,m,j',m'} = 1 \Leftrightarrow$ starting time of operation (j', m') is strictly less than completion time of (j, m);

► $g_{j,m,j',m'} = 1 \Leftrightarrow$ completion time of operation (j', m') is greater or equal than the completion time of (j, m).

The model IP1 also makes use of some integer nonnegative variables, namely:

► partial completion time variables $C^p_{j,m}$, $j \in J$, $m \in O_j$, $p \in P$, the date of period p at which operation (j, m) completes;

► time consumption variables $\overline{C}^p_{j,m}$, $j \in J$, $m \in O_j$, $p \in P$, the portion of period p used to complete processing of operation (j, m), equal to $\min\{C^p_{j,m}, q_{j,m}\}$.

It is worth giving some insight on how variables $C^p_{j,m}$, $\overline{C}^p_{j,m}$, $f_{j,m,j',m'}$ and $g_{j,m,j',m'}$ behave to help the reader to understand how they are used to model the problem. Let us look at Figure 6.

In the upper part of the figure we have two operations (j', m') and $(j", m")$. For operation (j', m'), $S_{j',m'} = 0$ holds as it spans over periods $p - 1$ and p $(p > 1)$ and $C^p_{j',m'} = \overline{C}^p_{j',m'} = t_3 - C^p < q_{j',m'} = t_3 - t_1$, and $q_{j',m'} - \overline{C}^p_{j',m'} = C^p - t_1$ is the quantity of period $p - 1$ spent for the first part of the operation. On the other hand, operation $(j", m")$ has $S_{j",m"} = 1$ as it begins and ends in the same period p, and we have $C^p_{j",m"} = t_4 - C^p \geq q_{j",m"} = t_4 - t_2 = \overline{C}^p_{j",m"}$. In both cases, terms $\overline{C}^p_{j,m}$ and $q_{j,m} - \overline{C}^p_{j,m}$ allow to compute, along with $S_{j,m}$, the time spent by an operation in each period, hence its energy cost.

The bottom part of Figure 6 shows all possible cases of time overlap between two operations. In we consider operations (j, m) and (j', m'), then from the definition of variables $f_{j,m,j',m'}$ and $g_{j,m,j',m'}$, we can see that they are both equal to 1 only in cases d) and f), whereas in the other cases a), b), c), g), h) one of the two variables is equal to 0. The other overlap situation, c), can be detected analogously by using the symmetric variables $f_{j',m',j,m}$ and $g_{j',m',j,m}$. The last overlap possibility occurs with operations that are shorter of (j, m), like $(j", m")$ in e): this also can be detected since variables $f_{j",m",j,m}$ and $g_{j",m",j,m}$ will be both equal to 1. Overlap variables f and g can therefore be used to measure the cumulative use of the power resource, and ultimately to enforce the power peak constraint.

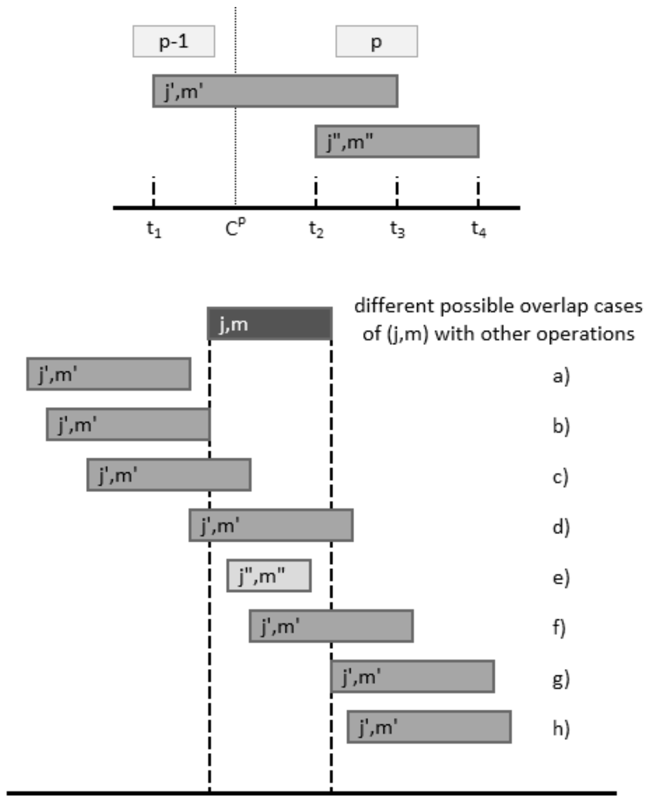

Figure 6: Graphical explanation of how S, C, $\overline{C}$ variables and f, g variables can help computing the overall energy cost and enforcing the maximum power limit constraint, respectively.

4.1.2. Objective function and constraints

The objective function of model IP1 is then the following:

$$\min z = \sum_{\substack{j \in J \\ m \in O_j}} \Big(cp^1 \ \phi_m \ q_{j,m} \ X^1_{j,m} + \sum_{\substack{p \in P \\ p > 1}} \big(cp^{p-1} \ \phi_m \ q_{j,m} \ X^p_{j,m} + ((cp^p - cp^{p-1}) \ \phi_m \ \overline{C}^p_{j,m}) \big) \Big) \tag{1}$$

In it, the overall energy cost is computed as the sum of the cost terms associated with each operation. Such terms are computed as follows and similarly as explained in Figures 1 and 6. When an operation (j, m) ends during period $p = 1$, $X^1_{j,m} = 1$ and $(\forall p > 1)$ $X^p_{j,m} = 0$, hence from (1) the energy cost is $cp^1 \cdot \phi_m \cdot q_{j,m}$. If operation (j, m) ends during period $p > 1$, then $X^p_{j,m} = 1$ and $(\forall p' \neq p)$ $X^{p'}_{j,m} = 0$, and either it is processed entirely during period p, or its processing begins at period $p-1$. In the first case, the term $cp^{p-1}\phi_m q_{j,m} + (cp^p - cp^{p-1})\phi_m \overline{C}^p_{j,m}$ correctly reduces to $cp^p \phi_m \overline{C}^p_{j,m}$ as $q_{j,m} = \overline{C}^p_{j,m}$; in the second case, the same term reduces to $cp^{p-1}\phi_m(q_{j,m} - \overline{C}^p_{j,m}) + cp^p \phi_m \overline{C}^p_{j,m}$, which is a weighted sum of the energy unit costs of the two periods according to the time spent by (j,m) in each one (see Section 4.1.1).

The constraints to be enforced are the following:

$$\sum_{p\in P}(C^p \cdot X^p_{j,m} + C^p_{j,m}) \leq \overline{C}_{\max} \qquad , \ \forall j \in J, m \in O_j \tag{2}$$

Constraints (2) state that each operation must be completed before the timespan limit $\overline{C}_{\max}$. To this end, we note that term $\sum_{p\in P}(C^p \cdot X^p_{j,m} + C^p_{j,m})$ represents the completion time of operation (j,m), and hence $\sum_{p\in P}(C^p \cdot X^p_{j,m} + C^p_{j,m}) - q_{j,m}$ is its starting time.

$$\sum_{p\in P}(C^p \cdot X^p_{j,m} + C^p_{j,m}) \leq \sum_{p\in P}\left(C^p \cdot X^p_{j,s_j(m)} + C^p_{j,s_j(m)} - q_{j,s_j(m)} \cdot X^p_{j,s_j(m)}\right) \qquad , \ \forall j \in J, m \in O_j \tag{3}$$

Constraints (3) guarantee the execution sequence of each job, as a time gap is imposed between the end of an operation (j,m) and the end of the following operation of the same job, $(j, s_j(m))$, and such gap is larger of equal to the processing time $q_{j,s_j(m)}$ of the latter.

$$\begin{aligned}&\sum_{p\in P}(C^p \cdot X^p_{j',m} + C^p_{j',m} - q_{j',m} \cdot X^p_{j',m}) - \\ &\sum_{p\in P}(C^p \cdot X^p_{j,m} + C^p_{j,m} - q_{j,m} \cdot X^p_{j,m}) \geq q_{j,m} + \overline{C}_{\max} \cdot (y_{j,j',m} - 1) \\ &\qquad , \ \forall j, j' \in J : j < j', m \in O_j \cap O_{j'}\end{aligned} \tag{4}$$

$$\begin{aligned}&\sum_{p\in P}(C^p \cdot X^p_{j,m} + C^p_{j,m} - q_{j,m} \cdot X^p_{j,m}) - \\ &\sum_{p\in P}(C^p \cdot X^p_{j',m} + C^p_{j',m} - q_{j',m} \cdot X^p_{j',m}) \geq q_{j',m} - \overline{C}_{\max} \cdot y_{j,j',m} \\ &\qquad , \ \forall j, j' \in J / j < j', m \in O_j \cap O_{j'}\end{aligned} \tag{5}$$

Constraints (4) and (5) are disjunction constraints: for each two jobs j and j' that at some point must be processed on the same machine m, such constraints are both defined once – which explains the condition $j < j'$. If $y_{j,j',m} = 1$, i.e. operation (j,m) executes before operation (j',m), then the associated constraint (4) imposes a minimum time gap of $q_{j,m}$ between the starting time of (j,m) and that of (j',m), while (5) is redundant. The two constraints behave in a complementary way if $y_{j,j',m} = 0$.

$$\sum_{p\in P} X^p_{j,m} = 1 \qquad , \ \forall j \in J, m \in O_j \tag{6}$$

Constraints 6 enforce the scheduling of each operation at some point of the time horizon.

$$C^p_{j,m} \le cap^p \cdot X^p_{j,m} \qquad , \forall j \in J; m \in O_j, p \in P \tag{7}$$

$$C^1_{j,m} \ge q_{j,m} \cdot X^1_{j,m} \qquad , \forall j \in J, m \in O_j \tag{8}$$

$$C^p_{j,m} \ge X^p_{j,m} \qquad , \forall j \in J, m \in O_j, p \in P : p > 1 \tag{9}$$

Constraints (7) to (9) link schedule and partial completion time variables: for each operation (j, m) and period p, they impose $X^p_{j,m} = 0 \Rightarrow C^p_{j,m} = 0$ and $X^p_{j,m} = 1 \Rightarrow C^p_{j,m} > 0$, with (8) representing the special case of period $p = 1$.

$$C^p_{j,m} \le q_{j,m} + (cap^p - q_{j,m}) \cdot S_{j,m} \qquad , \forall j \in J, m \in O_j, p \in P \tag{10}$$

$$\overline{C}^p_{j,m} \le C^p_{j,m} \qquad , \forall j \in J, m \in O_j, p \in P \tag{11}$$

$$\overline{C}^p_{j,m} \le q_{j,m} \qquad , \forall j \in J, m \in O_j, p \in P \tag{12}$$

$$\overline{C}^p_{j,m} \ge C^p_{j,m} - cap^p \cdot S_{j,m} \qquad , \forall j \in J, m \in O_j, p \in P \tag{13}$$

$$\overline{C}^p_{j,m} \ge q_{j,m} \cdot (S_{j,m} + X^p_{j,m} - 1) \qquad , \forall j \in J, m \in O_j, p \in P \tag{14}$$

Constraints (10)-(14) link interperiod variables, partial completion time variables and time consumption variables as explained in Section 4.1.1. More in detail, let (j, m) be a given operation, and p the period in which it completes, i.e. s.t. $X^p_{j,m} = 1$. If the operation spans over periods $p - 1$ and p, then $S_{j,m} = 0$, so constraints (14) are redundant while (11) and (13) jointly impose $(\forall p' \in P)$ $\overline{C}^{p'}_{j,m} = C^{p'}_{j,m}$, but p is the only one in which they are not equal to 0 (due to (7)) but bounded by $q_{j,m}$ instead (due to (10) and (12)). Conversely, if (j, m) also starts in p, then $S_{j,m} = 1$, constraints (10) are redundant while constraints (12) and (14) jointly impose $\overline{C}^p_{j,m} = q_{j,m}$ ($0 \le \overline{C}^{p'}_{j,m} \le q_{j,m}$ for periods $p' \neq p$).

$$\begin{aligned}&\sum_{p \in P} (C^p \cdot X^p_{j,m} + C^p_{j,m}) - \\ &\sum_{p \in P} (C^p \cdot X^p_{j',m'} + C^p_{j',m'} - q_{j',m'} \cdot X^p_{j',m'}) \le \overline{C}_{\max} \cdot f_{j,m,j',m'} \\ &\qquad , \forall j, j' \in J, m \in O_j, m' \in O_{j'}, m \neq m'\end{aligned} \tag{15}$$

$$\sum_{p\in P}(C^p\cdot X^p_{j',m'}+C^p_{j',m'})-\sum_{p\in P}(C^p\cdot X^p_{j,m}+C^p_{j,m})\leq \overline{C}_{\max}\cdot g_{j,m,j',m'}-1 \\ ,\ \forall j,j'\in J, m\in O_j, m\in O_{j'}, m\neq m' \tag{16}$$

$$\phi_m\cdot\sum_{p\in P}X^p_{j,m}+\sum_{j'\in J}\sum_{\substack{m'\in O_{j'}\\ m'\neq m}}(f_{j,m,j',m'}+g_{j,m,j',m'}-1)\cdot\phi_{m'}\leq W_{\max} \\ ,\ \forall j\in J, m\in O_j \tag{17}$$

Given operations (j,m) and (j',m'), the corresponding constraints (15) forces the variable $f_{j,m,j',m'}$ to value 1 in case the completion time of (j,m) is strictly greater than the starting time of (j',m'), i.e. accordingly to its definition. Similarly for the corresponding constraint (16) w.r.t the variable $g_{j,m,j',m'}$. Then, the corresponding constraint (17) prevents operation (j,m) from completing during period p if some overlaps with other operations occur such that the total used power exceeds the maximum authorized value $W_{\max}$.

4.2. Time-Indexed formulation IP2

The time-indexed formulation for the job-shop scheduling problem is based on Bowman [1959] and is mainly used to model problems with cumulative resources. A well-known drawback is that the time index causes the number of variables to grow rapidly with the length of the time horizon. However, time-indexed-based models can usually yield a good linear relaxation; moreover, the additional assumption required to apply the disjunctive formulation for the job-shop scheduling problem, i.e. $\max_{j,m}\{q_{j,m}\}\leq\min_p cap^p$ (see Section 4.1) is no longer necessary and can be discarded.

In this section, we extend the base time-indexed model of Bowman [1959] with the power and energy cost features.

4.2.1. Decision variables and additional notations

The time-indexed formulation for the job-shop scheduling problem with energy consideration is a 0-1 Linear Program (01LP), which we name IP2, with only one family of decision variables:

- ▶ binary schedule variables $X^t_{j,m}$, $j\in J$, $m\in O_j$, $t\in\{1,\ldots,\overline{C}_{\max}\}$, $X^t_{j,m}=1\Leftrightarrow$ operation (j,m) is completed at time t.

The key idea of time-indexed models like IP2 is to explicitly divide the planning horizon in unit time slots, hence decisions concern how to allocate operations to slots. To this end, specify the last occupied time slot of an operation is sufficient, e.g. $X^t_{j,m} = 1$ states that operation (j, m) is deployed over slots $t - q_{j,m} + 1,\ldots,t$. Let us denote for the sake of simplicity the time horizon $\{1,\ldots,\overline{C}_{\max}\}$ with T and the per-energy-unit cost at time $t \in T$ with $ct^t = cp^p \ \forall t \in [C^p, C^{p+1}[$.

4.2.2. Objective function and constraints

The objective function of model IP2 is the following:

$$\min z = \sum_{t \in T} ct^t \Big(\sum_{m \in M} \phi_m \Big(\sum_{\substack{j \in J: \\ m \in O_j}} \sum_{\substack{t' \in T: \\ t \leq t' < t + q_{j,m}}} X^{t'}_{j,m} \Big) \Big) \tag{18}$$

Relation (18) can be explained as follows. The nominal power ϕ_m of machine $m \in M$ is equal to the per-time-unit energy consumption of m when it is turned on. Term $ct^t \cdot \phi_m$ is hence the energy cost of having m operative at time $t \in T$, which we pay for each operation (j, m) (j being a job that must be processed on m) s.t. $X^{t'}_{j,m} = 1$, $t' \in \{t,\ldots,t + q_{j,m} - 1\}$, as it is explained in Figure 7.

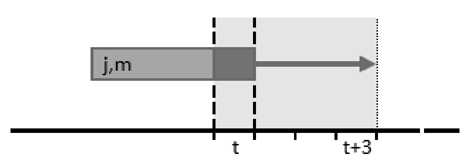

Figure 7: The energy cost of an operation (j, m) is the sum of the per-time-unit energy costs of the time slots involved in its deployment. In this example, $q_{j,m} = 4$: the energy cost of using machine m at time t is paid if the operation is completed at any time slot $t' \in \{t,\ldots,t + 3\}$.

The constraints to be enforced are the following:

$$\sum_{t \in T} t \cdot X^t_{j,m} \leq \overline{C}_{\max} \quad , \forall j \in J, m \in O_j \tag{19}$$

Similarly to (2), constraints (19) force each operation to be completed within the timespan limit $\overline{C}_{\max}$.

$$\sum_{t \in T} t \cdot X^t_{j,m} \leq \sum_{t \in T} (t \cdot X^t_{j,s_j(m)} - q_{j,s_j(m)} \cdot X^t_{j,s_j(m)}) \quad , \forall j \in J, m \in O_j \tag{20}$$

Constraints (20), as constraints (3) for IP1, enforce the execution sequence of each job by separating the completion times of two successive operations (j, m)

and $(j, s_j(m))$ of a same job j by a time gap greater or equal to the processing time of the second one.

$$\sum_{\substack{j\in J:\\ m\in O_j}} \sum_{\substack{t'\in T:\\ t\leq t'<t+q_{j,m}}} X_{j,m}^{t'} \leq 1 \qquad , \forall t \in T, m \in M \tag{21}$$

Disjunction constraints (21) assert that at most one operation can be processed at a given date on a machine. The underlying principle is similar to that of objective function (18) illustrated in Figure 7.

$$\sum_{\substack{t\in T:\\ t\geq q_{j,m}}} X_{j,m}^{t} = 1 \qquad , \forall j \in J, m \in O_j \tag{22}$$

$$\sum_{\substack{t\in T:\\ t<q_{j,m}}} X_{j,m}^{t} = 0 \qquad , \forall j \in J, m \in O_j \tag{23}$$

Constraints (22) impose the execution of all tasks, while relations (23) assert that operation (j, m) cannot complete sooner that a date equal to its processing time.

$$\sum_{\substack{j\in J\\ m\in O_j}} \sum_{\substack{t'\in T:\\ t\leq t'<t+q_{j,m}}} \phi_m \cdot X_{j,m}^{t'} \leq W_{\max} \qquad , \forall t \in T \tag{24}$$

Constraints (24) enforces at each date the maximum power limit, again similarly to (18) and (21).

4.3. Comparison of the size of models IP1 and IP2

Before describing in detail the computational session conducted to assess the performance of the two proposed models, it is useful to compare their size in terms of both number of variables and number of constraints, as summarized by Table 4. The main reason of this comparison is that, as was pointed out at the beginning of Section 4.2, one of the main drawbacks of time-indexed formulations that are found in the literature is the large number of variables.

The number of variables of IP2 is greater than that of IP1 when the following condition (25) is fulfilled:

$$|T| > 2|J||M| - \tfrac{3|J|+1}{2} + 3|P| + 1 \tag{25}$$

Indeed, the number of variables of IP2 is not much higher than that of IP1 as usually with these types of scheduling models. The reason probably lies in the

model	# of binary v.	# of nonnegative integer v.	# of constraints
IP1	$\lvert J\rvert\lvert M\rvert\ (2\lvert J\rvert\lvert M\rvert - \frac{3\lvert J\rvert+1}{2} + \lvert P\rvert + 1)$	$1 + 2\lvert J\rvert\lvert M\rvert\lvert P\rvert$	$\lvert J\rvert\lvert M\rvert\ (2\lvert J\rvert\lvert M\rvert - \lvert J\rvert + 7\lvert P\rvert + 3) - \lvert J\rvert + 1$
IP2	$\lvert J\rvert\lvert M\rvert\lvert T\rvert$	0	$\lvert J\rvert\ (4\lvert M\rvert - 1) + \lvert T\rvert\ (\lvert M\rvert + 1)$

Table 4: Comparison of the proposed two models for the job-shop scheduling problem with energy consideration. $|J|$, $|M|$, $|P|$, $|T|$ denote the number of jobs, the number of machines, the number of periods of the fee schedule and the number of time slots, respectively.

overlap variables of IP1, f and g, which are required to model the cumulative use of the power resource and hence enforce the limit on the power consumption. The number of such variables, which are not present in basic disjunctive models for the job-shop scheduling problem, grow as fast as $2|J|^2|M|^2$. This can become quickly very huge even for medium-sized job-shop problem instances.

4.4. A heuristic algorithm based on the time-indexed ILP formulation

Both the two formulations IP1 and IP2 give an exhaustive representation of the studied problem, in the sense that for both of them, each feasible solution corresponds to a feasible schedule, and each schedule can be represented by at least one feasible solution. This means that any Branch&Bound (B&B) algorithm based on one of these two models is an exact algorithm, as it can implicitly explore the set of all the feasible schedules and yield an optimal solution.
An interesting aspect of IP2 is that it can easily give rise to a heuristic algorithm by simply considering a subset of dates $\overline{T} \subset T$ instead of the whole set. By doing so, we obtain a restricted model IP2($\overline{T}$) whose set of schedule variables $X_{j,m}^t$ decreases proportionally in size, as do some families of constraints (namely (21) and (24)). Clearly, not every feasible schedule has a corresponding solution of such a restricted model: more exactly, every schedule with at least one operation (j, m) having a completion time at a date $t \in T\backslash\overline{T}$ cannot be represented, thus a B&B algorithm based on IP2($\overline{T}$) cannot find it. For this reason, such an algorithm is heuristic, and the solution it yields comes with no proof of optimality: nevertheless, it can be interesting to use it in order to quickly find feasible solutions, as it certainly ends in a shorter time than a B&B algorithm based on IP2.

Let us now define the restricted date set $\overline{T_n} = \{t \in T : \frac{t}{n} = \lfloor \frac{t}{n} \rfloor\}$, i.e. the subset of dates that are multiples of a given positive integer n. We denote with IP2H$_n$ the heuristic algorithm obtained by solving with a B&B the restricted model IP2($\overline{T_n}$).

5. Computational experiments

This section provides the computational experiments that have been conducted to evaluate the performances of the proposed formulations. Both models have been implemented with the commercial solver CPLEX 12.6 which uses B&B.

In the following, for the sake of brevity, we will use the notation IP1 to denote both the disjunctive formulation for the studied problem (see Section 4.1) and the algorithm that consists in solving such formulation by B&B, and similarly for notations IP2 and IP2H$_n$ w.r.t the time-indexed formulation (see Section 4.2) and its heuristic version defined on a subset of dates (see Section 4.4).

Tests have been run on an Intel Xeon E5530 with a 2.39 GHz CPU and 62.75 GB RAM. For a fair comparison, all the algorithms have had imposed the same running time limit of one hour.

5.1. Instance generation

We generate a set of 90 instances starting from two well-known benchmark instances, namely $mt06$ (taken from Fisher and Thompson [1963]) with 6 jobs and 6 machines, and $la04$ (taken from Lawrence [1984]) with 10 jobs and 5 machines, and extending them by adding machine power values, overall power consumption peak, timespan, period durations and electricity prices.

Five sets of machine power values ϕ_m are randomly generated from the uniform distribution $U(5, 10)$. We denote the resulting datasets with $DS1_\phi$ to $DS5_\phi$. For each dataset, we consider 3 different value of power peak $W_{\max} = \alpha \cdot \sum_{m \in M} \phi_m$, with $\alpha \in \{0.7, 0.9, 1.0\}$, and 3 different values of planning horizon duration $\overline{C}_{\max} = \lambda \cdot C^*_{max}$, with $\lambda \in \{1.0, 1.1, 1.2\}$. C^*_{max} is the optimal makespan obtained by imposing the power limitation constraint and minimizing the makespan instead of the overall energy cost (as done e.g. in Figure 3). As shown in Figure 8, for each initial benchmark instance, we have 5 groups of instances, each descending from a machine power values dataset and counting 9 instances which differ in the values of parameters α and λ. Conversely, each result will be presented as the average value among the five instances which descend from the same benchmark and have same values of α and λ, i.e. that differ in the dataset they belong to. This allows to mitigate the effects of the random generation of ϕ_m values.

As to electricity prices, we consider for each instance a succession of ON/OFF-peak periods. Values are inspired by the French case (see EDF [2017]), where the duration of ON-peak periods is 16 hours with an electricity price of 0.159 €/KWh, while the duration of OFF-peak periods is 8 hours and the electricity price is 0.13 €/KWh. As a consequence, the generated machine power values are meant to be expressed in KW, whereas the duration of operations (as they are taken from the literature) are considered to be expressed in hours. These duration values for periods and operations comply with the assumption $\max\limits_{j,m} q_{j,m} \leq \min\limits_{p} cap^p$ required to apply formulation IP1(see Section 4.1).

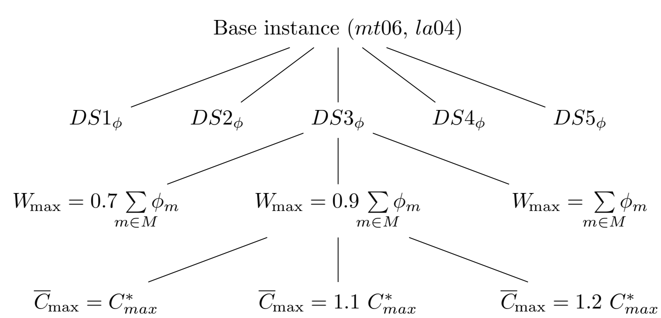

Figure 8: Sketch of the instance generation scheme.

5.2. Computational results and detailed analysis

Table 5 shows the results of the computational session. Each line represents the average values over five instances that descend from the same benchmark instance and have the same values of parameters α and λ (see Section 5.1), shown in columns 3 and 4. Lines 1-9 and 10-18 refer to the instances descending from benchmark job-shop scheduling problem instances $mt06$ and $la04$, respectively. The size of instances, i.e. $|J|$ and $|M|$, is shown in column 2. For each line, and for both IP1 and IP2, we report in columns 5-12 the number *#opt* of proven optimal solutions (within time limit), the number *#fea* of instances for which a feasible solution has been found (within time limit), the average relative optimality gap $\overline{gap}$ and the average computational time $\overline{CPU}$ (in seconds) over the instance set associated with the line.

The relative optimality gap is the percentage gap between the value of the best found feasible solution and the lower bound obtained with successive linear relaxations of the formulation. In minimization problems like the one studied here, the former is an upper bound on the value of the optimal solution and is often denoted as UB, while the latter is denoted as LB. The relative optimality gap is then computed as $100 \cdot \frac{\text{UB}-\text{LB}}{\text{UB}}$; its value is first computed when the first feasible solution is found and evolves during the execution of a B&B algorithm, which terminates when the gap is closed, i.e. becomes null. If time limit is reached before the optimality gap is closed, its value allows to estimate how far we are from optimality proof. If no feasible solution is found within the time limit, the gap cannot be computed. In Table 5, $\overline{gap}$ denotes the relative optimality gap at time limit, hence a value larger than zero means that either the optimum has not been found yet, or that its optimality has not been proved yet.

The linear relaxation (LR) of an Integer Linear Program is the Linear Program resulting from the relaxation of the integrality constraint on integer variables. During the resolution process, the LR is used to evaluate the nodes of the search tree and allow to prune non interesting subtrees. Therefore, the tighter is the lower bound provided by the LR, the more efficient is the solving.

inst	$\lvert J\rvert, \lvert M\rvert$	α	λ	IP1				IP2				IP2H$_5$			$\overline{R}$
				#opt	#fea	$\overline{gap}$	$\overline{CPU}$	#opt	#fea	$\overline{gap}$	$\overline{CPU}$	#fea	$\overline{gap\star}$	$\overline{CPU}$	
1	6, 6	0.7	1.0	5	5	0.00	58	5	5	0.00	10				1.21
2	6, 6	0.7	1.1	0	5	1.89	3,600	5	5	0.00	15				1.24
3	6, 6	0.7	1.2	0	5	3.71	3,600	5	5	0.00	33				1.27
4	6, 6	0.9	1.0	5	5	0.00	14	5	5	0.00	2				1.18
5	6, 6	0.9	1.1	5	5	0.00	1,462	5	5	0.00	3				1.21
6	6, 6	0.9	1.2	0	5	1.41	3,600	5	5	0.00	4				1.22
7	6, 6	1.0	1.0	5	5	0.00	15	5	5	0.00	1				1.17
8	6, 6	1.0	1.1	5	5	0.00	309	5	5	0.00	7				1.20
9	6, 6	1.0	1.2	0	5	0.94	3,600	5	5	0.00	7				1.21
10	10, 5	0.7	1.0	0	0	-	3,600	0	0	-	3,600	0	-	3,600	1.30
11	10, 5	0.7	1.1	0	3	12.05	3,600	0	2	0.21	3,600	5	0.24	2,010	1.30
12	10, 5	0.7	1.2	0	5	12.08	3,600	0	5	0.57	3,600	5	0.24	1,510	1.30
13	10, 5	0.9	1.0	0	5	10.80	3,600	0	0	-	3,600	4	0.18	3,054	1.27
14	10, 5	0.9	1.1	0	5	11.52	3,600	2	5	0.01	3,151	5	0.06	1,572	1.27
15	10, 5	0.9	1.2	0	5	11.66	3,600	3	4	0.12	2,371	5	0.06	174	1.27
16	10, 5	1.0	1.0	4	5	0.14	1,685	0	0	-	3,600	0	-	3,600	1.25
17	10, 5	1.0	1.1	0	5	8.46	3,600	4	4	0.00	1,726	5	0.13	2,024	1.26
18	10, 5	1.0	1.2	0	5	9.80	3,600	5	5	0.00	918	5	0.08	270	1.26

Table 5: Performances comparison of the disjunctive and time-indexed models.

Column 16 reports the ratio $\overline{R}$ between the value of the LR of IP2 at the root node of the B&B tree, and the same value for IP1. Such ratio is computed as $\overline{R}= \frac{\text{LR(IP2)}}{\text{LR(IP1)}}$ and provides a good indicator of the relative quality of the two linear relaxations during the whole resolution process. For the considered instances, the values of $\overline{R}$ are all greater than 1 since the LB of IP2 is always tighter than that of IP1. Actually, the LB of IP2 is always between 25 and 30% higher. This means that the LR of the time-indexed formulation provides better lower bounds.

Average optimality gaps are computed w.r.t the instances for which at least one feasible solution has been found. For example, for the instance set descending from $la04$ with $\alpha = 0.7$ and $\lambda = 1.1$ (line 11) IP1 was able to find a feasible solution for 3 instances out of five: the average gap over them is 12.05%. Similarly, the average gap of 0.21% of IP2 is computed over two instances. This example also shows that althought IP2 has found less feasible solutions than IP1, they are of much better quality. As we will see, this is a general trend. Average computational times are computed over all the five instances associated with a line.

Results show that IP1 has difficulties to find or to prove optimal solutions. For example, in the instances descending from $mt06$ it only finds 25 optimal solutions out of 45 instances and is clearly outperformed by IP2 which finds all the 45 optimal solutions in less than 35 seconds. More generally, the average gap of the time-indexed model is always less than 1%, while it can go up to 12% for the disjunctive model, which is coherent with the values of $\overline{R}$ observed. The only advantage of IP1 lies in its ability to find feasible solutions more easily (e.g. 38 versus 25 for the instances descending from $la04$).

In most cases, for a given instance, the average gap increases with λ, regardless of α, for both IP1 and IP2. In more detail, for instances derived from mt06 (lines 1-9), IP2 always finds the optimal solution, whereas IP1 either finds it or reaches a small gap (under 2% in all cases but one) within the time limit. A different behaviour occurs for instances descending from la04. IP2 cannot find feasible solutions when $\lambda = 1.0$, while when $\lambda > 1$, feasible solutions are often found (in 25 instances out of 30), and they are either optimal (14/25 cases) or near-optimal (overall average gap under 0.16%). IP1 can find a good-quality solution only for $\lambda = 1.0$ and $\alpha = 1.0$, whereas in other cases the gap at time limit is important. The time-indexed model tends to find optimal solutions more easily, and in general more feasible solution with λ increasing. The disjunctive one, on the other hand, finds more difficulties in finding the optimum when λ grows, as the relative optimality gap becomes more relevant.

Since IP2 seems to struggle to find feasible solutions, particularly for instances with a tight constraint on makespan ($\lambda = 1.0$), it is reasonable to consider the idea to warmstart it, i.e. to feed it with an initial solution obtained beforehand in order to intensify the pruning effect in the B&B tree search and ultimately to accelerate the resolution. A good warmstart solution may be offered by the algorithm IP2H$_n$. Columns 13-15 of Table 5 report the results obtained by IP2H$_5$

(i.e. obtained from model IP2 with dates that are multiples of 5) on the instances derived from $la04$. The term $\overline{gap\star}$ is the average (over the instance set associated with a line) of the percentage gap computed as $100 \cdot \frac{\mathrm{UB}(\mathrm{IP2H}_5)-\mathrm{LB}(\mathrm{IP2})}{\mathrm{UB}(\mathrm{IP2H}_5)}$, i.e. the gap between the best solution of $\mathrm{IP2H}_5$ with the best lower bound of IP2, so as to allow the comparison of $\overline{gap\star}$ with $\overline{gap}$ of IP2. Hence, a null value for $\overline{gap\star}$ means that the solution obtained with the heuristic is actually an optimal solution.

We run $\mathrm{IP2H}_5$ only on the instances derived from $la04$. The number of feasible solutions generated by the heuristic is equal to 34, versus 25 when considering only the basic time-indexed model. When $\lambda = 1.0$, in most of the cases, $\mathrm{IP2H}_5$ is not able to find feasible solutions within the time limit. However, when $\lambda \in \{1.1, 1.2\}$, solutions of very good quality ($\overline{gap\star} \leq 0.24$) are found in a reasonable time, with several cases where the solution yielded by $\mathrm{IP2H}_5$ at time limit is better than the solution found by IP2 (i.e. , $\overline{gap} > \overline{gap\star}$).

The complexity of the problem could also be affected by the number of variables. For the instances derived from $mt06$ the average number of variables is equal to 2,792 and 2,259 for IP1 and IP2 respectively. For the instances derived from $la04$, this value is equal to 5,101, 39,073 and 7,794 for IP1, IP2 and the heuristic respectively.

5.3. Overall analysis

The computational experiments show that the time-indexed model IP2 is more efficient than the disjunctive model IP1, as when the former succeeds in finding feasible solutions, then it is capable in most cases of proving optimality, or to achieve a much smaller gap within the same time limit. This is probably due to the fact that the time-indexed model is structurally tighter, leading to a stronger lower bound. A well-known limit of time-indexed models for job-shop scheduling problems is that they tend to suffer from the enlargement of the time horizon, due to the proportional increase of the number of variables. However, in the case of the problem studied here, the size of model IP2 is very similar to that of IP1, due to the cumulative resource constraints (17) of the latter as explained in Section 4.3. This means that the performance of B&B algorithms based on these two formulations will be comparable in terms of computation burden for the solver engine. This contributes to make the overall performances of the time-indexed model better.

The only weak point of the time-indexed model seems to be its greater difficulty to determine feasible solutions. However, this can be tackled by coupling it with a heuristic algorithm in order to provide warmstart solutions. Incidentally, the model itself easily gives rise to a simple heuristic by reducing the number of available time slots to deploy operations, as it has been explained in Section 4.4 and exploited in Section 5.2. By choosing the set and number of usable time slots, one can tune this heuristic algorithm and either obtain very good quality solutions or quicker solutions to warmstart the solving of the complete model.

6. Conclusion and future work

Due to the increase of energy prices during the last decades, it is more and more important to take electricity price into account when minimizing the overall cost of a production system. In this paper, a job-shop scheduling problem with energy consideration is studied. The objective is to find a schedule that respects a power limitation during the planning horizon with a variable electricity cost profile while minimizing the energy cost. Two integer programming models, namely a disjunctive and a time-indexed formulations, are proposed to tackle this job-shop scheduling problem variant. The computational results have shown that the time-indexed formulation outperforms the disjunctive one. The time-indexed model has also proven to be more generic. Indeed, extending the disjunctive model to consider non-constant energy cost profiles and machine power consumption profiles can be a hard task. Moreover, such a model requires making some additional assumption (namely, to have the longest operation shorter than the shortest period). The time-indexed model does not suffer from these limitations, being in particular more adaptable to the generalization of energy cost profiles and machine consumption profiles. As to energy cost profiles, one could for instance consider *Critical Peak Pricing*. As to the power consumption of machines, non-constant profiles can be integrated, as well as different profiles, one per operation. A set of possible consumption profiles could be also associated with each operation. This is for example what occurs with machines with different processing speeds. In this case, the decisions about the processing of an operation would concern both the starting time and the chosen speed, which determines the machine consumption and ultimately impacts the overall power consumption. For all of these reasons, the time-indexed model is preferrable.

Since the proposed problem is NP-hard, a heuristic algorithm has been developed starting from the time-indexed formulation and simply reducing the number of available time slots to deploy operations, in order to provide warmstart solutions to the Branch&Bound (B&B) algorithm and accelerate its convergence. However, the development of heuristic algorithms is of interest mostly because it allows to tackle instances of greater size. To this end, the literature of approximating methods for job-shop scheduling problems includes many works that could help in defining an efficient heuristic approach for the proposed variant. The most promising direction seems that of local search-based metaheuristics, which are known to better fit strongly constrained problems like the proposed one with respect to for example population-based methods.

Another promising direction comes from the development of valid inequalities that could help strengthen the lower bound of the proposed time-indexed integer linear programming model, as well as problem-specific branching strategies that could improve the exploration of the B&B tree. Both techniques would allow to accelerate the solving of the associated B&B method. A more general conclusion of the present work is that researchers working on scheduling with energy considerations should in our opinion give more attention to time-indexed models rather than to the more usual disjunctive models, as it can be seen from this work and as noted by Merkert et al. [2015] who worked on industrial challenges and opportunities for scheduling problem with energy considerations.

This paper tackles a deterministic version of the job-shop scheduling problem

with energy aspects, but a future effort should be oriented to versions that incorporate non-deterministic features. The consideration of uncertain or variable data could lead to the development of robust optimization algorithms, as well as methods for the sensisivity analysis of the solution returned by the deterministic algorithm. At the same time, one could develop stochastic optimization approaches so as to take into account machine failures for example.
Finally, switching to another price-driven demand response program such as *Real-Time Pricing*, or to an event-driven program, could lead to Real-Time (or Quasi Real-Time) Optimization methods, for which even finding feasible solutions could be hard.

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