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# Gaussian Processes Indexed by Clouds of Points: a study

Babacar SOW (EMSE, LIMOS), Rodolphe LE RICHE (CNRS, LIMOS)

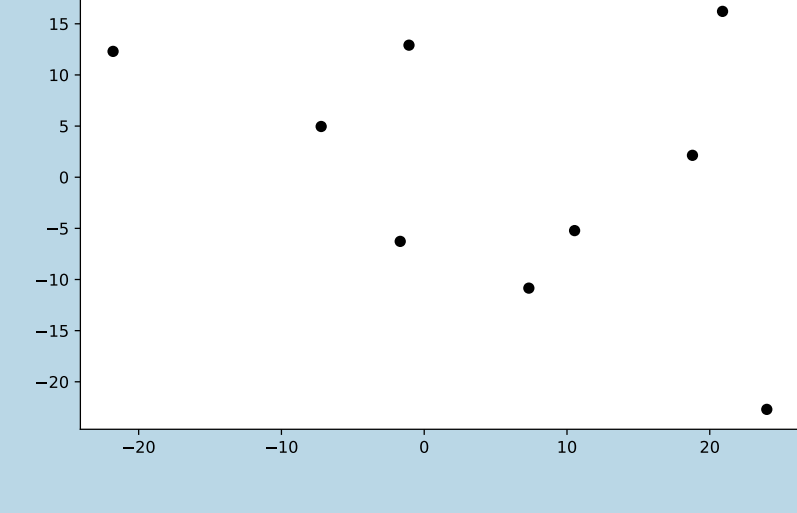


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## Context And Problematic



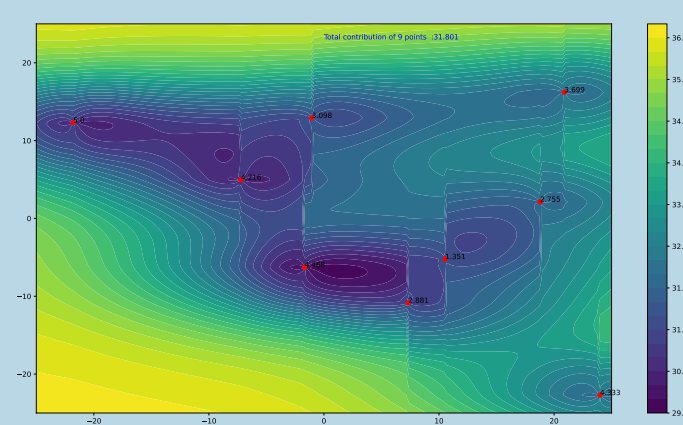
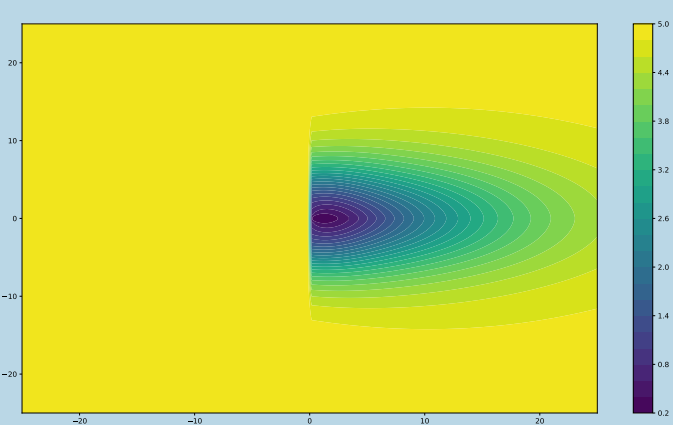
- Metamodel a function over clouds of points using Gaussian process.
- A cloud is a set of points invariant under permutation  $\{x_1, \dots, x_n\}$  with  $x_i \in \mathbf{R}^{dim}$

## Test Function

The following test function mimicks a wind-farm production:

$$F(\{x_1, \dots, x_n\}) = \sum_{i=1}^n \sum_{\substack{j \\ x_{j,1} \leq x_{i,1}}} f_p(x_j, x_i) f_0(x_i)$$

where  $f_p(x_j, x_i)$  expresses the energy loss over  $x_i$  that is caused by  $x_j$  and  $f_0$  is a constant.



## Kernels

### Substitution kernel with MMD

- We want to construct a kernel between two clouds of the form  $K(X, Y) = \sigma^2 \exp(-\frac{d^2(X, Y)}{2\theta^2})$  where  $d$  is an Hilbertian [2] distance.
- For two clouds  $X = \{x_1, \dots, x_n\}$  and  $Y = \{y_1, \dots, y_m\}$ ,  $P_X = \frac{1}{n} \sum_{i=1}^n \delta_{x_i}$  and  $P_Y = \frac{1}{m} \sum_{j=1}^m \delta_{y_j}$  are the respective associated empirical uniform distributions.
- There exists a Reproducing Kernel Hilbert Space,  $\mathcal{H}$  with a characteristic kernel such as  $k_{\mathcal{H}}(x, \cdot) = \exp(-\frac{\|x - \cdot\|_{\mathcal{H}}^2}{2\theta^2})$ .
- The characteristic nature guarantees the injectivity of the embedding map [1]:  $P_X \mapsto \mu_X = \int P_X(x) k_{\mathcal{H}}(x, \cdot) dx$ .
- $MMD^2(P_X, P_Y) = \|\mu_X - \mu_Y\|_{\mathcal{H}}^2$
- For any kernel  $k_{\mathcal{H}}$  of the RKHS, the uniform empirical laws gives  $MMD^2(P_X, P_Y) = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n k_{\mathcal{H}}(x_i, x_j) + \frac{1}{m^2} \sum_{i=1}^m \sum_{j=1}^m k_{\mathcal{H}}(y_i, y_j) - \frac{2}{nm} \sum_{i=1}^n \sum_{j=1}^m k_{\mathcal{H}}(x_i, y_j)$
- The correlation kernel  $K_{sub\_mmd}(X, Y) = \sigma \exp(-\frac{\|\mu_X - \mu_Y\|_{\mathcal{H}}^2}{2\theta^2})$  is symmetric and definite positive.

## References

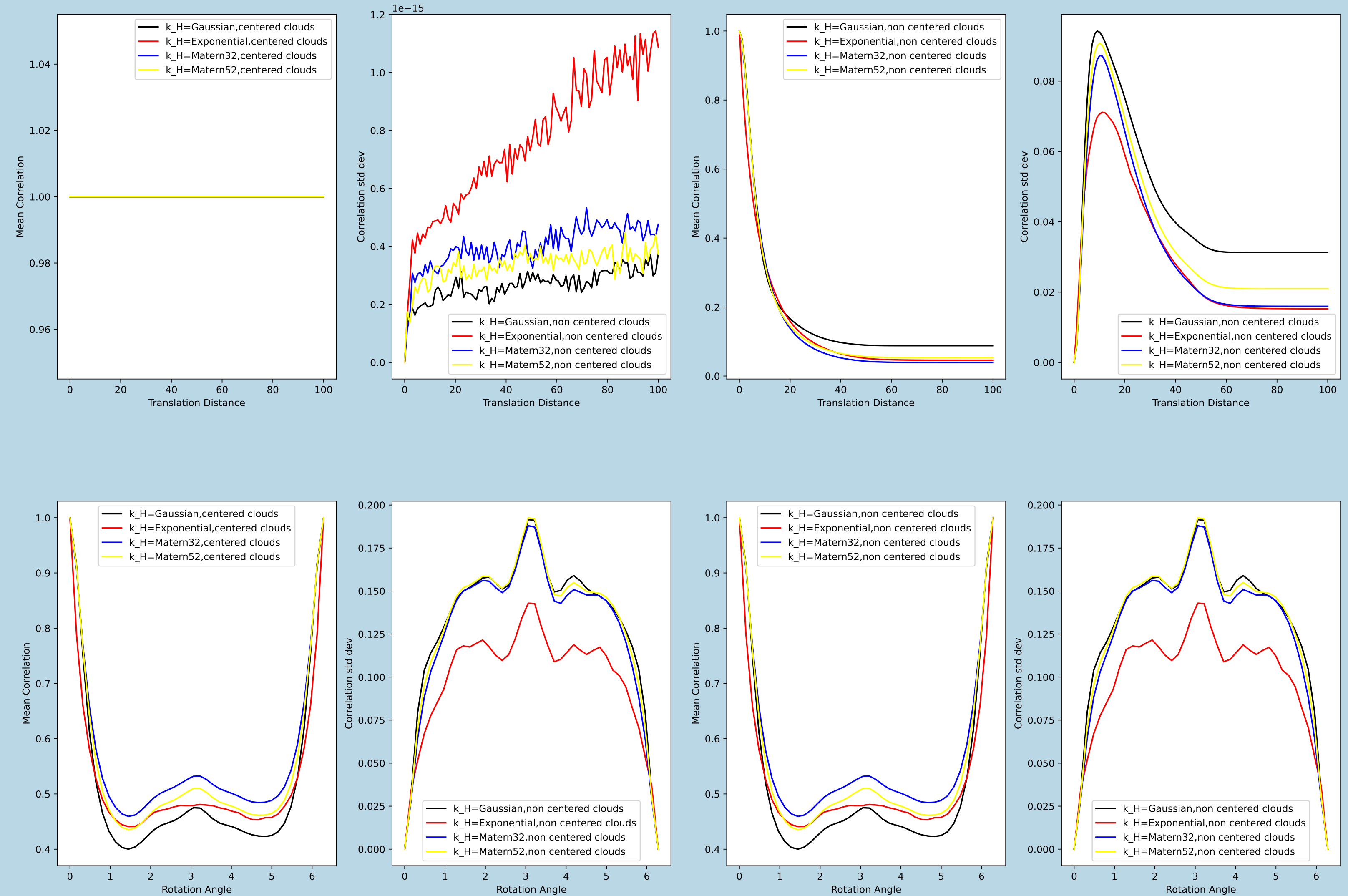
- [1] Muandet, Krikamol and Fukumizu, Kenji and Sriperumbudur, Bharath and Schölkopf, Bernhard and others: *Kernel mean embedding of distributions: A review and beyond*, Foundations and Trends® in Machine Learning
- [2] Haasdonk, Bernard and Bahlmann, Claus: *Learning with distance substitution kernels*, Springer

## Geometrical Properties of the kernels

- Below is represented the correlation between a cloud and its image by a geometric transformation. Considered transformations are rotations and translations.
- We compare two scenarios: centered clouds and non-centered ones.
- The different kernels of the Hilbertian Space are the Exponential, the Gaussian(Squared Exponential), the Matern32 and the Matern52.

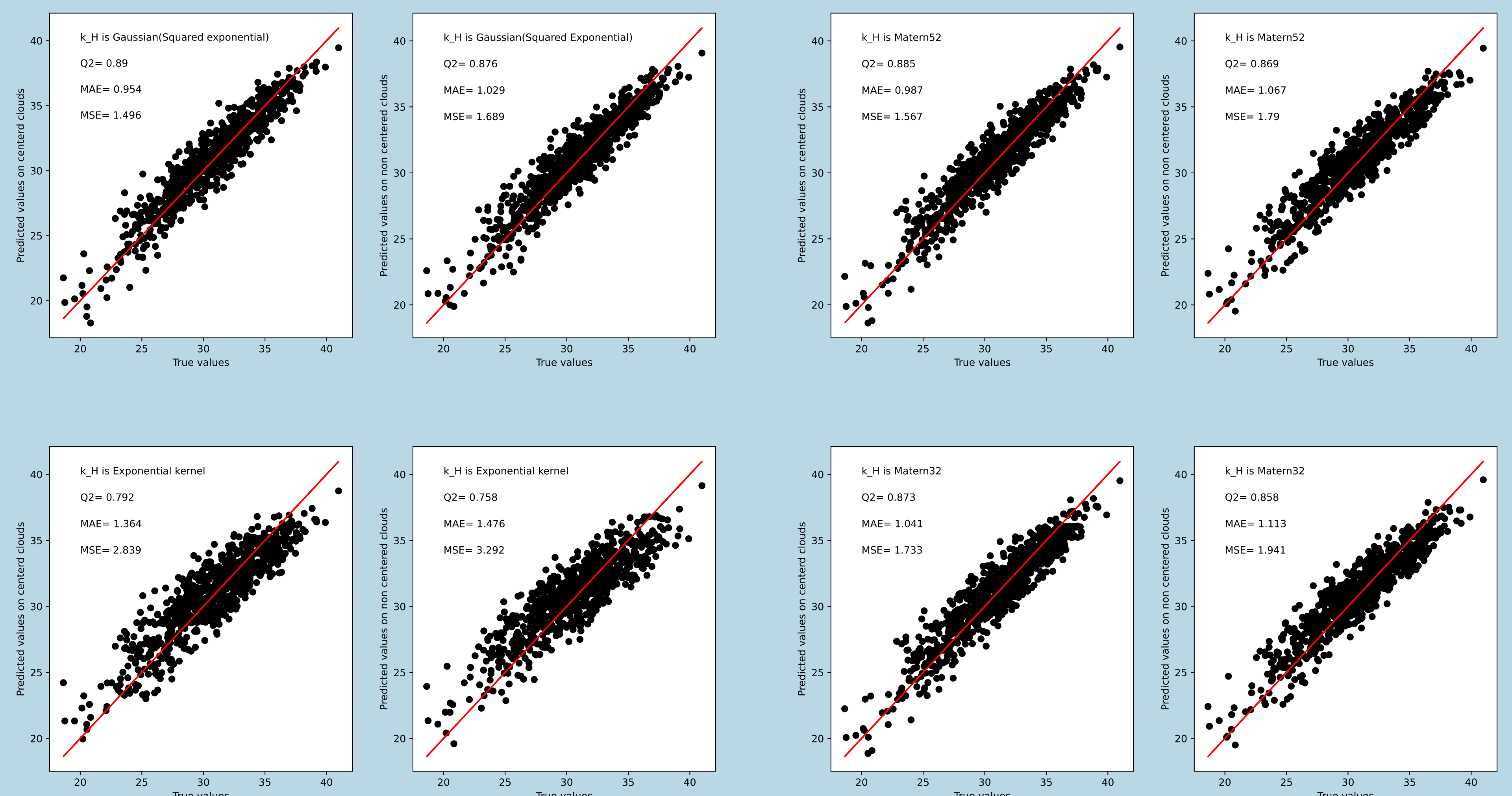
### centered cloud

### non-centered cloud



## Prediction Results on the analytical Function F

- We metamodel the wind-farm proxy function  $F$  with a Gaussian process of kernel  $K_{sub\_mmd}$
- We consider a set of 1000 clouds of 10 points each.
- Each point of a cloud is drawn uniformly in a square.
- The kernel parameters are learned using 200 clouds by maximizing log-likelihood with BFGS.
- On each plot, we represent predicted values vs. true ones on the remaining clouds, obtained with the different kernels.
- The corresponding Q2, MAE and MSE are also displayed.



- The best results are obtained when  $k_H$  is the Squared Exponential and clouds are centered. This can be explained by the fact that the quadratic modified distance with two length scales of this  $k_H$  complies better with the geometric properties of  $F$ .