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# Bounding transverse permeability of fibrous media: a statistical study from random representative volume elements with consideration of fluid slip

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## Abstract

In this article, a statistical study on transverse permeability of random fibrous medium is performed. For that purpose, numerous random numerical microstructures are generated with constant or randomly varying fibre radii. Their statistical representativity with respect to experimental data is first briefly discussed. Flow simulations are then performed on these digital microstructures to retrieve their full transverse permeability tensor. The representative volume element (RVE) size is determined by studying convergence of permeability distribution when domain size increases. This allows to characterise the medium isotropy as well as the impact of geometrical randomness on permeability. The approach also integrates Gaussian process regression, that is a Bayesian machine-learning model, to consider variability within interpolation in the proposed permeability predictive model. In addition, this paper considers the impact of fluid slip at liquid/fibre interface on permeability for random fibrous media. An analytical expression is proposed to describe precisely the transition from a no-slip to a free-slip regime. This allows us to propose a probabilistic model that links permeability to both the fibre volume ratio and slip length. This finally yields two bounds for transverse permeability of fibrous media: a first related to statistical scattering and a second purely linked to fluid slip.

*Keywords:* Numerical permeability, Statistics, Fluid slip, Gaussian process regression

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## 1. Introduction

Permeability is a key notion for describing flows within porous materials. As a consequence, this tensorial characteristic has been extensively studied in numerous scientific communities [21, 82] including modelling of manufacturing processes such as Liquid Composite Moulding (LCM) for long carbon fibre reinforced polymers wherein a viscous fluid  
5 impregnates fibrous preforms [13, 30, 62, 76].

Classically, flows in porous media can be studied at various scales. Experimental approaches to describe permeability of fibrous materials operate at a macroscopic scale (i.e. the laboratory scale). They were found to suffer from a lack of reproducibility as well as  
10 a high sensitivity to laboratories settings [5, 79]. On the contrary, analytical and computational approaches consist in local studies of flows across fibrous microstructures. Both local and macroscopic descriptions can be connected through an upscaling procedure from which permeability naturally shows up. Considering local scale approaches, analytical models that are convenient to use [11, 14, 29] lean on geometrical simplifications  
15 that do not allow to consider realistic systems with intrinsic variabilities (e.g. random fibre radius, random fibre placement,...) as observed in manufacturing processes. These limitations lead researchers to develop numerical strategies to characterise permeability more accurately [2, 20, 58]. This approach first requires virtual geometries. Those can be obtained from digitisation of real-world materials thanks to recent tomography  
20 techniques [3, 89] or in a pure numerical way with textile modelling software [26, 50] or random non-overlapping disk algorithms. The latter will be detailed in Section 2.1. **This work is primarily oriented towards modelling the manufacturing processes of composite materials, although these results can be relevant wherever a viscous flow in a fibrous medium is observed: textile, biomechanics,... In this study, no direct comparison with experimental permeability measurements will be carried out. However, comparisons with other numerical results — which has been found to be consistent with the experiment — will be performed.**

From those fibrous geometries, mass (Eq.1) and momentum (Eq.2) local conservation equations for a newtonian incompressible viscous fluid (i.e. Stokes equations) are classi-

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cally solved to first represent the fluid flow across the fibrous arrangements and then to assess their permeability. This is done through various numerical methods: Finite Element Method (FEM) [73], Lattice Boltzman Method (LBM) [35], Fast Fourier Transform (FFT) [1, 83],...

$$\nabla \cdot \mathbf{v} = 0 \quad (1)$$

$$\mu \Delta \mathbf{v} - \nabla p = \mathbf{0} \quad (2)$$

where  $\mathbf{v}$  is the fluid velocity,  $p$  the fluid pressure and  $\mu$  the fluid viscosity. As indicated previously, those local equations can be upscaled to get a macroscopic description of the fluid flow within the porous structure. This corresponds to Darcy law [6, 21, 82]:

$$\nabla \cdot \mathbf{v}_D = 0 \quad (3)$$

$$\mathbf{v}_D = -\frac{1}{\mu} \mathbf{K} \cdot \nabla p_D \quad (4)$$

where  $\mathbf{v}_D$  (resp.  $p_D$ ) is the upscaled velocity (resp. pressure) and  $\mathbf{K}$  the second-order permeability tensor. This permeability term naturally comes from the upscaling procedure: it encompasses all the effects leading to the fluid energy loss, i.e. fluid viscosity effects and its contact with fibres [82]. Hence both contributions of flow channels formed between fibres and fluid/fibres interaction should be investigated. **It has been also highlighted that permeability may be affected by thermal effects or by the type of fluid under consideration [70]: those contributions will be neglected here.** In practice, solution fields at the local scale are integrated to then compute permeability from Eq.4 (Section 2.3). Permeability tensor  $\mathbf{K}$  is generally represented by a diagonal matrix since fluid is assumed to flow along the principal directions of the medium under consideration. This assumption is however rarely verified [56] as computing off-diagonal components of the permeability tensor may be challenging. In this paper, geometrical periodicity allows to provide periodic boundary conditions leading to a direct computation of the off-diagonal components. The importance of those terms will be discussed.

That upscaling procedure must be performed on a representative volume element (RVE) that is the smallest geometry for which permeability becomes independent of the

domain size [9, 32, 33, 36, 72, 75]. Numerical permeability of fibrous media is then generally studied from single deterministic unit cells that are supposed to be representative of real-world materials [30, 60]. Yet, fibrous materials display a natural geometrical complexity that justifies a statistical modelling [17, 86, 87, 90]. As a consequence, for a given fibre volume fraction  $V_f$  value, permeability tensor of numerous random RVEs should be computed to provide an accurate characterisation of the property. This is done in this paper where the tensors computed from microstructures are regarded as realisations of a permeability continuous random variable. Only few papers can be found with a similar methodology for permeability study of fibrous materials [8, 90]. This approach differs from [74, 75] where a statistical RVE (SRVE) is defined as a geometry sufficiently large to capture both physical and geometrical information.

As noticed previously, permeability can be related to fluid/fibres interaction. As a consequence, the condition provided at the liquid/solid interface has a direct impact on the permeability. As carbon fibre radius is around  $3.3 \mu\text{m}$  [77] and industrial requirements for  $V_f$  may be close to 0.6, flow channels between fibres may become submicrometric. At this scale, the usual no-slip condition at the liquid/solid interface should be reconsidered to take fluid slip into account [59, 71, 91]. Fluid slip is an active research topic in various communities [16, 46, 47, 66, 68]. It can be explained by surface roughness effects as well as chemical fluid/solid affinity [41]. This is in line with industrial observations according to which permeability measurements may be sensitive to carbon fibres surface treatment [25]. Fluid slip consideration is also convenient when it comes to modelling moving fluid front in transient **multiphase** flows as it allows to alleviate Huh-Scriven paradox [39]. Details on fluid slip origins and terminologies can be found in [81].

Mathematically, fluid slip is generally expressed through Navier slip condition [57] at the liquid/solid interface  $\Gamma_{LS}$ . It relates tangential fluid velocity to shear components of fluid Cauchy stress tensor  $\boldsymbol{\sigma}$  (Fig.1):

$$\mathbf{v} \cdot \mathbf{t} = -\frac{\ell_s}{\mu} \mathbf{t} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} \quad \text{on } \Gamma_{LS} \quad (5)$$

where  $\ell_s$  is the slip length, and  $\mathbf{t}$  (resp.  $\mathbf{n}$ ) is a unit tangential (resp. normal) vector to  $\Gamma_{LS}$ . This generalises no-slip ( $\ell_s \rightarrow 0$ ) and free-slip conditions ( $\ell_s \rightarrow \infty$ ) [78, 80]. Many articles can be found on experimental or numerical determination of slip length for various fluid/solid couples [42, 43, 63]. It is generally found to span from  $10^{-9}m$

to  $10^{-6}m$ . Maali *et al.* [51] estimate the slip length for water/graphite couple to be  $8 \pm 2$  nm. It can be then remarked that those orders of magnitude can be comparable to the characteristic size of flow channels within our microstructures. Slip length will be considered here as a constant parameter even if its dependencies to other characteristics of the flow might be complex and still on study [4, 41, 52, 66]. However, as slip length mainly acts as an intrinsic property of the fluid/solid couple, it seems appropriate to consider it as independent of the imposed pressure drop — at least for a newtonian fluid. The influence of slip length value on permeability of idealised geometries has been investigated in few articles [7, 27, 28, 45]. One of the novelties of this paper is to consider fluid slip on numerous RVEs of fibrous media on top of other sources of variability.

We will first describe the microstructure generation method and its representativity (Section 2.1). Full permeability tensor (Section 2.3) of numerous generated geometries will be then computed from finite element solving (Section 2.2) of Stokes equation. This will allow to determine the domain size to reach a RVE (Section 3.1). Then isotropy of the RVEs will be discussed (Section 3.2). The influence of fibres centre randomness (Section 3.3) and radius variability (Section 3.4) on permeability will be also detailed. Gaussian process regression (Section 2.4) will be integrated into the statistical analysis. In the end, fluid slip will be considered and its influence on permeability will be characterised (Section 3.5). A probabilistic model that links permeability to  $V_f$  and slip length will be finally proposed. This will give boundings to stochastic permeability. Those results will be then discussed (Section 4): comments will be first made about the variability of permeability. Then the influence of fluid slip on permeability — with slip lengths consistent with available experimental measurements — will be discussed.

## 2. Materials and methods

### 2.1. Microstructure generation algorithm and representativity

Various numerical methods for microstructure generation can be found in the literature [55, 84, 85]. Those consist in non-overlapping disk placement methods that are finally supposed to be representative of the transverse section of long fibre composite materials. The most straightforward algorithm is named hard-core method and consists in placing fibre centers totally randomly before performing a non-overlapping test of

2D cross-sections [12]. This method is highly limited by jamming effect that prevents from reaching high  $V_f$ . The algorithm can be adapted to reduce this effect and provide geometrical periodicity [55].

Another technique consists in starting from an initial configuration that is subsequently perturbed randomly by picking a random displacement vector for each fibre [15, 34, 84]. Potential-based methods that consider fibres as a set of interacting particles seem to be the most efficient to reach high  $V_f$  [40, 49]. In spite of the efficiency of the presented algorithms, the question of representativity still remains. This is regularly assessed through Complete Spatial Randomness (CSR) characterisation thanks to related tools such as Ripley's function [65]. However, such a consideration does not lie on experimental observations. In addition, total randomness cannot be observed at high  $V_f$  due to the non-overlapping condition. Indeed, it entails forbidden positions for fibre placement that result in a biased CSR: this is comparable to an interaction potential between fibres [8, 31, 33]. To overcome this issue, generation algorithms that are directly based on experimental parameters distribution can be also proposed [77]. Despite the representativity of these approaches, it has been finally shown in [87] that fibrous geometries and permeability results slightly depend on generation algorithm, at least for moderate  $V_f$  values. At last, only few algorithms take radius randomness into account [49, 54] while it may entail an additional source of variability. Its influence will be investigated in this paper.

In the context of this study, the requirements related to the generation method were multiple. First, the algorithm should reach  $V_f$  values around 0.6 to be relevant for industrial applications. Then the generated microstructures should display a variability  $C_r$  of fibre radius  $r$  that is set at 10% in this work. Fibre radius should follow a normal distribution:  $r \sim \mathcal{N}(\bar{r}, C_r \bar{r}) = \mathcal{N}(\bar{r}, 0.1\bar{r})$ . The microstructures should also be periodic to provide periodic boundary conditions. Indeed, this allows to be consistent with rigorous homogenisation procedures and to reach a RVE for smaller domain size since edge effects are prevented [88]. At last, the method should be computationally efficient enough to generate numerous microstructures.

For all these reasons, this work focus on a simple method that generates microstructures within a square domain of size  $L^2$  by perturbing an initially dense quadratic

packing (Fig.2). It is based on an algorithm proposed in [34] with some differences. Fibre centres are first placed according to a quadratic packing of maximum compacity. Then random radii are picked in a Gaussian law  $\mathcal{N}(\bar{r}, C_r \bar{r})$  and a non-overlapping condition is enforced. The value of  $\bar{r}$  can be directly computed from both domain size and  $V_f$ :

$$\bar{r} = L \sqrt{\frac{V_f}{N\pi}} \quad (6)$$

130 where  $N$  is the number of fibre contained inside the domain that is explicitly calculated from  $V_f$  and  $L$  considering a quadratic packing. Then a random displacement vector is applied on each fibre. This displacement is kept if it does not lead to fibre overlapping or to a distance between fibres so small that it could cause mesh refinement issues: it is here chosen to reject distances inferior to  $0.07\bar{r}$ . This is repeated  $N_p$  times. That step also  
 135 provides geometrical periodicity as a fibre going out of the domain reenters on the opposite side. The algorithm is also convenient to generate microstructures with a constant fibre radius by taking  $C_r \ll 1$ . An example of numerically generated microstructure with varying radius can be observed in Fig.3.

Other generation algorithms have also been tested. It has been remarked that the  
 140 targeted fibre radius variance could be difficult to retrieve, especially for high  $V_f$  values. This is directly related to the non-overlapping rejecting test. The generation algorithm described previously has been found to be suitable to limit this problem. Indeed, for all the  $V_f$  considered here, a Gaussian radius variability with  $C_r \sim 10\%$  can be retrieved. Through this algorithm, the obtained  $V_f$  value may slightly differ from the targeted one,  
 145 especially for  $V_f > 0.55$ . Indeed, a limit is observed for  $V_f \sim 0.62$ : higher  $V_f$  values cannot be reached. Even if the maximum compacity for a quadratic packing is close to 0.78, a compromise between obtaining very high  $V_f$  values and mesh issues has to be achieved. Indeed, for very high  $V_f$  values, flow channels become so narrow that ensuring a sufficient number of finite elements to represent the flow profile correctly may  
 150 be complex. It should be remarked that this issue could be alleviated by using anisotropic mesh adaptation techniques [23] that would help to reach higher  $V_f$  values.

Representativity of the generated microstructures should be next assessed. The CSR criterion is here rejected due to the arguments developed previously : the non-overlapping condition makes irrelevant complete spatial randomness as it acts as an interaction po-



155 tential between fibres. We choose to compare first and second nearest neighbour (NN)  
distance distributions of the generated microstructures to experimental ones. Further  
microstructural parameters are commonly used in literature : Delaunay triangulation  
[87], Voronoi tessellation characteristics [67],... However, first and second NN distances  
are one of the most straightforward tools to characterise fibrous microstructures on a  
160 first approach. These morphological parameters measure short-range interactions of the  
particles.

Experimental microstructure analyses are so rare that we struggled to find some well  
described data for comparing with our virtual microstructures. In Fig.4, we compared  
our NN distributions to results from Vaughan and McCarthy [77] that consider experi-  
165 mental distributions of carbon fibres for high performance composite materials with high  
 $V_f$  value. First NN distribution can be reasonably modeled with a Gaussian law, while  
2nd NN distribution fit a nearly symmetric lognormal law. It can be observed that both 1st  
NN distributions are centred on close values while differences can be seen for the 2nd  
NN. Furthermore, variance is significantly higher for numerically generated microstruc-  
170 tures. However, it should be remarked that the experimental distribution relies on a  
single microstructure while several fibrous geometries should be analysed to retrieve a  
more representative distribution. Further comparisons should be carried out when sup-  
plementary documented data are made available. Nonetheless, mean NN distances of our  
generated microstructures are rather close to experimental ones which give us confidence  
175 on the relevance of representativity of the generation algorithm. By comparing 1st NN  
distribution for different numbers of perturbations  $N_p$ , it can be seen in Fig.5 that dis-  
tributions become stable from  $N_p = 1000$ . In a similar fashion, it has been observed that  
permeability distributions are nearly unchanged from 1000 perturbations, for a same set  
of input parameters. For the rest of the study,  $N_p = 5000$  will be chosen.

## 180 2.2. Numerical strategy

Stokes equations are solved on fluid domain  $\Omega_L$  (Fig.6) using a stabilised FEM ap-  
proach implemented in Z-Set software<sup>1</sup> which has been widely presented in former articles

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<sup>1</sup><http://www.zset-software.com/>

[2, 18, 61]. An Algebraic Sub-Grid Scale (ASGS) [19, 38] stabilisation scheme is implemented to circumvent Ladyzhenskaya–Babuška–Brezzi [10] condition with a piecewise  
185 linear approximation of both velocity and pressure fields.

Accounting for Navier slip in the weak formulation can be now briefly described, further details are presented in [18]. The weak formulation of the problem is obtained by multiplying Stokes equations by appropriate test functions and then intergrating by parts. This leads to a surface integral term  $\mathcal{T}$  on the boundaries of the fluid domain,  $\partial\Omega_L$ . For single-phase flow, the integral is decomposed into two terms: a Neumann boundary condition term on RVE boundaries (i.e. the imposed stress vector on RVE boundaries),  $\mathcal{T}_{\Gamma_N}$ , and a complementary term on the fibre contours,  $\mathcal{T}_{\Gamma_{LS}}$ , that naturally reveals the Navier slip condition:

$$\mathcal{T} = \mathcal{T}_{\Gamma_N} + \mathcal{T}_{\Gamma_{LS}} \quad (7)$$

with:

$$\begin{aligned} \mathcal{T}_{\Gamma_{LS}} &\equiv \int_{\Gamma_{LS}} \boldsymbol{\sigma} \cdot \mathbf{n} \cdot \mathbf{v}^* d\Gamma \\ &= \int_{\Gamma_{LS}} -\frac{\mu}{\ell_s} \mathbf{v} \cdot \mathbf{v}^* d\Gamma \end{aligned} \quad (8)$$

by considering Eq.5 and where  $\mathbf{v}^*$  is a test function with suitable properties.

Computational times strongly depend on both domain size and  $V_f$ . For  $V_f = 0.5$  and  $L/\bar{r} = 60$ , the mesh around  $10^6$  nodes leading to  $3 \times 10^6$  degrees of freedom (two per nodes for velocity and one for pressure) (Fig.7). The number of nodes has been optimised  
190 using an adaptative mesh: 8 points are regularly placed on the contour of each fibre. For each point, the first NN inter-fibre distance is computed to define a mesh size map (Fig.7). This finally permits to retrieve the minimum number of elements (i.e. 8 elements from our tests) within each flow channel between fibres to ensure a good representation of the parabolic Poiseuille-like flow. The full computation chain, from microstructure  
195 generation to flow simulations, has been performed on a desktop computer (CPU: i5-8500, 6×3.0 GHz ; RAM: 16 Gb) and lasts approximately 10 minutes with a direct solver for the finite element linear system: around 80% of the computation time is taken by the resolution of the FEM problem. Computational times are thus reasonable enough to generate data massively.

The permeability tensor  $\mathbf{K}$  (Eq.4) is considered as a continuous random variable: each permeability tensor  $\mathbf{k}$  computed from a generated microstructure is regarded as a realisation of  $\mathbf{K}$ . The computation of  $\mathbf{k}$  is detailed in the following.

Considering a given microstructure, constant pressures  $p_1$  and  $p_2$  are applied on opposite boundaries entailing a pressure difference  $\Delta p = p_2 - p_1$  (Fig.6). Pressure is weakly imposed through the Cauchy stress tensor:  $\mathbf{n} \cdot \boldsymbol{\sigma} \cdot \mathbf{n} = -p_1$  or  $-p_2$  on  $\Gamma_N$ . Note that for this linear approach the computed permeability does not depend on the pressure differential. Since the mesh is periodic, a periodic velocity is easily prescribed on perpendicular edges (Fig.6):

$$\left. \begin{array}{l} \mathbf{v}(x, 0) = \mathbf{v}(x, L) \\ \mathbf{v}(0, y) = \mathbf{v}(L, y) \end{array} \right\} \text{for a pressure drop along } \left\{ \begin{array}{l} x \text{ direction} \\ y \text{ direction} \end{array} \right. \quad (9)$$

This allows the full permeability tensor  $\mathbf{k}$  to be computed from outgoing flows (Fig.8). Let's assume that a pressure difference  $\Delta_j p$  is imposed over a length  $L_j$ , along the  $j$  direction characterised by a unit vector  $\mathbf{e}_j$ . The upscaled pressure gradient is usually linearised:  $\nabla p_D = (\Delta_j p / L_j) \mathbf{e}_j$ . The upscaled velocity component  $v_{D_i}$  is generally defined as  $v_{D_i} = Q_i / A_i$  where  $Q_i = \int_{A_i} \mathbf{v} \cdot \mathbf{e}_i dA_i$  is the flow-rate through cross-section  $A_i$ . Darcy law can thus be rewritten (Eq.4) so that permeability can be computed:

$$k_{ij} = -v_{D_i} \frac{\mu L_j}{\Delta_j p} = -\frac{Q_i}{A_i} \frac{\mu L_j}{\Delta_j p} \quad (10)$$

It is important to notice that the computation of off-diagonal components of permeability tensor from perpendicular outgoing flow is allowed by periodic boundary conditions. A comparable method for permeability calculation can be found in [56]. As a general notation, the overscript bar (e.g.  $\bar{k}$ ) will denote for the rest of the paper the empirical mean computed from realisations.

#### 2.4. Gaussian process regression

Several kinds of variability affect the permeability values in this study. As a consequence, it is relevant to consider the resulting permeability through a regression method that naturally takes into account those variabilities and proposes an uncertainty estimation for interpolation. Moreover, as computation may be expensive, the regression should

be efficient even for small databases. For all these reasons, a Bayesian machine-learning approach, namely Gaussian Process Regression (GPR), is selected here [64]. The basics of GPR are now detailed. GPR is equivalent to kriging [44, 48, 53] even if both approaches are different: GPR uses a Bayesian framework while kriging is based on the best linear unbiased predictor.

A dataset  $D$  is considered :

$$D = \{(X_1, Y_1), \dots, (X_N, Y_N)\} = \{\mathbf{X}_D, \mathbf{Y}_D\} \quad (11)$$

From  $D$ , the image  $Y^*$  of a new input  $X^* \notin D$  through the unknown function to model  $f$  is sought. For sake of simplicity, the case  $f : \mathbb{R} \rightarrow \mathbb{R}$  is presented here even though the approach can be generalised for higher dimension spaces. GPR is based on a probabilistic interpolation of data:  $f$  is seen as a random variable. In other terms, the probability of a model *knowing* the dataset  $D$  is considered. The latter can be rewritten using Bayes formula. A hypothesis on the prior probability distribution should be then proposed. GPR assumes that the model probability follows a Gaussian process:

$$\forall \mathbf{f}_{X_1, \dots, X_n} = (f(X_1), \dots, f(X_n)), \quad \mathbf{f}_{X_1, \dots, X_n} \sim \mathcal{N}(0, \mathbf{C}) \quad (12)$$

where  $\mathbf{C}$  corresponds to the covariance matrix that can be expressed with a kernel function  $c$  according to Mercer's theorem. The choice of the kernel function makes an assumption on the regularity of the function to model. In the context of this study, variations are supposed to be rather smooth leading to consider a common radial basis function:

$$C_{ij} = c(X_i, X_j) = \sigma \exp\left(-\frac{(X_i - X_j)^2}{2\lambda^2}\right) \quad (13)$$

where  $\sigma$  and  $\lambda$  are two hyper-parameters. The previous assumptions on the Gaussian behaviour allows to use related powerful properties. They especially state that  $Y^*$  *knowing*  $D$  and  $X^*$  (i.e. the new wanted output knowing the related new input and the dataset) follows a Gaussian trend with explicit formula for parameters:

$$p(Y^*|D, X^*) \sim \mathcal{N}(\mu^*, \Sigma^*) \quad (14)$$

with:

$$\mu^* = c(X^*, \mathbf{X}_D)^T \cdot (c(\mathbf{X}_D, \mathbf{X}_D) + \alpha \mathbf{I})^{-1} \cdot \mathbf{Y} \quad (15)$$

and

$$\Sigma^* = c(X^*, X^*) - c(X^*, \mathbf{X}_D)^T \cdot (c(\mathbf{X}_D, \mathbf{X}_D)^{-1} + \alpha \mathbf{I}) \cdot c(X^*, \mathbf{X}_D) \quad (16)$$

with  $c(X^*, \mathbf{X}_D) = (c(X^*, X_1), \dots, c(X^*, X_N))$  and  $c(\mathbf{X}_D, \mathbf{X}_D) = (c(X_i, X_j))_{ij}$ . Eq.15  
 220 should be seen as the most probable output for  $X^*$  while Eq.16 is the related uncertainty  
 on interpolation. This is illustrated in Fig.9 as an example where the dataset contains  
 only six  $(X_i, Y_i)$  couples: Eq.15 corresponds to the blue continuous line while Eq.16  
 corresponds to the light blue shaded zone. A noisy GPR formulation is considered here  
 (Eq.15, 16): a diagonal perturbation term  $\alpha \mathbf{I}$  is added. Such a term has a double interest.  
 225 First, it adds a flexibility on dataset points. This prevents from overfitting but also allows  
 taking dataset variability into account (Fig.9, 10). Second, the noisy formulation has also  
 a purely numerical role. Indeed, Eq.16 relies on inverting a matrix that can be poorly  
 conditioned due to the use of a quadratic exponential function (Eq.13). Noise terms  
 thus make inversion stable. In the context of this work, we select the noise intensity  $\alpha$   
 230 to have a physical meaning, it will be equal to the variability observed at each dataset  
 point. At last, it should be remarked that the derivation of hyper-parameters  $\sigma$  and  $\lambda$   
 have not been detailed yet. Those are classically computed afterwards by maximisation  
 of the marginal likelihood function.

### 3. Results

#### 3.1. RVE determination

The first step of our approach consists in determining the RVE size to ensure the valid-  
 ity of the upscaling process. Classically, permeability is computed for increasing domain  
 size. The minimum length to retrieve a reasonable convergence towards an asymptotical  
 value corresponds to the RVE size  $\ell_{RVE}$ . The convergence pattern is characterised by  
 240 a clear convergence of mean value as well as a decrease of the variability [24]: the do-  
 main becomes large enough to capture the statistical information of the medium. The  
 value of  $\ell_{RVE}$  clearly depends on  $V_f$ . It can be expected that  $\ell_{RVE}$  diminishes when  
 $V_f$  increases as geometrical arrangement tends towards a hexagonal pattern that can be  
 described with a well-known unit cell [29]. In addition,  $\ell_{RVE}$  can be related to the type  
 245 of boundary condition that are applied. Indeed, periodic boundary conditions are known  
 to help convergence as it gets rid of edge effects since the space is toroidal [88].

Permeability tensor is thus computed for various domain size  $\ell$  at different  $V_f$  values. Domain size is here expressed as a dimensionless quantity :

$$\ell = \frac{L}{\bar{r}} \quad (17)$$

A no-slip condition is first considered at fluid-solid interface as Navier slip is supposed not to have a significant impact on RVE size. For each  $(\ell, V_f)$  couple, 30 microstructures are considered. This allows to determine the RVE related to a given  $V_f$  value. We have considered  $V_f \in \{0.3, 0.4, 0.5, 0.6\}$  to describe situations from highly porous cases to the ones observed in the manufacture of high performance composite materials. An example of convergence plot for permeability of diagonal terms is represented in Fig.11 for both constant and random Gaussian radii. As indicated earlier, microstructures with a constant radius are obtained by setting a coefficient of variability for the fibre radius close to zero (i.e.  $C_r \ll 1$ ).

Those curves follow the expected trend as mean values converge towards a constant value and variability decreases. When convergence is reached for diagonal terms, off-diagonal components are found to be two orders of magnitude lower than diagonal ones. It has thus been chosen to neglect them. Those results follow the conclusions of [56]. It should be reported that, for a given microstructure, discrepancies are observed between off-diagonal terms whereas permeability tensor is generally considered as symmetric. Those differences can be attributed to numerical artefacts or permeability computing method. This is not really problematic as far as the weight of those terms is irrelevant.

The value of  $\ell_{RVE}$  is now estimated. For all  $V_f$  values, the coefficient of variability  $C_k$  (i.e. the ratio between the standard deviation and the mean value) decreases when  $\ell$  increases until being stable at around 3%. Similar observations can be found in [87] for constant fibre radii. Based on this observation, we first choose to define  $\ell_{RVE}$  as the first value from which  $C_k$  becomes lower than 5%. Estimations of  $\ell_{RVE}$  from this criterion are proposed in Table 1. As convergence patterns are similar for both diagonal terms, a single  $\ell_{RVE}$  is indicated for both  $k_{xx}$  and  $k_{yy}$ . Results follow the expectations as  $\ell_{RVE}$  decreases when  $V_f$  value increases. It is also relevant to notice that, for a same  $V_f$  value,  $\ell_{RVE}$  is higher for a random radius. Indeed, radius variability brings an additional statistical information: the RVE is then larger in order to grasp it.

However, the convergence criterion may be limited. Indeed, even when dispersion is

275 stable, distributions and especially mean values can be locally sensitive to the domain size. Broadly speaking, giving a precise value of  $\ell_{RVE}$  is not necessarily straightforward and relevant due to the stochastic character of the study. To circumvent this problem, it has been chosen to consider  $\ell_{RVE} \sim 100$  for all  $V_f$ . Indeed, this value seems to be large enough to provide convergence in any case. It is also rigorously consistent with the 280 scale separation hypothesis which is the basement of homogenisation procedures [82]. This choice is critical in terms of computation effort.

### 3.2. RVEs isotropy

From Fig.11, it can be noticed that  $k_{xx}$  and  $k_{yy}$  distributions at convergence are very close: this is more clearly illustrated in Fig.12. Both seem to follow a Gaussian trend [8, 90]. A Kolmogorov-Smirnov statistical hypothesis test [37] can be performed to justify rigorously the similarity of the distributions. This test is based on the comparison between the cumulative distribution function of both samples. The associated null hypothesis supposes an equality between both cumulative distribution function. A risk of 5% has been chosen. For all  $V_f$  values,  $p$ -values are retrieved : all are superior to 0.53. As they are significantly higher than the risk, this leads to conserve the null hypothesis:

$$K_{xx} \stackrel{d}{=} K_{yy} \quad (18)$$

where  $\stackrel{d}{=}$  means "equal in distribution". Considering that the perturbation step of generation algorithm is isotropic, it can be definitively stated that:

$$K_{xx} = K_{yy} \quad (19)$$

This means that microstructures are statistically isotropic in the plane  $(e_x, e_y)$  which is consistent with both ergodicity hypothesis and the generation method. Similar observations have been made in [87]. This also explains the irrelevance of off-diagonal terms for 285 RVEs. In the following, permeability will thus be considered as a scalar. In a similar fashion,  $k$  will denote realisation of a random permeability variable  $K$ .

### 3.3. Variability related to fibre centre randomness

We first describe the influence of fibre centre location, induced by the proposed generation method, on permeability. For that purpose, permeability for a constant radius 290

can be compared with various analytical robust models. Those models generally consider unit cells for which fibre centres are placed periodically allowing geometrical simplifications. Gebart permeabilities [29] have thus been considered. A comparison is observed in Fig.13. It can be seen that analytical models do not manage to model the data properly for the  $V_f$  values considered. Indeed, geometrical randomness decreases considerably the permeability values. This could have been inferred since the flow channels formed between fibres become more tortuous than for regular packings. The difference between the results from analytical models and those obtained from random microstructures can be measured by studying the mean value,  $\bar{\varepsilon}$ , of all relative differences  $\varepsilon_i$ . Mean relative differences between data mean values and Gebart permeabilities (Table 2) are both significantly high. It can be remarked that  $\bar{\varepsilon}$  is lower for Gebart's quadratic model. Moreover, it can be seen in Fig.13 that this model is more efficient than the hexagonal one for high  $V_f$  value. This is explained by the selected generation algorithm: for high  $V_f$  values, the memory of the initial packing still remains. Nevertheless, with little computational efforts, the geometrical stochastic character of the permeability can be highlighted.

The results are now compared to permeability values from similar studies that has been shown to be in agreement with experimental data [8, 87]. The microstructures of these works also show a constant fibre radius while the generation algorithms are different. Results are presented in Fig.14. A good agreement is globally observed as it can be confirmed by computing  $\bar{\varepsilon}$ . It should be noted that  $\bar{\varepsilon}$  is computed from mean values. As a result, it does not take into account the observed permeability distribution. For instance, the mean relative difference for [8] seems to be high while both distributions (Fig.14) seem to be consistent. It should be remarked that results from [8] show a high dispersion ( $\sim 15\%$ ) that may question the representativity of the geometries in consideration. Yet, all the results show a clear linear trend between the logarithm of mean permeability and  $V_f$  as it is confirmed by the GPR and its uncertainty (Fig.14). The mean relative difference between data mean values and a linear model is close to 0.5%. As a consequence, for the range of  $V_f$  values considered (i.e.  $V_f \in [0.3, 0.6]$ ), it can be assumed:

$$\log\left(\frac{\bar{k}}{\bar{r}^2}\right) \propto V_f \quad (20)$$

As noticed previously, for a given  $V_f$ , permeability distribution follows a Gaussian



law with a low coefficient of variability. Values of  $C_k$  are presented in Table 3 for various  $V_f$  values. The interest of considering a coefficient of variability rather than a standard deviation is to reason with a dimensionless quantity that can be compared for different  $V_f$ , which is necessary since permeability spans several orders of magnitude. It can be seen from Table 3 that this variability is globally stable w.r.t. to  $V_f$  and close to 3%. This statistical variability can be integrated into the model by multiplying the relation between permeability and fibre volume ratio (Eq.20) by a random variable  $\gamma \sim \mathcal{N}(1, 3\%)$ :

$$\frac{K}{\bar{r}^2} = \gamma \exp(c_1 + c_2 V_f) \quad (21)$$

where  $c_1$  and  $c_2$  are the coefficients from linear regression (Eq.20) ( $c_1 \approx 1.18$ ,  $c_2 \approx -11.8$ ). This allows a probabilistic modelling of the permeability behaviour, involving an explicit random variable. This finally leads to a first statistical bounding for permeability of random fibrous media with circular cross-sections.

310 It should be noticed from Fig.14 that all the results match well for high porosity values. This shows that the generation algorithms are equivalent for low  $V_f$ . Indeed, few fibres are observed within the RVEs and consequently, the non-overlapping condition is no more critical. At the opposite, when  $V_f$  value is high, the specificities of the selected generation algorithms may affect more directly permeability distributions [87]. **However,**  
 315 **the overall good agreement between the presented models gives us confidence about the universality of coefficients  $c_1$  and  $c_2$  in similar cases.**

#### 3.4. Variability related to radius randomness

The influence of radius randomness on permeability is here investigated. Most of the studies in literature are performed with constant radius which is not consistent with  
 320 experimental observations. Yet, radius randomness changes flow channel width and tortuosity: it can be expected to alter permeability. As introduced earlier, fibre radii are supposed to follow a normal law with of 10% variability. Permeability values are computed from numerous RVE for constant and random radius. Both histograms are compared in Fig.15. The fibre radius variability does not seem to have a significant impact  
 325 on permeability distribution. This observation can be confirmed with a Kolmogorov-Smirnov test. For all  $V_f$  values considered, each  $p$ -value is higher than 20% which leads

to conserve the null hypothesis according to which both samples follow the same distribution. This allows to show that fibre radius variability does not affect significantly permeability. This can be explained as the radii follow a Gaussian law centered in  $\bar{r}$ .

330 As a result, even if the flow channels width is altered, overall contribution equilibrates which leads to an insignificant global effect. Such an observation may thus justify the consideration of a constant fibre radius which has several important implications. First, it makes possible to consider a simpler generation algorithm with smaller computation domains (Section 3.1). Then, Eq.21 remains valid for random radii with a variability of

335 10%. For a higher radius variability (e.g. glass fibres) or for non-circular cross-sections (e.g. flax fibres), those results should be reconsidered.

### 3.5. Variability related to slip length consideration

We now consider Navier slip (Eq.5) at the fluid-solid interface. As explained in introduction, both analytical and computational upscaling procedures usually assume a no-slip condition while fibre treatment has been experimentally shown to alter fluid flow within the porous medium [25]. For a given microstructure, fluid slip is expected to increase permeability as it gets easier for the liquid to flow across the porous medium. This can be simply highlighted considering a Poiseuille flow between two planes separated by a distance  $a$  (Fig.16). Permeability  $\kappa$  can be computed analytically by solving Stokes equations, integrating velocity field and then identifying with Darcy law:

$$\kappa = \frac{a^2}{12} + \frac{a\ell_s}{2} \quad (22)$$

When slip length vanishes, Eq.22 corresponds to no-slip permeability. In a free-slip scenario, permeability becomes infinite. Despite the simplicity of this example, it allows to

340 underline the significant importance of slip length on permeability. As a consequence, permeability cannot be only considered as an intrinsic geometrical property like it is quasi-systematically assumed in literature. On the contrary, permeability should be related to fluid shear that has two origins: the tortuosity of flow channels (i.e. a geometrical contribution) and the condition at liquid/solid interface. This especially explains why

345 permeability cannot be defined in a Poiseuille flow with free-slip (Eq.22) as the fluid is not sheared. However, when the fluid is newtonian with a no-slip condition, the interface

contribution does no longer induce variability and permeability can be considered as a morphological property of the medium.

The influence of slip length on permeability is represented in Fig.17 for a given random microstructure with random radii. Permeability computed with fluid slip  $K_s$  is bounded between no-slip  $K$  and free-slip permeabilities  $K_{fs}$  as Navier slip (Eq.5) generalises both conditions. We have considered slip length values significantly superior to the ones encountered experimentally [42, 43] (typically  $10^{-9} - 10^{-6}$  m) in order to capture and characterise precisely the transition between regimes. GPR has been performed to model the trend: the related uncertainty is relatively low. Mean value of GPR perfectly match a sigmoid model. The following relation can thus be proposed:

$$K_s = K + \frac{K_{fs} - K}{1 + \frac{1}{2} \frac{\bar{r}}{\ell_s}} = K \left[ 1 - \frac{1 - \alpha}{1 + \frac{1}{2} \frac{\bar{r}}{\ell_s}} \right] \quad (23)$$

where  $\alpha = K_{fs}/K$  represents the amplitude of fluid slip phenomenon.

350 The influence of slip length on permeability can be now expressed for numerous microstructures at a given  $V_f$  value (Fig.18). This allows to integrate the statistical variability into the study of fluid slip. It is relevant to notice in Fig.18 that microstructure statistical variability, even with random radii, is relatively low in comparison with the curve amplitude due to the slip effect. The coefficient of variability is globally similar for  
 355 all the  $\ell_s$  values. Once again, GPR is performed from mean values with a noise equal to the variance. It also follows a sigmoid trend which confirms the validity of Eq.23. In addition, Eq.23 seems to hold well for different  $V_f$  values without further fit parameters, as it can be observed in Table 4. This equation can also be useful in practice to determine slip length values as discussed in Section 4.2.

We expect the fluid slip importance to rise when fibre volume fraction gets higher as the slip surface increases. We thus consider  $\alpha$  as a function of  $V_f$  (Fig.19). It should be noticed that the ratio between no-slip and free-slip permeabilities can reach a decade for  $V_f$  of interest in high performance composites. As the relation between the logarithm of  $\alpha$  and  $V_f$  is linear in a first approach, we write:

$$\log(\alpha) = \beta V_f \quad (24)$$

where  $\beta$  is the slope of the linear regression ( $\beta \approx 3.65$ ) This relation can be injected to

Eq.23 to get:

$$K_s = K \left[ 1 - \frac{1 - \exp(\beta V_f)}{1 + \frac{1}{2} \frac{\bar{r}}{\ell_s}} \right] \quad (25)$$

Eq.25 requires no-slip permeability  $K$  which can be retrieved numerically or experimentally in practice. In the context of this work,  $K$  carries the stochastic information of the model: it is expressed as a random variable. Its expression is directly linked to  $V_f$  as proposed previously in Eq.21. This finally provides a general bounding of permeability including both fluid slip and statistical variability:

$$K_s = \bar{r}^2 \gamma \exp(c_1 + c_2 V_f) \left[ 1 - \frac{1 - \exp(\beta V_f)}{1 + \frac{1}{2} \frac{\bar{r}}{\ell_s}} \right] \quad (26)$$

360 It can be remarked from the mathematical expression of the sigmoid model (Eq.23) that the transition zone from no-slip to free-slip regime is not affected by  $V_f$ . This means that, independently from  $V_f$ , the regime transition always occurs for slip lengths between  $10^{-9}$ m and  $10^{-4}$ m. Those values are close to realistic values of  $\ell_s$  meaning that fluid slip may affect permeability in practice (Section 4.2). Finally, permeability can be expressed  
 365 as a function of the slip length and  $V_f$  that are two independent variables. Both analytical (Eq.26) and GPR approaches are in very good agreement and allow a good modelling of data. The mean relative difference between analytical model and data is about 2.35%. It should be remarked that Eq.26 only holds for the transverse permeability in a 2D case. Longitudinal permeability should also be considered [22, 69] to retrieve a more complete  
 370 and realistic model. Further studies should be performed to clarify this point, that is out of the scope of this work.

## 4. Discussion

### 4.1. On permeability variability

Variability of permeability values is now discussed. We have highlighted that common  
 375 analytical models based on geometrical simplifications cannot inherently model random fibrous medium permeability. As noticed before, for a given  $V_f$  value, this variability is approximately 3% regardless of radius randomness. It seems acceptable to consider this variability as low. This observation gives an interesting insight to studies that aim at

finding the best correlations between permeability and various microstructural parameters [54, 87]. Indeed, while  $V_f$  is the most straightforward parameter to describe a fibrous microstructures, it captures nearly 95% of the observed variability. This confirms that  $V_f$  is a first-order parameter for the characterisation of fibrous media permeability as it is generally assumed. Therefore further morphological parameters may allow to describe the remaining variability or else to propose a better correlation with permeability than  $V_f$ . Those observations regarding the low variability of permeability values computed from RVEs may also justify the use of deterministic relations. Depending on the context of further works, this may be justified as far as RVEs are supposed to grasp the statistical variability of a medium.

#### 4.2. Numerical application for fluid slip influence

In the previous sections, theoretical influence of slip length on permeability has been examined. However, as a wide range of  $\ell_s$  were considered, some values were considerably higher than the orders of magnitude that are typically found in literature. As a consequence, it seems relevant to study the influence of slip length for more realistic values.

As a numerical application, we consider here  $\bar{r} = 3.5\mu m$ ,  $V_f = 0.6$ , that is a typical value for high-performance applications and  $\ell_s = 8 \pm 2$  nm that was experimentally estimated in [51] for water on a plane graphite surface, assuming that this slip length can characterise carbon fibre/fluid interactions. The extreme values have been considered for slip length (i.e. 6 nm and 10 nm). It should be remarked that those values do not take surface roughness effects into account [51]: such additional effects would tend to increase slip length. By application of Eq.25, results presented in Table 5 are retrieved: assuming Gaussian distributions, intervals that contain 95% of the distributions can be proposed. This leads to a confidence interval  $[3.24, 3.71] \times 10^{-14} m^2$  for permeability. Considering mean values, it can be seen that the relative difference w.r.t. no-slip situation is near 3%. This is comparable with statistical variability (Section 4.1). In other words, taking fluid slip into account or not would lead, at least, to an uncertainty as large as statistical variability. **This can justify the importance of fluid slip in real-world applications, for example in composite materials manufacturing.** This also confirms experimental observations from [25] where sizing (i.e. a chemical surface treatment of fibres) can divide

410 permeability values up to 4 times. Moreover it should be noticed that slip length of  
10<sup>-6</sup> – 10<sup>-4</sup> m can be found in [66]. Even if those values were back-calculated for  
transient multiphase flow involving capillarity, they would lead to huge differences be-  
tween permeability computed with fluid slip or no-fluid slip conditions (Table 5). This  
highlights the necessity of accurate slip length value for the resin/carbon couple under  
415 consideration in LCM processes. It should be also noticed that the difference between  
free-slip and no-slip permeabilities is close to one order of magnitude (Table 5, Fig.19).  
As a consequence, the bounding of permeability related to fluid slip is particularly wide  
in comparison with statistical bounds.

Finally, Eq.23 can also be useful for experimental determination of slip length from  
420 real-world fibrous materials. Indeed, the equation does not show any tuning parameter  
for a given microstructure. Thereby, permeability  $K$  should be first measured experimen-  
tally. Then, digitisation of the medium (e.g. through X-ray tomography) may allow to  
estimate  $K_{fs}$  with numerical simulation. This may lead to retrieve the slip length from  
Eq.23. Naturally, this would rely on the validity of Eq.23: further studies should be car-  
425 ried out to show the relevance of the expression especially for other radius distributions  
or for random slip length values within a microstructure.

## 5. Conclusion

The main results of the study are summarised here:

- Numerous periodic fibrous microstructures have been generated numerically through  
430 a simple and fast algorithm. The RVE size has been determined through a con-  
vergence study of permeability components. To simplify the determination of RVE  
size, it has been set to meet the scale separation hypothesis.
- A statistical study has been performed using several data-related tools such as  
Kolmogorov-Smirnov statistical hypothesis testing or Gaussian process regression:  
435 those allowed to take variability into account in a response model. The permeability  
tensor can thus be reduced to a single scalar value as long as isotropy is met  
statistically.

- Two kinds of geometrical variability have been considered. Fibre center location randomness has been shown to alter significantly permeability values, highlighting the importance of a statistical modelling. Considering a Gaussian distribution for fibre radius — with a radius variability up to 10% — has been shown to have a negligible impact on permeability.
- Permeability has been then found to follow a Gaussian distribution with a relatively low coefficient of variability ( $\sim 3\%$ ) providing a narrow confidence interval for permeability. This has also lead to consider  $V_f$  as a relevant morphological parameter for permeability characterisation as it explains on its own more than 95% of the variability.
- Fluid slip at fibre/liquid interface has been taken into account. A sigmoid transition has been observed between no-slip and free-slip regimes. This model has been then connected to the early statistical study to get a global equation that encompasses both phenomena. This has finally provided a double bounding for permeability.

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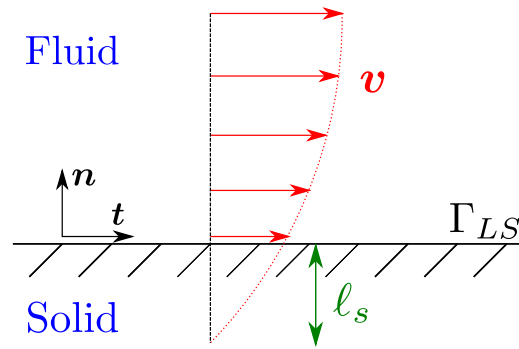


Figure 1: Fluid slip and slip length

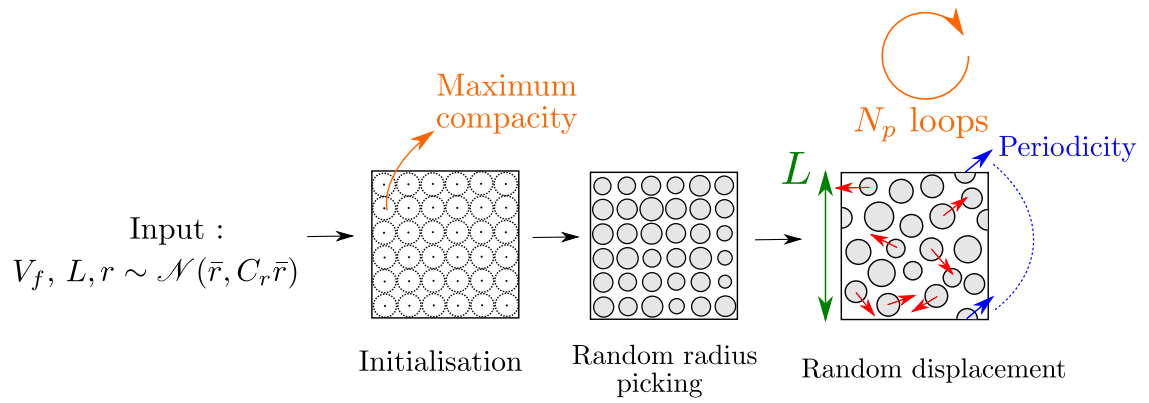


Figure 2: Algorithm for microstructure generation

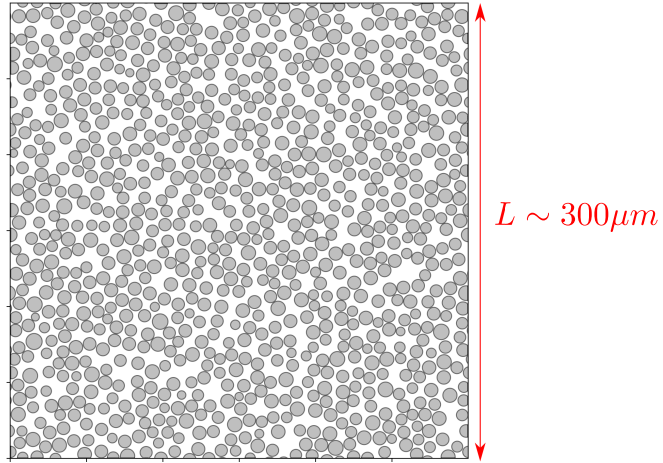


Figure 3: A numerically generated microstructure ( $V_f = 0.5$ ,  $r \sim \mathcal{N}(\bar{r}, 0.1\bar{r})$ ,  $L/\bar{r} \sim 85$ )

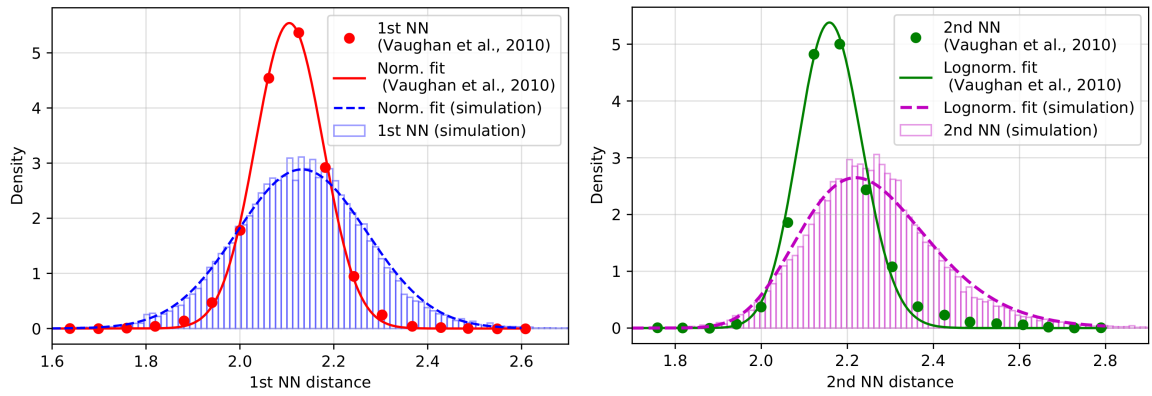


Figure 4: Nearest neighbour distance distributions normalised by  $\bar{r}$  ( $\sim 2.3 \times 10^4$  generated microstructures,  $V_f = 0.59$ ,  $N_p > 1000$ )

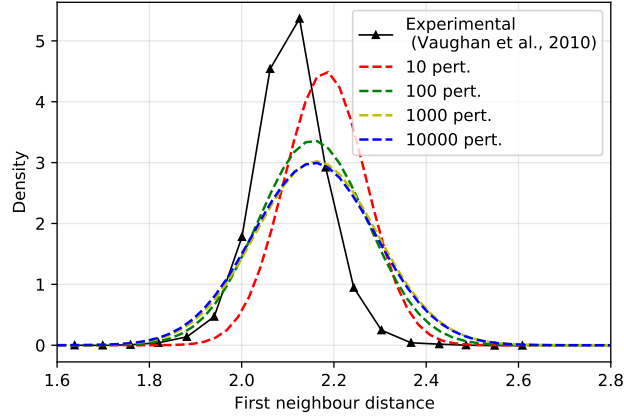


Figure 5: Gaussian fit of first nearest neighbour distance distribution for various  $N_p$  values ( $V_f = 0.59$ )

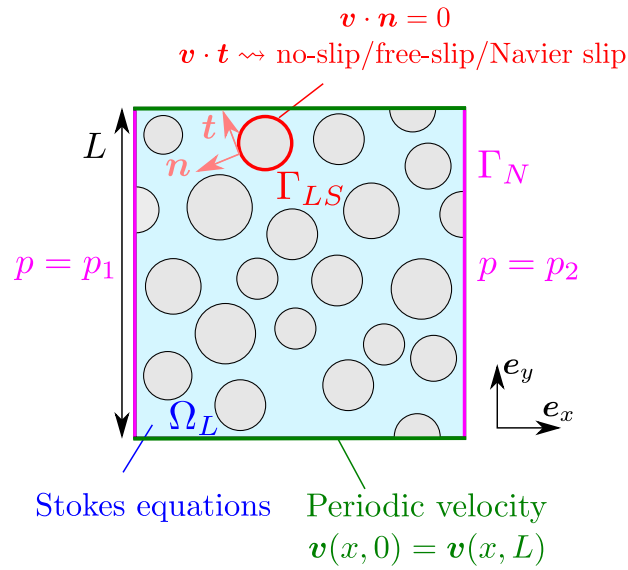


Figure 6: Domains, equations and boundary conditions for a flow in the  $x$  direction

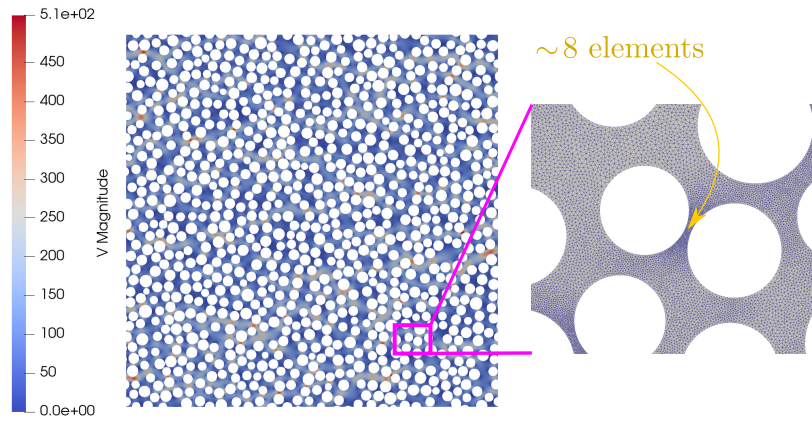


Figure 7: Numerical flow simulation (velocity magnitude field,  $\times 10^{-3}$  mm/s) and adapted mesh

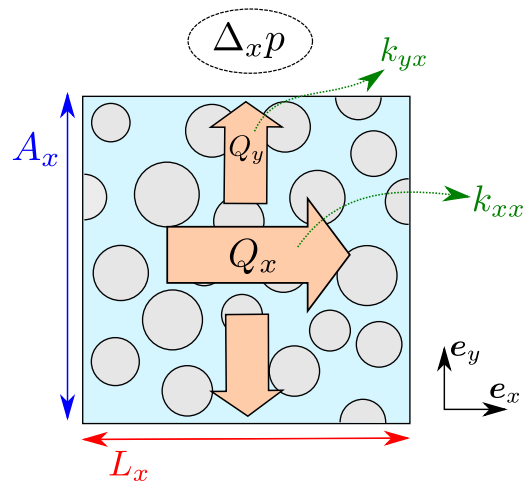


Figure 8: Illustration of the permeability computation method for a pressure gradient along  $x$  direction



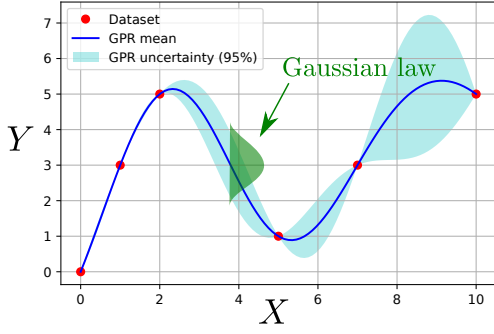


Figure 9: Illustration of noiseless GPR

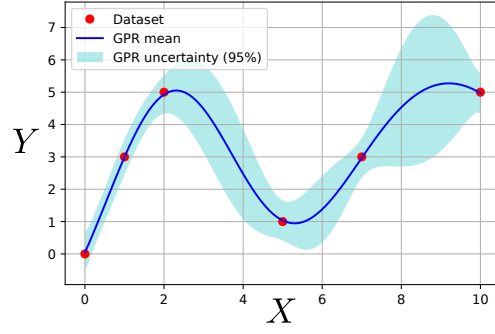


Figure 10: Illustration of noisy GPR

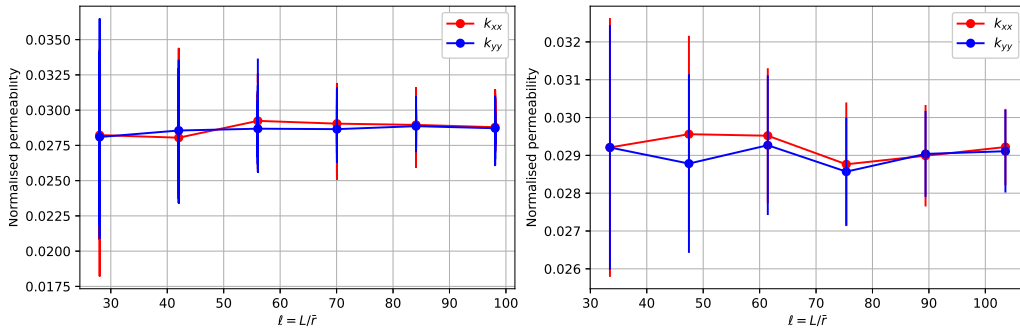


Figure 11: Convergence plot for diagonal terms: mean values and standard deviations ( $V_f = 0.4$ , left: constant radius, right: random radius)

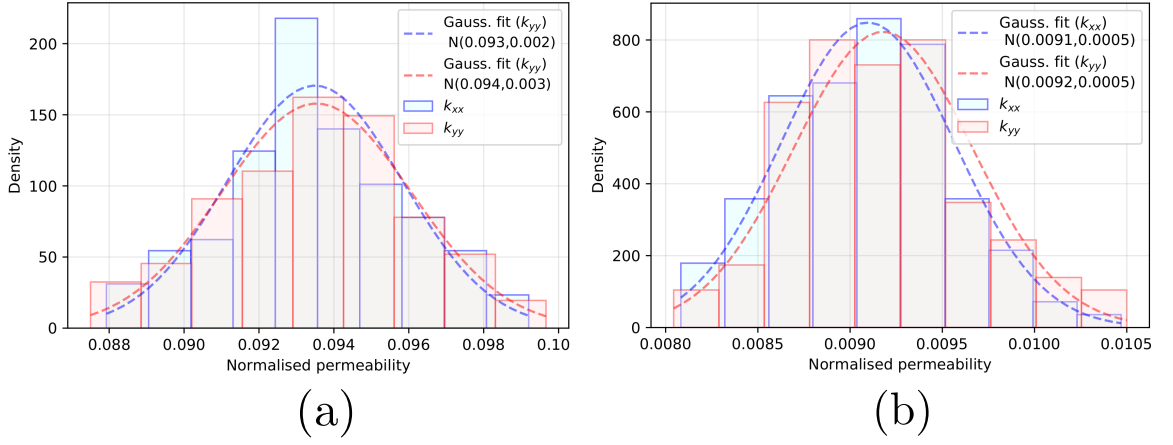


Figure 12: Histograms of diagonal permeabilities normalised by  $\bar{r}^2$  ((a)  $V_f = 0.3$ , (b)  $V_f = 0.5$ )

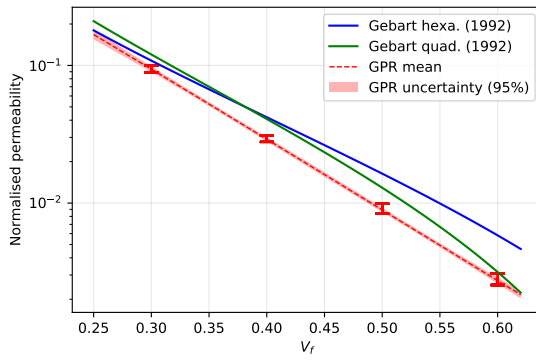


Figure 13: Permeability normalised by  $\bar{r}^2$  as a function of  $V_f$ : influence of fibres center randomness

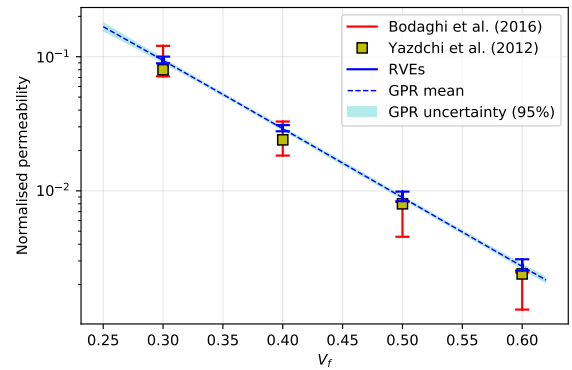


Figure 14: Permeability normalised by  $\bar{r}^2$  as a function of  $V_f$ : comparison with other studies

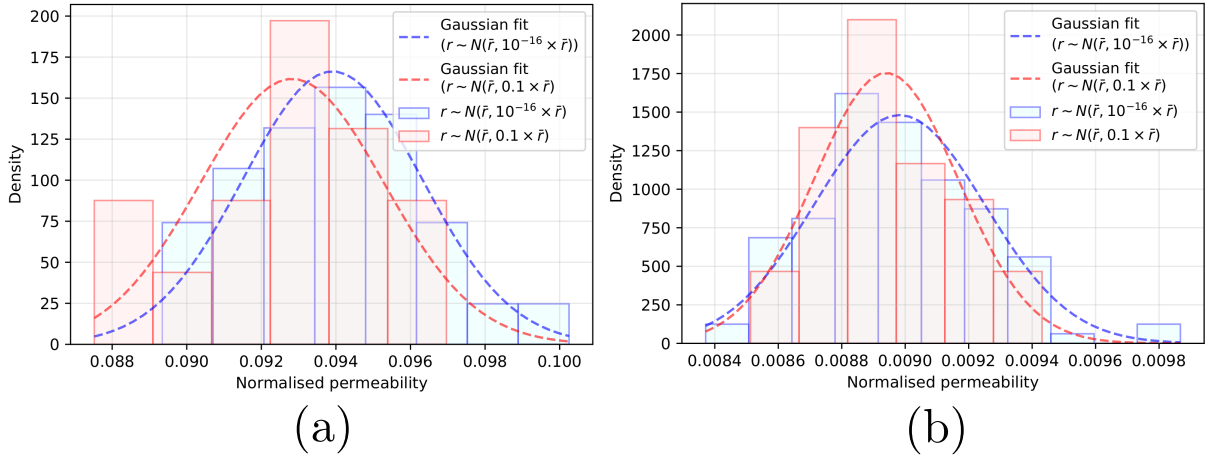


Figure 15: Histogram of permeability normalised by  $\bar{r}^2$  for constant and random radii ((a):  $V_f = 0.3$ , (b):  $V_f = 0.5$ )

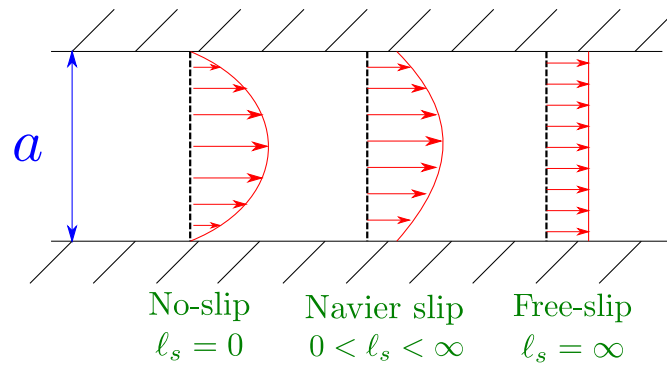


Figure 16: Poiseuille flow and slip length

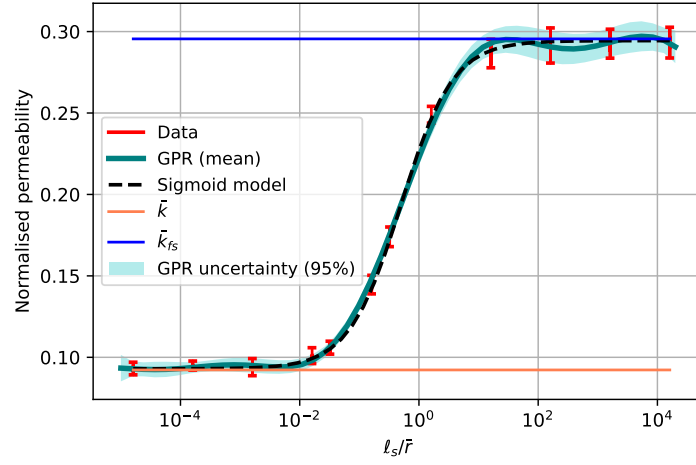


Figure 18: Permeability normalised by  $\bar{\tau}^2$  as a function of slip length with consideration of statistical variability ( $V_f = 0.3$ , 30 RVEs for each  $\ell_s$  value)

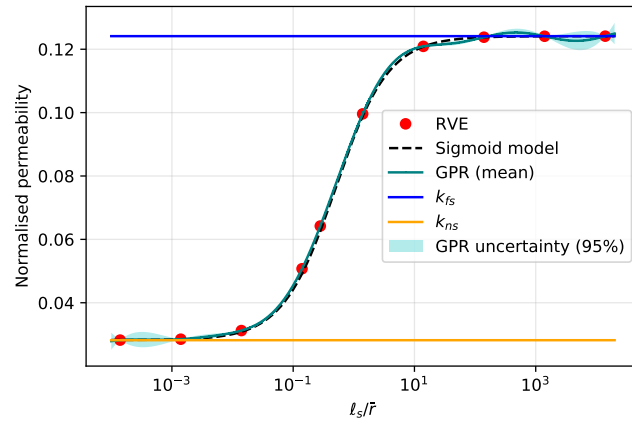


Figure 17: Permeability normalised by  $\bar{\tau}^2$  as a function of slip length for a given microstructure ( $V_f = 0.4$ )

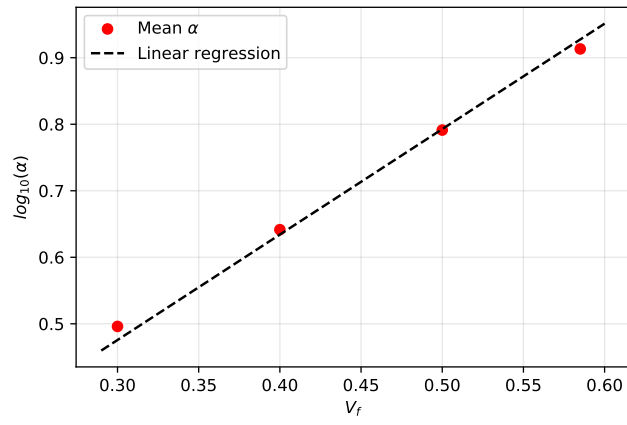


Figure 19: Decimal logarithm of mean  $\alpha$  as a function of  $V_f$

	$V_f = 0.3$	$V_f = 0.4$	$V_f = 0.5$	$V_f = 0.6$
$\ell_{RVE}$ [constant radius]	80	85	75	60
$\ell_{RVE}$ [ $r \sim \mathcal{N}(\bar{r}, 0.1\bar{r})$ ]	95	90	90	70

Table 1: Estimation of  $\ell_{RVE}$  for different  $V_f$  values

	Gebart (hexa.)	Gebart (quad.)	Bodaghi et al. (2016)	Yazdchi et al. (2012)
$\bar{\varepsilon}$ (%)	78.4	39.9	19.4	14.4

Table 2: Mean relative difference w.r.t. mean permeability values

$V_f$	0.3	0.4	0.5	0.6
$C_k$ (%)	2.6	2.6	3.0	2.9

Table 3: Coefficient of variability of permeability for different  $V_f$  values

$V_f$	0.3	0.4	0.5	0.6
$\bar{\varepsilon}$ (%)	0.75	1.2	0.91	1.3

Table 4: Mean relative difference between data mean values and sigmoid model

	No-slip	$\ell_s = 6$ nm	$\ell_s = 10$ nm	$\ell_s = 1$ $\mu\text{m}$	$\ell_s = 100$ $\mu\text{m}$	Free-slip
95% of permeability distribution ( $\times 10^{-14}$ $m^2$ )	[3.15 , 3.55]	[3.24 , 3.65]	[3.30 , 3.71]	[12.3 , 13.8]	[27.8 , 31.2]	[28.2 , 31.7]
Relative difference between $\bar{k}$ and mean value (%)	-	2.7	4.5	$\sim 300$	$> 700$	$> 700$

Table 5: Mean permeability (no-slip and realistic slip length)