On the Application of some Plasticity Laws

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Abstract. Three main rheological laws are found in the literature to describe the strain hardening of materials at high temperatures. The choice of the most suited law to describe a flow stress curve is often discussed as a function of the nature of the material; but it still remains difficult to choose the most appropriate one. These semi-empirical laws systematically comprise two main terms linked either to the dislocations generation or their annihilation.
The objective of this paper is to determine by an inverse method which law appears to be the most suited. It is finally demonstrated that the application of one law is mostly equivalent to another. The various laws are overall equivalent and do not help to describe some peculiar physical mechanism of plasticity.

Introduction

There are different models that describe the evolution of the dislocation density ρ.
These relations allow to translate the physical behavior observed on the stress-strain σ-ε curves and can be written as:

$$\frac{d\rho}{d\varepsilon} = \left(\frac{d\rho}{d\varepsilon}\right)^{+} - \left(\frac{d\rho}{d\varepsilon}\right)^{-} \tag{1}$$

This expression includes a positive term, which corresponds to the strain hardening, and thus to the introduction of dislocations in the system. The negative term corresponds to a dynamic recovery term. In general, the term associated with the work hardening, which will be called h, is proportional to $1/b\lambda$, where λ represents the mean free path of dislocations and b is the Burgers vector. Two cases can be distinguished: the mean free path does not depend on the density of dislocations but only on the size of grains, sub-grains and/or the distance between particles. In this case, it can be written as [1] :

$$\left(\frac{d\rho}{d\varepsilon}\right)^{+} = h(\dot{\varepsilon}, T) \tag{2}$$

- If the mean free path depends on the density of dislocations, it means that the dislocations are the ones that are locking between themselves. In this case, the term associated with the work hardening depends on the dislocation density according to the relation [2], [3]:

$$\left(\frac{d\rho}{d\varepsilon}\right)^{+} = h\sqrt{\rho} \tag{3}$$

The recovery term is generally proportional to ρl/b, where l corresponds to the length of dislocation annihilated during each encounter. Similarly to the work hardening, there are two completely analogous cases in which l depends on the dislocation density or not [4]:

- Independent of ρ, l here is fixed by the size of the crystallites:

$$\left(\frac{d\rho}{d\varepsilon}\right)^{-} = r(\dot{\varepsilon}, T)\rho \tag{4}$$

- Dependent of ρ, which is the parameter of the Frank network [3]:

$$\left(\frac{d\rho}{d\varepsilon}\right)^{-} = r\sqrt{\rho} \tag{5}$$

Two popular models are usually met in the literature: Kocks-Mecking model, and Laasraoui-Jonas model. Both models take the assumption that l is independent of ρ, following equation (4). However, the first one assumes a hardening depending on the dislocation density, while the second one assumes a constant h value for a given set of $(\dot{\varepsilon}, T)$.

Kocks-Mecking model [3]

$$\frac{d\rho}{d\varepsilon} = h\sqrt{\rho} - r\rho \tag{6}$$

Besides, the flow stress can be related to the density of dislocations during the deformation with the following relation [3]:

$$\sigma = M\alpha\mu b\sqrt{\rho} \tag{7}$$

where M is the Taylor factor, α is a constant (dimensionless), μ the shear modulus (Pa) and *b* the modulus of the Burgers vector (m). So, the equation (7) can be rewritten as:

$$\sigma = \left(M\alpha\mu b\frac{h}{r}\right) - \left[\left(\left(M\alpha\mu b\frac{h}{r}\right) - \sigma_0\right)\right].e^{\frac{-r\varepsilon}{2}} \tag{8}$$

with σ_0 the initial yield stress at ε=0. In this model, the term h is considered as an athermal constant while r depends strongly on the thermal activation.

The second model, developed by Laasraoui and Jonas [2] describe the rheology as:

Laasraoui-Jonas model [2]

$$\frac{\mathrm{d}\rho}{\mathrm{d}\varepsilon} = \mathrm{h} - \mathrm{r}\rho \tag{9}$$

Similarly, by using equation (7), one can deduce the following expression for the flow stress:

$$\sigma = \left(\left[(M\alpha\mu b\sqrt{\frac{h}{r}})^2 - ((M\alpha\mu b\sqrt{\frac{h}{r}})^2 - \sigma_0^2)\right].e^{-r\varepsilon}\right)^{\frac{1}{2}} \tag{10}$$

This model assumes that subgrains and precipitates play a major role on the dislocation density. The thermomechanical tests carried out by Laasraoui and Jonas in the context of the validation of the model were usually performed on hot deformed low carbon steels.

A third model generalize the two first ones, by the introduction of a new adjustable parameter.

Generalized model

The uniform model generalizes the consideration of dislocation density in the general equation as a parameter ζ (dimensionless) from 0 to 0.5 such that:

$$\frac{d\rho}{d\varepsilon} = h\rho^{\xi} - r\rho \tag{11}$$

hence,

$$\sigma = M\alpha\mu b\left(\frac{h}{r} + \left[(-\frac{h}{r} + \frac{\sigma_0}{M\alpha\mu b})^{2(1-\xi)} \right] . e^{r\varepsilon(\xi-1)}\right)^{\frac{1}{2(1-\xi)}} \quad (12)$$

This model aims to balance the Laasraoui-Jonas and Kocks-Mecking visions by introducing an adjustable parameter influencing the regulation of the dislocation density introduction in the system, during the deformation.

Choice of the plasticity law

The use of a preferred law over another is often mentioned, with arguments on the physical reasons that may lead to this type of choice. Thus (although this is not an absolute rule), one often observes in the literature materials with low stacking faults energy being characterized with Kocks-Mecking type laws [3], [5] , while for materials with high stacking faults energy like the case for ferritic materials (as α-iron and aluminum alloys), the use of the Laasraoui-Jonas law is often favored [2], [6], [7]. A comparative study of the Kocks-Mecking, Laasraoui-Jonas and a power law was formerly conducted [5]. It was shown that strain hardening during hot working could be described with an equivalent precision using these three equations [8] . In addition, the sets of parameters determined for each law are not independent, and it is possible to establish relations between these sets. This conclusion seems to be valid for a large range of materials.

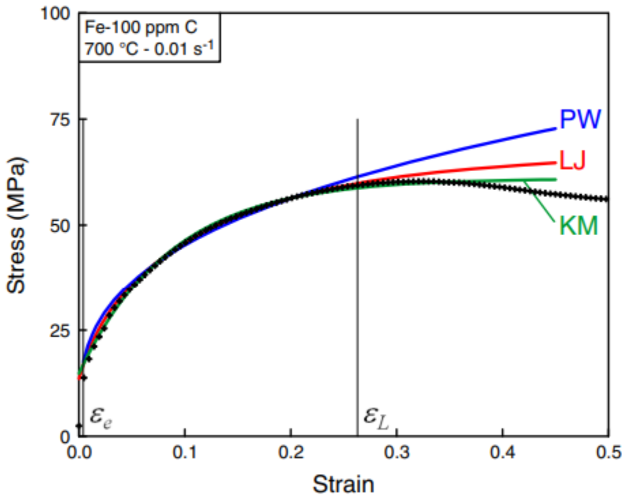


Figure 1: — Uniaxial compression stress–strain curve of a high purity base iron alloy containing 100 wt. ppm carbon deformed at 973 K (700 C) and 0.01 s1 (crosses). Three fits of the experimental curve are shown (solid lines), using the Laasraoui-Jonas (LJ), Kocks-Mecking (KM), and power law (PW) equations, respectively [8]

This work proposes to assess the different plasticity laws, and to determine whether one should be preferred to another. First, the determination of rheological parameters is done by numerical adjustment, based on a method that offers a certain robustness and repeatability that can dismiss some findings made on previous works [1], [7]. In a second step, the method is applied on pure copper, which is generally the reference material to explain the Kocks-Mecking law [3], and a ferritic stainless steel, where the Laasraoui-Jonas law is more generally applied [2]

Finally, different values for the parameter ξ are tested and the capability to determine a specific optimal value of this parameter is discussed.

Results

In order to avoid any bias in the processing of the results, a numerical algorithm is built with Python using Scikit libraries, allowing to adjust numerically the fitting parameters for the experimental flow stress curves. The data processing follows different steps. First, the yield stress must be carefully determined: indeed, the plasticity equation obviously does not hold in the elastic domain, and a yield point has to be determined, beyond which the plasticity equation holds. Then, the parameters of the plasticity law can be fitted by a least-squares method.

The determination of the elastic limit by hot compression is actually not straightforward due to the blurred boundary between the elastic and plastic zones. In this work, the yield point is determined in a robust way by considering the deviation from a linear behavior. An initial set of experimental points is selected from $\varepsilon=0$ to a threshold value $\varepsilon= \varepsilon_{th}$. For a small window $[0, \varepsilon_{th}]$ the behavior is perfectly linear, and fitting with a straight line yields to a coefficient of determination $r^2=1$. Then ε_{th} is progressively increased until the r^2 value drops to a threshold value, arbitrarily defined by $r^2=0.95$ in this work. The final value of ε_{th} corresponds to the limit of the elastic domain. The yield stress was defined as the value of flow stress for a strain $\varepsilon= \varepsilon_{th}+0.02\%$.

Figure 2 illustrates in orange the domain of the curve concerned by strain hardening, starting from the yield point and ending at the peak stress. This range of experimental points is selected to determine the parameters from equation (12).

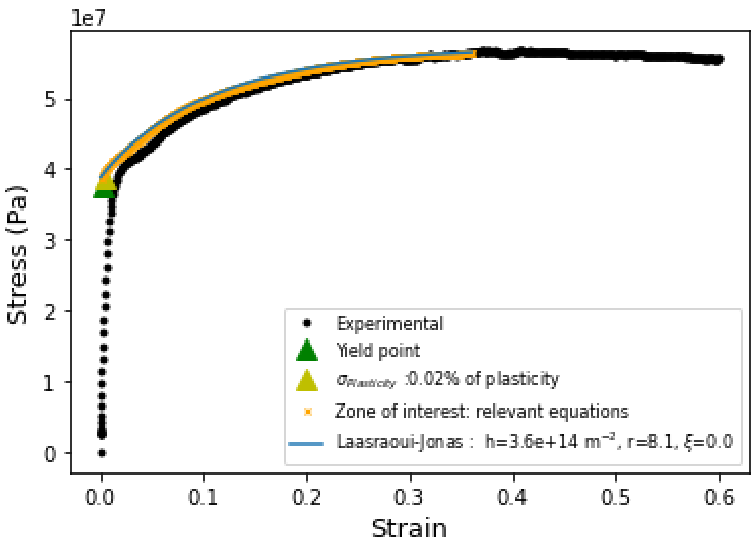


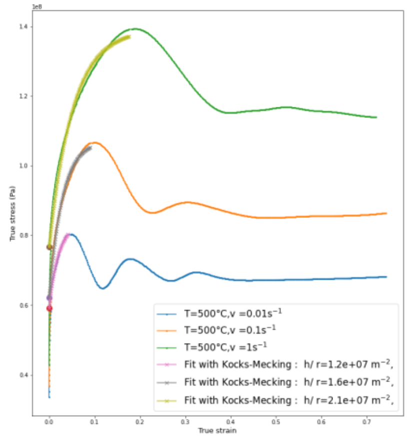
Figure 1: Example of numerical treatment of an experimental curve. The orange zone corresponds to the true zone of interest applicable for the plasticity equations

To determine whether it exists an optimal value of ξ parameter for a specific material, two cases will be studied. First, the flow stress of an stabilized ferritic steel, the composition of which is given in Table 1, will be examined: for this class of alloys, it is usually reported in literature [1] that Laasraoui-Jonas law performs better, with $\xi=0$. As a second case study, a simple metal as pure copper was selected: it was shown in literature that Kocks-Mecking models perform well for this metal [3], with $\xi=0.5$.

Element	C	Si	Mn	Ni	Cr	Mo	V	Cu	S	P	N	Nb	O (ppm)
Wt (%)	0.006	0.337	0.33	0.143	16.15	0.022	0.059	0.021	0.0032	0.021	0.014	0.229	26

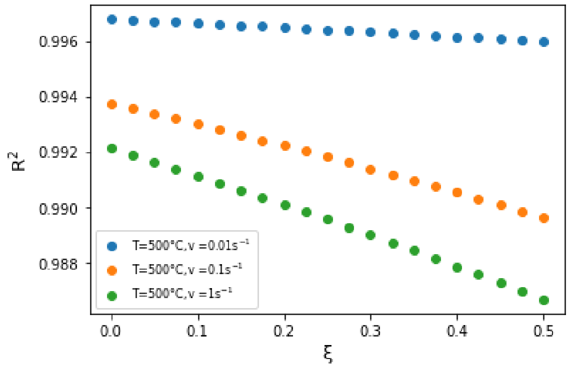
Table 1 : the grade was grown in the laboratory steel shop from Ugitech's research center, equivalent to grade 1.4511

For both metals, the full range of $\xi \in [0, 0.5]$ will be tested to assess whether the usual choice made is actually optimal, and whether an optimum actually exists or not.



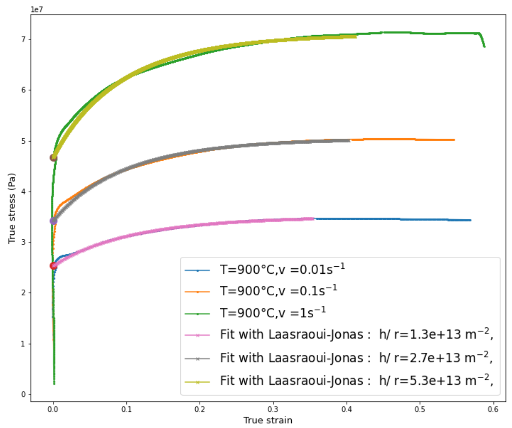
Cu

(a)



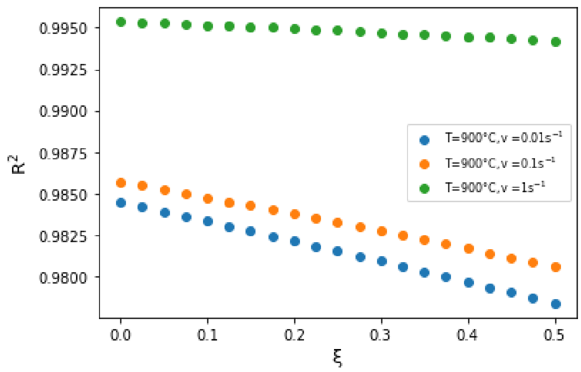
Cu

(b)



α-iron

(c)



α-iron

(d)

Figure 3 : (a) Numerical fit on data taken from the literature for hot deformed pure copper [9], (b) Corresponding values swept by ξ and its associated coefficient of determination for copper, (c) Numerical fit on a strain applied on ferritic stainless (d), Corresponding values swept by ξ and its associated coefficient of determination for ferritic steel. The various curves on the left show the elastic limits and the associated numerical adjustments, once the elastic part has been subtracted.

Figure 3 (a) shows that Kocks-Mecking law, corresponding to ξ=0.5, performs well for pure copper, in agreement with literature. In spite of that, Figure 3 (b) shows an increase of the coefficient of determination r^2 when ξ tends toward 0. Therefore, Laasraoui-Jonas tends to provide slightly better results than other equations such as Kocks-Mecking. However, the difference is extremely weak, with a difference of R^2 below 0.06% (for the most critical variation), therefore it is not really significant.

A similar result is found for ferritic stainless steel (Table 1): a suitable fit is obtained with ξ=0, corresponding to the Laasraoui-Jonas law. This fit loses slightly accuracy when ξ increases, with a monotonous decrease of R^2 of 0.001% only. Here again, the variation is not significant.

More generally, the conclusions obtained are similar to those observed by Montheillet et al.[8]; it remains important to note that the differences of laws fitting are nearly negligible. Therefore, analyzing the flow stress in this regime does not provide sufficient information to identify the value of ξ. It can be concluded that there is a very limited capability of determination of the ξ parameter, and both Kocks-Mecking and Laasraoui-Jonas laws could be used indifferently.

Conclusions

Numerical fitting allows to numerically attest the relevance of an equation and to ensure the relevance of the choice of one law compared to another. This note has been written to assess the different plasticity laws. It is finally demonstrated that the choice among these laws is not crucial, as they both yield to a same level of accuracy. The discussion on the exact hardening and recovery mechanisms based on the analysis of the flow stress curve only is not a robust approach as the coefficient ξ cannot be determined with sufficient accuracy.

Perspectives

The description of plasticity has admitted many laws that have been used to describe many grades of materials. It is natural that in order to advance the hypothesis of interchangeability of one law with another, it could be relevant to carry out a study on a set of materials of different nature, in order to verify that the experimental description that can be made of the thermomechanical paths followed can be carried out with one law as well as with another.

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