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Abstract

Industrial symbiosis is promoted as a sustainable option to convert production residues into added-value products. The by-product synergy is a particular configuration of an industrial symbiosis system, where the by-products generated by a production unit are used as raw materials by another production unit. The by-product exchange takes place between two or several companies (related or autonomous), which requires the alignment of lot sizing decisions of each involved actor. To cope with this joint production planning problem, we focus on the framing of symbiotic partnerships within a by-product synergy network, involving one supplier and one receiver of by-products. The collaboration policies can differ from one industrial symbiosis to another. In this paper, we investigate several collaboration policies for different levels of information sharing designed by using various approaches: **(i)** centralized and decentralized collaboration policies based on mixed-integer programming for full and none information sharing, **(ii)** a contractual-based collaboration policy based on game theory for one-sided asymmetric information sharing, **(iii)** a contractual-based collaboration policy obtained via a negotiation-based scheme managed by a blind mediator for multilateral asymmetric information sharing.

Keywords:
Production planning, Information sharing, Industrial symbiosis, Negotiation, Mixed-integer programming, Game theory, Monte Carlo approximation

1. Introduction

Industrial symbiosis is one of the sustainable ways to convert production residues into useful and added-value products. This is a form of collaboration between companies based on the *exchange of physical flows*, such as production residues or other secondary resources (e.g., water and/or energy), and/or the *sharing of services* like knowledge, logistics, expertise (Lombardi and Laybourn, 2012). According to the Waste Framework Directive[2], *production residues* are defined as materials that are not deliberately produced during a production process, and can be divided into two broad classes of products: **(i)** *by-products*, i.e., lawful production residues obtained unavoidably as an integral part of a production process, ready for a certain use without further transformation, and **(ii)** *wastes*, i.e., production residues, which are not by-products.

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[2]The communication from the Commission to the Council and the European Parliament on the Interpretative Communication on waste and by-products, number 52007DC0059: `https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A52007DC0059` Access: 26 August 2021

The particular configuration of an industrial symbiosis system, where the by-products generated by a production unit are used as raw materials by another production unit, is called *by-product synergy*. The by-product exchange can take place either within a single parent entity or between two or several different autonomous companies. The resulting network includes at least two actors: **(i)** a *supplier*, who generates by-products, and **(ii)** a *receiver*, who uses them. In the absence of a single parent entity, the intervention of a third party can be required to ensure the coordination between the supplier and the receiver of a by-product, by means of collaboration policies.

We distinguish two main types of collaboration policies specific to industrial symbiosis with respect to their time frames: opportunistic (short-term/one-time) and symbiotic (long-term/perennial) (Suzanne et al., 2021). The current paper focuses on the symbiotic linkage between one supplier and one receiver of by-products, i.e., on long-term collaborations. As production timing and quantity are directly linked to the by-product synergy, we take an interest in lot-sizing decisions.

Albrecht (2009) discusses different collaboration mechanisms, that can be encountered in a supply chain for long-term production collaborations. In the following, we classify the collaborative mechanisms according to the level of information sharing:

- **Full information sharing**: A centralized collaboration policy is possible in the case of full trust between actors. Generally, it happens when actors belong to the same parent company. A production plan designed under full information sharing has the advantage of being globally optimal from an economic point of view.

- **Partial information sharing:** Under these settings, some sensitive information is kept private. Various mechanisms can be applied, which are classified into three groups: contracts (usually managed via game theoretic-based collaboration policies), negotiation (conducted with or without a mediator, i.e., a third party), and auctions (Vosooghidizaji et al., 2020; Albrecht, 2020).

- **No information sharing:** is often encountered in hierarchical collaboration policies based on a single contract and presupposes a leader and a follower. It leads to a lack of flexibility for the actor that follows the contract.

In general, the full information sharing between actors may be difficult to be considered for different reasons such as the requirements to keep sensitive information private or not to reveal the risks related to production disruptions or production recipes of products. Within an industrial symbiosis system, the lack of information sharing can be too restrictive for all actors, in the sense that it may lead to no collaboration if the production plan of the follower does not include the reuse of by-products. Between these two extremes, partial information sharing represents a compromise and practical solution as attested by the existing IT platforms. Nowadays, a growing number of IT platforms are implementing not only: **(i)** to facilitate access to information about the location and availability of by-products, but also **(ii)** to support the framing of collaboration schemes. Let us mention several platforms dedicated to fostering the industrial symbiosis (Vladimirova et al., 2019): **(i)** SYNERGie 4.0 Platform and Database, promoted by International Synergies[3], **(ii)** MAESTRI Toolkit, EPOS Toolbox, Sharebox or SYNERGie 2.0 Platform, developed

[3] International Synergies https://www.international-synergies.com/ Access: 26 August 2021

in the framework of three European projects (MAESTRI[4], EPOS[5] and Sharebox[6]), and **(iii)** Industrial Symbiosis Data Repository Platform[7], an open source platform. These IT platforms are often secured and the information shared by one actor is not accessible to other actors. Nevertheless, some companies choose not to disclose their sensitive information, even if it may harm them.

Given these practical considerations, consider the specification of a symbiotic production plan within a by-product synergy network under the form of a contract approved by the involved actors. A *contract* is supposed to specify the by-product exchange plan consisting of timing and quantity. To maximize the chances of satisfying the interest of each actor, a third party (e.g., blind mediator, platform) may be requested to propose a large number of attractive and fair contracts based on partially shared information.

In practice, only part of the information related to lot-sizing decisions is really sensitive (Fraccascia and Yazan, 2018). By nature, an industrial symbiosis network is based on a win-win collaboration between actors, where actors are interested in sharing non-sensitive information. Depending on the sensitive information of each actor, three types of partial information sharing can occur (Vosooghidizaji et al., 2020): **(i)** *one-sided (or unilateral) asymmetric:* one actor has superior knowledge of an element affecting decisions, **(ii)** *multilateral (or bilateral in a network with two actors) asymmetric:* both actors have different information levels that can be about the same element or different elements, and **(iii)** *symmetric:* one actor has the information required to make an optimal production plan for the network.

In the case of one-sided asymmetric information sharing, the actor who proposes a set of contracts is determined by its bargaining power on the other actor. For multilateral asymmetric information sharing, an impartial IT platform may be implemented to manage contract proposals and pilot the negotiation process. To encourage actors to accept a given contract, contracts can include incentives like side payments (i.e., amount of money).

The current paper extends the work of Suzanne et al. (2021) by studying the collaborative lot-sizing problem in an industrial symbiosis framework. More precisely, the contributions of this paper are the following:

- Investigating several collaboration policies in an industrial symbiosis framework for different levels of information sharing designed by appropriate mathematical methods: **(i)** contractual-based collaboration policies based on game theory for one-sided asymmetric information sharing, **(ii)** a contractual-based collaboration policy obtained via a negotiation scheme managed by a blind mediator for bilateral asymmetric information sharing (see Section 2).
- Analyzing these collaboration policies for partial information sharing compared to several baseline centralized and decentralized collaboration policies for full and no information sharing, using mixed-integer linear programming. These collaboration policies are discussed according to three dimensions: the satisfaction in terms of involved actors defined as the profit level of each actor compared to his/her objective, the environmental impact, and economic benefits.

The remainder of this paper is organized as follows. Section 2 reviews the literature on **(i)** game-theoretic collaboration policies for one-sided asymmetric information sharing in

[4]MAESTRI project. `https://maestri-spire.eu/` Access: 26 August 2021
[5]EPOS project. `https:/www.spire2030.eu/epos` Access: 26 August 2021
[6]Sharebox project. `http://sharebox-project.eu` Access: 26 August 2021
[7]Industrial Symbiosis Data Repository Platform. `http://isdata.org` Access: 26 August 2021

the framework of lot-sizing decisions, and **(ii)** negotiation schemes in lot-sizing problems. The problem under study is stated, and the sensitivity of the parameters is discussed in Section 3. Section 4 introduces the centralized policy for full information sharing associated with the problem under study. A game-theoretic collaboration policy for one-sided asymmetric information sharing is explored in Section 5. Section 6 describes the proposed negotiation-based scheme managed by a blind mediator for bilateral asymmetric information sharing. Managerial implications of the collaboration policies introduced in the current paper are discussed in Section 7. Finally, concluding remarks and perspectives are provided in Section 8.

2. Literature review

The consideration given to industrial symbiosis networks in the production planning literature is growing (Daş et al., 2024) although still poor regarding the lot-sizing literature (Daquin et al., 2019; Suzanne et al., 2021; Daquin et al., 2023). Although the authors of these papers discuss the interactions between actors, they do not deal with partial information sharing and advanced collaboration policies. More broadly, the literature review proposed in this section focuses on the collaboration policies in classical supply chains for different types of asymmetric information sharing managed by the corresponding mechanism, namely one-sided asymmetric information sharing managed by contracts (see Section 2.1) and bilateral asymmetric information sharing managed via negotiation (see Section 2.2). For more details on the production planning problems in general, the reader is referred to the literature reviews of Quadt and Kuhn (2008); Buschkühl et al. (2010); Díaz-Madroñero et al. (2014); Brahimi et al. (2017); Melega et al. (2018).

2.1. One-sided (or unilateral) asymmetric information sharing: Leader-follower

One-sided asymmetric information sharing means that one actor, called a *leader*, is more powerful than the other ones, called *followers*. In other words, the leader possesses the necessary elements to establish and propose a contract and the followers adapt subsequently. The collaboration policy based on leadership under no information-sharing settings is well-known in the lot-sizing literature. As far as partial information-sharing settings are concerned, the literature is scarce. Generally, the unknown sensitive information is related to local costs. Among these local costs, some of them can be estimated. For others, intervals within which the costs belong can and have to be estimated.

Regarding the collaboration mechanisms, the majority of related papers dealing with one-sided asymmetric information sharing study a problem with two actors: a supplier and a retailer. The existing problems for supply chain coordination with one-sided asymmetric information sharing usually minimize the supplier's costs, who plays the role of the leader and proposes a side payment option (see e.g., Sucky (2006); Pishchulov and Richter (2016)). In the lot-sizing literature, Kerkkamp et al. (2019) formalize a lot-sizing problem with continuous estimated intervals of the unknown parameters, while Phouratsamay et al. (2021) and Mobini et al. (2019) study a lot-sizing problem with unknown parameters defined on discrete domains.

The problems studied in the current paper for one-sided asymmetric information sharing are adaptations from the literature to an industrial symbiosis network including **(i)** the integration of side payments, **(ii)** discrete time-space, **(iii)** discrete and time-dependent costs of the leader, and **(iv)** unknown costs of the follower considered constant by the leader. Contrary to classical studies, each actor can be the leader in the framework of industrial symbiosis. This paper deals with the minimization of the supplier's costs and the receiver's costs alternately.

2.2. Bilateral asymmetric information sharing: Negotiation process

Recall that the coordination mechanisms between two or more actors are complex and can be classified into three categories depending on the level of information sharing: **(i)** full information sharing (centralized collaboration policies), **(ii)** partial information sharing (decentralized collaboration policies), and **(iii)** no information sharing (decentralized collaboration policies). For a complete literature review of these coordination mechanisms with mathematical programming models for decentralized decision-making, the reader is referred to (Rius-Sorolla et al., 2020).

The current paper introduces a negotiation scheme between two actors. This negotiation can take two forms. In the first one, one of the actors manages the collaboration, i.e., she/he proposes the contracts and chooses when the negotiation is finished (see e.g., Dudek and Stadtler (2007); Li et al. (2011); Ogier et al. (2015); Schoenmeyr and Graves (2022)). In this configuration, there is a leader and the other actors do not have the same decisional power. Alternatively, an impartial mediator (for instance, an IT platform) can manage the negotiation procedure, having the asset of being fair.Complementary to the aforementioned existing studies and consistent with the emergence of IT platforms, let us focus in this paper on a negotiation procedure managed by an automatic impartial mediator.

In the lot-sizing literature dealing with a negotiation procedure managed by a mediator, the blind mediator knows the demands but does not know the local costs. Generally, the negotiation procedure is based on heuristics and meta-heuristics. The main steps of a conventional negotiation procedure are:

1. **Define the contract content and generate the first contract:** Generally, in the lot-sizing literature dealing with a negotiation procedure managed by a mediator, a contract corresponds to a complete solution specifying production quantities. The first contract is generated by the mediator, by randomly choosing the values of decision variables. The solution thus obtained is then sent to each actor.
2. **Provide feedback to the mediator:** To converge towards a viable contract, the mediator needs feedback on each proposed contract by each actor. This feedback can take the form of **(i)** an appreciation of the quality of the contract (e.g., a vote) or, **(ii)** a counter-proposal. The configuration, where the actors offer counter-proposals, is scarce in the negotiation-based schemes managed by a mediator (Reiß and Buer, 2014). Generally, only the mediator provides contracts.
3. **Generate a new contract based on feedback:** In the literature, feedback is used to update the parameters of the solution approach, generally a meta-heuristic or heuristic. It allows, for instance, to update the pheromone matrix in ant colony-based approaches (Homberger and Gehring, 2011) or the temperature in simulated annealing approaches (Eslikizi et al., 2015) or, to generate a new population in a genetic algorithm (Gansterer and Hartl, 2020).
4. **Choose the best contract (acceptance rule):** Generally speaking, the negotiation procedure with a mediator stops after a fixed number of iterations or a computational time limit, or when no improvement occurs after a given number of iterations. The contract chosen at the end of the procedure is the current economically best one or the best-rated one.

   To encourage an actor to choose a contract, which may not be interesting for him but beneficial from a global point of view, side payments can be computed after the recovery of the feedback of each actor (Buer et al., 2015; Eslikizi et al., 2015; Homberger et al., 2015).

Contributions of this paper. The work proposed in the current paper addresses symbiotic partnerships within a by-product synergy network under full and partial information-sharing settings. To better fit the requirements of industrial production systems, the following choices have been made:

- *Allow more flexibility in the decision-making process of actors:* In this paper, the contracts proposed by the mediator correspond to partial lot-sizing solutions. This is also the case in the negotiation procedure without a third party proposed in (Dudek and Stadtler, 2005, 2007; Li et al., 2011; Ogier et al., 2015). Contrary to the existing literature, our approach is based on the relative estimation of costs, and not on the absolute values of decision variables. This allows the mediator, in our case, to send to each Production Unit (PU) the by-product exchange plan (the core of a contract).

- *Model unknown data transparently:* Contrary to the literature, in which the feedback of the actors is used to update the hyper-parameters of the solution approach, we model the unknown parameters explicitly in the mediator's optimization program. This makes the proposed approach more: **(i)** *Generic*: Any optimization solver/solution approach can be used to solve the mediator's optimization program, and **(ii)** *Non-biased*: Since the solution approach is not necessarily feedback-dependent, the mediator has the possibility to make more accurate the correspondence between the received feedback and estimations of unknown data.

- *Pursue augmented goals:* To take advantage of the industrial symbiosis network structure, the choice of the final contract is not made based only on an economic criterion, but it is also discussed under the lens of three criteria: economic, environmental, and the satisfaction of PUs, i.e., the profit level of each actor compared to his/her objective. Note that, contrary to existing studies on classical supply chains, if the proposed contracts are not satisfactory for all actors, the collaboration can be rejected.

3. Problem statement

This paper continues the investigation of the lot-sizing problem for an industrial symbiosis (ULS-IS) introduced by Suzanne et al. (2021), where two production units (PU1 and PU2) have to plan their production over a planning horizon of T periods, as illustrated in Figure 1. Let $\mathcal{T} = \{1, 2, \ldots, T\}$ denote the set of periods. Each production unit produces a product to meet a deterministic demand. Denote by d_t^1 (resp. d_t^2) the demand of PU1 (resp. PU2) in period $t \in \mathcal{T}$ and $d_{tt'}^1$ (resp. $d_{tt'}^2$) the cumulative demand of PU1 (resp. PU2) from period $t \in \mathcal{T}$ to period $t' \in \mathcal{T}$. In addition, during the process of producing a quantity of X_t^1 units of the main product in PU1, a quantity of X_t^1 units of by-products is generated. In PU2, to produce X_t^2 units of the main product at period $t \in \mathcal{T}$, X_t^2 units of raw materials are required. The by-product generated by PU1 can be assimilated as the raw material needed to produce the main product of PU2.

To ensure the procurement of raw materials, PU2 can supply its production either with the by-products generated by PU1 or with the raw materials from an external supplier. The quantity of by-products, which is not used by PU2, can be disposed of or stored by PU1, as long as the stored quantity does not exceed a limited capacity B in each period $t \in \mathcal{T}$. The quantity of by-products, transported from PU1 to PU2 at each period $t \in \mathcal{T}$, is denoted by W_t. The quantity of raw materials bought from an external supplier at period $t \in \mathcal{T}$ is denoted by Z_t. Let L_t be the quantity of by-products disposed of in period $t \in \mathcal{T}$. The quantity of by-products stored at the end of period $t \in \mathcal{T}$ is denoted J_t.

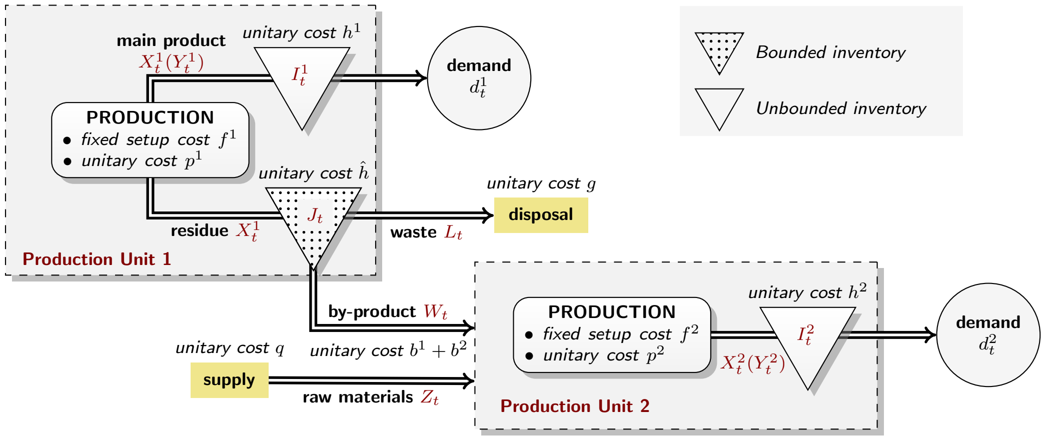

Figure 1: Process flow diagram of the ULS-IS problem, $\forall t \in \mathcal{T}$

The management of the exchange of by-products and the supply of raw materials induce the following unitary costs in each period $t \in \mathcal{T}$:

- A unitary disposal cost g, paid by PU1,
- A unitary inventory holding cost $\hat{h}$ paid by PU1 to store the generated by-products,
- A unitary cost of reusing by-products of PU1 by PU2, decomposed into two unitary costs: b^1 (resp. b^2) paid by PU1 to prepare by-products for further use (resp. paid by PU2 to transport by-products from PU1 to PU2),
- A unitary purchasing cost q of raw materials supplied from an external supplier, paid by PU2.

Moreover, each PU pays the classical lot-sizing costs per period $t \in \mathcal{T}$, namely: a unitary production cost p^1 (resp. p^2), a fixed setup cost f^1 (resp. f^2), and a unitary holding cost h^1 (resp. h^2), paid by PU1 (resp. PU2). The binary setup indicators of production for PU1 and PU2 are denoted by Y_t^1 and Y_t^2, respectively. Let I_t^1 be the inventory level of the main product in PU1 and I_t^2 be the inventory level of the product in PU2, at the end of period t. The parameters and variables are summarized in Table 1.

In what follows, a number of assumptions are made:

(A.1) The by-product inventory is null at the end of the planning horizon, i.e., $J_T = 0$.

(A.2) The treatment or transportation cost of the by-product imputed to PU1 is lower than its disposal cost paid by PU1, i.e., $b^1 \leq g$.

(A.3) The treatment or transportation cost of the by-product imputed to PU2 is lower than its purchasing cost, i.e., $b^2 \leq q$.

(A.4) On average, the by-product inventory holding cost is small enough to encourage the storage of by-products instead of their disposal of, i.e., $\hat{h} < g - b^1$. Otherwise, the problem to solve can be reduced to the problem without intermediate storage of the by-product.

(A.5) The need for raw materials in PU2 cannot trigger the production in PU1, i.e., $q \leq p^1 + b^1 + b^2$.

(A.6) A by-product availability in PU1 cannot trigger the production in PU2, i.e., $g \leq p^2 + b^1 + b^2$.

(A.7) The by-product has a lower value than the main product, i.e., $\hat{h} < h^1$.

Table 1: Summary of the problem parameters

Parameters:	
T	Number of time periods
$\mathcal{T}$	Set of periods $\mathcal{T} = \{1, 2, \ldots, T\}$
d_t^1 (d_t^2)	Demand for the main product of PU1 (PU2) in period t
p^1 (p^2)	Unitary production cost for PU1 (PU2) in period t
f^1 (f^2)	Fixed setup cost for PU1 (PU2) in period t
h^1 (h^2)	Unitary holding cost for the main product of PU1 (PU2) in each period t
$\hat{h}$	Unitary holding cost for the by-product of PU1 in period t
q	Unitary purchasing cost of raw materials by PU2 from an external supplier in period t
b^1 (b^2)	Unitary treatment or transportation cost imputed to PU1 (PU2) for the by-product in period t
g	Unitary by-product disposal cost of PU1 in period t
B	By-product inventory capacity in PU1 in each period
$d_{tt'}^1$ ($d_{tt'}^2$)	Cumulative demand of PU1 (PU2) between periods t and t', i.e., $d_{tt'}^1 = \sum_{i=t}^{t'} d_i^1$ $\left(d_{tt'}^2 = \sum_{i=t}^{t'} d_i^2 \right)$
M_t^1 (M_t^2)	Big number with $M_t^1 = d_{tT}^1$ $\left(M_t^2 = d_{tT}^2\right)$
Decision variables:	
X_t^1 (X_t^2)	Production quantity in PU1 (PU2) in period t
Y_t^1 (Y_t^2)	Binary setup indicator for PU1 (PU2) associated with period t
I_t^1 (I_t^2)	Inventory level of the main product of PU1 (PU2) at the end of period t
J_t	Inventory level of the by-product of PU1 at the end of period t
W_t	Quantity of by-products sent from PU1 to PU2 in period t
Z_t	Quantity of raw materials purchased at an external supplier by PU2 in period t
L_t	Quantity to be disposed of by-products in period t

(A.8) In accordance with the definition of a by-product, a quantity of the main product cannot be stored to admit a gain induced by a by-product, i.e., $(g - b^1) \leq h^1$ and $(q - b^2) \leq h^2$.

Note that, if one of Assumptions (A.2)-(A.4) is not met, the problem becomes trivial. If Assumptions (A.5)-(A.8) are not met the definition of a by-product is not fulfilled, and the problem can be reduced to a co-product management problem (Suzanne et al., 2020).

Recall that the current paper focuses on collaboration policies for partial information sharing. It assumes that some information is known while others are unknown, depending on their sensitivity degree. In the literature, several types of information can be kept private like costs, demands, capacities, objective functions, price, and quality (Vosooghidizaji et al., 2020). For the ULS-IS problem, let us consider that cost information is private. First, we assume that the demands for main products are not sensitive information as they can be estimated according to the market (which is known) and the proposed by-product exchange plan. In the same way, the by-product inventory capacity is supposed known, as this information is not really sensitive.

In accordance with Fraccascia and Yazan (2018), the non-sensitive information of the ULS-IS problem is the quantity of available by-products at the first PU (PU1) and the required quantity of raw materials at the second PU (PU2). This information is sufficient to calculate the environmental impact related to by-product exchanges and to identify the potential by-product synergies because the quantities of exchanged by-products are bounded by the demands of main products, which are supposed known.

In the following, we will classify the problem parameters into three categories: known,

Table 2: Classification of the information related to ULS-IS problem

Known parameters	**Estimated parameters**	**Neutral parameters**
• Demands: d_t^1, d_t^2, $\forall t \in \mathcal{T}$ • By-product inventory capacity: B	• Setup costs: f^1, f^2 (via TBO^1, TBO^2) • Inventory holding costs: h^1, h^2, $\hat{h}$ • Gains of reusing the by-product: $g-b^1$, $q-b^2$	• Production costs: p^1, p^2 • Disposal cost: g • Purchasing cost: q

to estimate, and optimization-neutral (i.e., parameters that do not impact the process of contract generation). As reported in Table 2, in the ULS-IS problem, we suppose that, as previously mentioned, the available information is: **(i)** demands d_t^1 and d_t^2 in each period $t \in \mathcal{T}$, and **(ii)** by-product inventory capacity B. Internal costs are considered sensitive. To improve the quality of proposed contracts (and, consequently, make possible the collaboration between PUs), the internal costs of each PU are expressed via intervals, estimated based on market knowledge. When trying to estimate the local costs of PU1, the objective function of PU1 can be expressed as follows:

$$\begin{aligned}
C^1 &= \sum_{t=1}^{T}(p^1X_t^1 + f^1Y_t^1 + h^1I_t^1 + \hat{h}J_t + gL_t + b^1W_t)\\
&= p^1\sum_{t=1}^{T}X_t^1 + g\sum_{t=1}^{T}L_t + b^1\sum_{t=1}^{T}W_t + \sum_{t=1}^{T}(f^1Y_t^1 + h^1I_t^1 + \hat{h}J_t)\\
&= p^1d_{1T}^1 + g\sum_{t=1}^{T}(X_t^1 - W_t) + b^1\sum_{t=1}^{T}W_t + \sum_{t=1}^{T}(f^1Y_t^1 + h^1I_t^1 + \hat{h}J_t)\\
&= (p^1 + g)d_{1T}^1 + \sum_{t=1}^{T}(f^1Y_t^1 + h^1I_t^1 + \hat{h}J_t - (g - b^1)W_t)
\end{aligned}$$

Similarly, when trying to estimate the local costs of PU2, the objective function of PU2 can be expressed as follows:

$$C^2 = \sum_{t=1}^{T}(p^2X_t^2 + f^2Y_t^2 + h^2I_t^2 + qZ_t + b^2W_t) = (p^2+q)d_{1T}^2 + \sum_{t=1}^{T}\left[f^2Y_t^2 + h^2I_t^2 - (q-b^2)W_t\right]$$

Production costs p^1 and p^2 can be neglected as they are constant. As long as quantity d_{1T}^1 (resp. d_{1T}^2) of main products is produced, $p^1d_{1T}^1$ (resp. $p^2d_{1T}^2$) has to be paid. Disposal of and purchasing costs g and q do not have a direct impact on the unknown parameter estimations since we estimate the gains of reusing the by-product, namely $(g - b^1)$ and $(q - b^2)$.

By virtue of the economic order quantity relationship (see e.g.,Trigeiro et al. (1989); Helber (1995)), the setup cost can be expressed as a function of the inventory holding cost, the average demand, and the Time Between Orders (TBO). The ratio between setup and holding costs is thus a critical problem parameter. Hence, the setup cost f can be approximated based on the average demand $\bar{d}$ known by the negotiator, inventory holding cost h, and the estimated TBO, as follows:

$$f = \frac{1}{2}h\Big(TBO\Big)^2\bar{d} \tag{1}$$

According to Assumptions (A.1)-(A.8), the definition domains of the main parameters, that can impact the contract generation, are defined as follows:

1. **Time Between Orders (TBO) of PU1 (resp. PU2)** TBO^1 **(resp.** TBO^2**):** take their values in the set $\{TBO^{min}, TBO^{min}+1, \ldots, TBO^{max}\}$.
2. **Inventory holding costs of the main products:** The absolute cost values being unimportant in the light of relationship (1), let us express the estimation domain of the other parameters in terms of h^1. The value of h^2 provides the significance of PU2 compared to PU1. h^2 is defined in the continuous interval $[h^{min}, h^{max}]$.
3. **Inventory holding cost of the by-product:** By Assumption (A.7), the by-product inventory holding cost in PU1 is lower than the main product inventory holding cost. To keep the scale, we fix $\hat{h} \in [0, h^1[$. This interval is continuous. Note that, when the by-product is unstorable, $\hat{h}$ is fixed to zero.
4. **Gain of reusing the by-product instead of disposing it of and purchasing the raw material (Estimation of** $g+q-b^1-b^2$**):** Note that $g+q-b^1-b^2$ is smaller or equal to 0 because it corresponds to a gain. The gain can be separated into two coefficients $g-b^1$ and $q-b^2$ such that: **(i)** $g-b^1$ corresponds to the estimation of the gain of PU1, and **(ii)** $q-b^2$ represents the estimation of the gain of PU2. In accordance with other coefficients, we define $g-b^1 \in [\hat{h}, h^1]$ given Assumptions (A.4) and (A.8), and $q-b^2 \in [0, h^2]$ given Assumption (A.8).
5. **Setup costs** f^1 **and** f^2 are calculated via formula (1), based on h^1, TBO^1 and $\bar{d}^1$ and, h^2, TBO^2 and $\bar{d}^2$, respectively.

In what follows, let us introduce and discuss collaboration policies for the ULS-IS problem for three levels of information sharing: **(i)** full information sharing, i.e., centralized collaboration policy (Section 4), **(ii)** one-sided asymmetric information sharing: game theory-based collaboration policies (Section 5), and **(iii)** bilateral asymmetric information sharing: negotiation process managed by a mediator (Section 6).

4. Full information sharing: Centralized collaboration policy

In this section, the centralized collaboration policy for full information sharing introduced in (Suzanne et al., 2021) is explicitly provided for reference. Using the notations given in Table 1, the centralized collaboration policy of the ULS-IS problem can be modeled via the following straightforward formulation:

$$\begin{aligned} \min \sum_{t=1}^{T} (f^1 Y_t^1 + h^1 I_t^1 + \hat{h} J_t - (g-b^1) W_t) \\ + \sum_{t=1}^{T} (f^2 Y_t^2 + h^2 I_t^2 - (q-b^2) W_t) \end{aligned} \tag{2}$$

$$
\begin{aligned}
\text{s.t. } & I^1_{t-1} + X^1_t - I^1_t = d^1_t, && \forall t \in \mathcal{T} && (3)\\
& I^1_0 = 0, && && (4)\\
& X^1_t \le M^1_t Y^1_t, && \forall t \in \mathcal{T} && (5)\\
& I^2_{t-1} + X^2_t - I^2_t = d^2_t, && \forall t \in \mathcal{T} && (6)\\
& I^2_0 = 0, && && (7)\\
& X^2_t \le M^2_t Y^2_t, && \forall t \in \mathcal{T} && (8)\\
& J_{t-1} + X^1_t = W_t + L_t + J_t, && \forall t \in \mathcal{T} && (9)\\
& J_0 = J_T = 0, && && (10)\\
& J_t \le B, && \forall t \in \mathcal{T} && (11)\\
& W_t + Z_t = X^2_t, && \forall t \in \mathcal{T} && (12)\\
& X^1_t, X^2_t, I^1_t, I^2_t, W_t, Z_t, J_t, L_t \ge 0, && \forall t \in \mathcal{T} && (13)\\
& Y^1_t, Y^2_t \in \{0, 1\}, && \forall t \in \mathcal{T} && (14)
\end{aligned}
$$

The objective function (2) minimizes the sum of the following costs: setup, inventory holding, and non-reuse of the by-products. Note that, costs of constant terms $(p^1 + g)d^1_{1T}$ and $(p^2+q)d^2_{1T}$ are added to the value of objective function. Constraints (3) and (6) model the flow conservation of the main products of PU1 and PU2, respectively. Constraints (4) and (7) empty the initial inventories of the main products of PU1 and PU2. Constraints (5) and (8) are production constraints, which ensure that the production launching at a given period entails a setup operation at the same period. Constraints (9) and (12) are flow conservation constraints of by-products and external raw material flows, which ensure that the production residues of PU1 are either disposed of, stored, or used, and raw materials required for the production of PU2 are bought. Constraints (10) fix the initial and ending inventories of by-products to zero. The inventory capacity of the by-product is limited by Constraints (11). Constraints (13) and (14) are the nonnegativity and binary requirement constraints.

5. One-sided asymmetric information sharing: Game-theoretic collaboration policy

This section explores lot-sizing problems for industrial symbiosis in the case of one-sided asymmetric information sharing. The production unit having the best bargaining power about lot-sizing decisions on the other one is called the *leader*. The production unit, which does not have information about the planning of its interlocutor, is called the *follower*. The leader and the follower aim to collaborate, i.e., to synchronize their production plans in order to reuse the by-product generated by the supplier. For this purpose and inspired by the field of mechanism designs from economics (see e.g. Laffont and Martimort (2009)), the leader proposes a menu of contracts, and the follower selects subsequently a contract from it or can choose to not cooperate. A *contract* is composed of a production plan, and a potential side payment, i.e., an amount of money given by the leader to the follower to encourage him/her to accept the contract. The challenge of the leader is to construct a menu of contracts, taking into account the uncertain parameters of the follower, such that his/her expected costs are minimized. Note that for each value of the uncertain parameter, only one contract is proposed. The following two configurations are studied:

- *PU1 has the leadership* and proposes a menu of contracts to PU2. This situation is realistic given that the by-product is created by PU1, which has to get rid of it, even if it has to pay for that.
- *PU2 has the leadership.* The relevance of this situation lies in the fact that PU1 can be interested in adapting its production according to the production of PU2 in order to get rid of the by-product.

Both configurations are explained more deeply in Sections 5.1 and 5.2. To make the problem tractable using mixed-integer programming, we suppose that the range of the unknown parameters is known to the leader and is given by the discrete set of scenarios Θ. Each scenario $\theta \in \Theta$ corresponds to a possible realization of the unknown parameters. The unknown parameters of the follower are indexed by scenario and take their values in range $\{\underline{\theta}, \dots, \overline{\theta}\}$.

We also suppose that from the point of view of the leader, a scenario $\theta \in \Theta$ has a probability $\mathbb{P}(\theta)$ of occurring. All decision variables introduced in Section 3 are also indexed by scenario. We also introduce new decision variables $z(\theta)$ associated with the side payment given by the leader to the follower, corresponding to scenario $\theta \in \Theta$.

We also introduce $C^*(\theta)$, which represents the optimal value of the follower without collaboration if scenario $\theta \in \Theta$ occurs. This value is obtained by solving the single-item lot-sizing problem of the follower without collaboration, by fixing the unknown parameters to the associated scenario. Recall that some information is kept hidden by each PU for privacy reasons. Then, the leader makes arrangements so that the follower is encouraged to stay honest. To do this, a variable measuring the honesty of the follower when choosing his/her contract is added. Let $C(\hat{\theta}|\theta)$ be the value of the optimal solution of the follower when she/he applies the production plan corresponding to scenario $\hat{\theta}$, whereas his/her true estimated parameters correspond to scenario $\theta \in \Theta$.

5.1. The supplier is the leader

In this section, the supplier of by-products is the leader and the receiver is the follower. As discussed in Table 2, the unknown parameters depend on scenario θ: $h^2(\theta)$, $f^2(\theta)$ and $(q - b^2)(\theta)$, $\forall \theta \in \Theta$.

The goal of the supplier is to propose a contract for each scenario realization, that minimizes his/her average estimated cost. The supplier has to ensure that each contact provides the receiver with a total production cost at least as good as the cost obtained without collaboration. The supplier should also ensure that the receiver does not lie about his/her true costs. The problem that the supplier has to solve can be formulated by the following mixed-integer linear program:

$$\min \sum_{\theta \in \Theta} \mathbb{P}(\theta) \left[\sum_{t=1}^{T} \Big(f^1 Y_t^1(\theta) + h^1 I_t^1(\theta) + \hat{h} J_t(\theta) - (g - b^1) W_t(\theta) \Big) + z(\theta) \right] \tag{15}$$

$$\text{s.t.} \quad C(\hat{\theta}|\theta) = \sum_{t=1}^{T} \Big[f^2(\theta) Y_t^2(\hat{\theta}) + h^2(\theta) I_t^2(\hat{\theta}) - (q - b^2)(\theta) W_t(\hat{\theta}) \Big], \qquad \forall \hat{\theta}, \theta \in \Theta \tag{16}$$

$$C(\theta|\theta) - z(\theta) \leq C^*(\theta), \qquad \forall \theta \in \Theta \tag{17}$$

$$C(\theta|\theta) - z(\theta) \leq C(\hat{\theta}|\theta) - z(\hat{\theta}), \qquad \forall \hat{\theta}, \theta \in \Theta \tag{18}$$

$$I_{t-1}^1(\theta) + X_t^1(\theta) - I_t^1(\theta) = d_t^1, \qquad \forall \theta \in \Theta, \forall t \in \mathcal{T} \tag{19}$$

$$I_0^1(\theta) = J_0(\theta) = J_T(\theta) = 0, \qquad \forall \theta \in \Theta \tag{20}$$

$$X_t^1(\theta) \leq M_t^1 Y_t^1(\theta), \qquad \forall \theta \in \Theta, \forall t \in \mathcal{T} \tag{21}$$

$$I_{t-1}^2(\theta) + X_t^2(\theta) - I_t^2(\theta) = d_t^2, \qquad \forall \theta \in \Theta, \forall t \in \mathcal{T} \tag{22}$$

$$I_0^2(\theta) = I_T^2(\theta) = 0, \qquad \forall \theta \in \Theta \tag{23}$$

$$Y_t^2(\theta) \leq X_t^2(\theta) \leq M_t^2 Y_t^2(\theta), \qquad \forall \theta \in \Theta, \forall t \in \mathcal{T} \tag{24}$$

$$J_{t-1}(\theta) + X_t^1(\theta) = W_t(\theta) + L_t(\theta) + J_t(\theta), \qquad \forall \theta \in \Theta, \forall t \in \mathcal{T} \tag{25}$$

$$W_t(\theta) + Z_t(\theta) = X_t^2(\theta), \qquad \forall \theta \in \Theta, \forall t \in \mathcal{T} \tag{26}$$

$$J_0(\theta) = J_T(\theta) = 0, \qquad \forall \theta \in \Theta \tag{27}$$

$$J_t(\theta) \leq B, \qquad \forall \theta \in \Theta, \forall t \in \mathcal{T} \tag{28}$$

$$X_t^1(\theta), X_t^2(\theta), I_t^1(\theta), I_t^2(\theta), W_t(\theta), Z_t(\theta), L_t(\theta) \geq 0, \qquad \forall \theta \in \Theta, \forall t \in \mathcal{T} \tag{29}$$

$$Y_t^1(\theta), Y_t^2(\theta) \in \{0, 1\}, \qquad \forall \theta \in \Theta, \forall t \in \mathcal{T} \tag{30}$$

The objective function (15) minimizes the total sum of costs related to PU1: fixed setup cost, inventory holding costs, the costs related to the by-products non-reuse and the side payment paid to PU1. The value of constant term $(p^1 + g)d_{1T}^1$ is added to the value of the objective function. The set of constraints (16) defines the sum of all the PU2's lot-sizing cost when selecting a contract designed for $\theta \in \Theta$ under scenario $\hat{\theta} \in \Theta$. Constraints (17), known as the individual rationality constraints, ensure that the contracts proposed to PU2 are always advantageous for PU2, i.e., at least as good as no contract at all. The set of constraints (18), known as the incentive compatibility constraints, pushes PU2 to accept the contract corresponding to its true costs, i.e., PU2 does not lie. The sets of constraints (19)-(21) correspond to the lot-sizing constraints related to PU1. The sets of constraints (22)-(24) are the classical lot-sizing constraints related to PU2. Note that the equation $Y_t^2(\theta) \leq X_t^2(\theta)$ is added to avoid the creation of a setup in certain periods, that can appear without minimization of PU2 costs. Constraints (25)-(26) are the flow conservation constraints of the by-product linking the two production units. The initial and ending by-product inventories are emptied by constraints (27). Constraints (28) limit the by-product inventory capacity in each period. Finally, constraints (29) and (30) define the decision variables.

5.2. The receiver is the leader

In this section, the receiver is the leader and the supplier is the follower. As discussed in Table 2, the unknown parameters depend on the scenario θ: $h^1(\theta)$, $f^1(\theta)$, $(g - b^1)(\theta)$ and $\hat{h}(\theta)$, $\forall \theta \in \Theta$.

Similar to the problem faced by the supplier when she/he has the lead, the problem, that the receiver has to solve when she/he has the lead, can be formulated as follows:

$$\min \sum_{\theta \in \Theta} \mathbb{P}(\theta) \left[\sum_{t=1}^{T} \Big(f^2 Y_t^2(\theta) + h^2 I_t^2(\theta) - (q - b^2) W_t(\theta) \Big) + z(\theta) \right] \tag{31}$$

$$\text{s.t. } (17)-(22),(24)-(30) \tag{32}$$

$$C(\hat{\theta}|\theta) = \sum_{t=1}^{T} \Big[f^1(\theta)Y_t^1(\hat{\theta}) + h^1(\theta)I_t^1(\hat{\theta}) + \hat{h}(\theta)J_t(\hat{\theta}) - (g-b^1)(\theta)W_t(\hat{\theta}) \Big], \quad \forall \hat{\theta},\theta \in \Theta \tag{33}$$

$$I_0^2(\theta) = 0, \quad \forall \theta \in \Theta \tag{34}$$

$$I_T^1(\theta) = 0, \quad \forall \theta \in \Theta \tag{35}$$

Model (31)-(35) is symmetrical with regard to Model (15)-(30). Objective function (31) minimizes the sum of costs associated with PU2. Value of constant term $(p^2+q)d_{1T}^2$ is added to the value of objective function. The set of constraints (33) defines the sum of all the PU1's costs when selecting a contract designed for $\theta \in \Theta$ given $\hat{\theta} \in \Theta$. Constraints (34) fix the initial inventory of the main product of PU2 to zero. The ending inventory of the main product of PU1 is emptied by constraints (35).

6. Bilateral asymmetric information sharing: Negotiation process managed by a mediator

Under one-sided asymmetric information sharing, it seems natural to consider that the leader manages the network. In the case of bilateral asymmetric information sharing, there is no perfect leader. In this section, a negotiation-based scheme managed by a blind mediator is proposed (see e.g. Homberger (2010); Gansterer and Hartl (2020)).

6.1. General scheme

As specified in Section 3, we suppose that the mediator knows only the demands of PU1 and PU2, d_t^1 and $d_t^2, \forall t \in \mathcal{T}$, and by-product inventory capacity B. The goal of the mediator is to propose contracts attractive for PU1 and PU2. Consistent with the sustainability goals underneath the industrial symbiosis, let us consider that the contracts proposed by the mediator have to be not only economically, but also environmentally attractive. To propose satisfactory contracts, the mediator has to estimate accurately the unknown parameters. Let us consider that the mediator assumes that all parameters are constant. The mediator has then to estimate the following parameters, under the form of a limited range:

- Time Between Orders (TBO) $TBO^1, TBO^2 \in \{TBO^{min}, TBO^{min}+1, \ldots, TBO^{max}\}$;
- Inventory holding costs: $h^2 \in [h^{min}, h^{max}]$ and $\hat{h} \in [0, h^1[$;
- Gains of reusing the by-product instead of disposing it of (PU1) or purchasing raw materials (PU2): $g - b^1 \in [\hat{h}, h^1]$ and $q - b^2 \in [0, h^2]$.

The steps of the negotiation process are provided in Algorithm 1. They can be separated into two main phases:

- **Phase 1** *(Estimation of TBO):* The first phase of Algorithm 1 aims at estimating the values of TBO^1 and TBO^2 by a brute force search. Contracts are generated for each value of TBO^1 and TBO^2, while all the other costs are fixed. After the evaluation of these contracts by each PU, the values of TBO^i leading to the higher score are kept for Phase 2. If several values of TBO^i provide the higher score, definition domain of TBO^i is reduced, $i \in \{1, 2\}$.

- **Phase 2** *(Monte Carlo sampling):* This phase is based on a crude Monte Carlo sampling process. It provides a large number of contracts generated by randomly choosing values of $h^2, g - b^1, q - b^2$, and $\hat{h}$, while TBO^1 and TBO^2 are chosen in their new definition domains reduced by Phase 1. These contracts are evaluated by each PU, and non-dominated scenarios are kept for the final choice.

Algorithm 1 Negotiation procedure operated by a mediator based on a Monte Carlo sampling

Create an empty list of contracts $\mathcal{S} = \emptyset$
Initialize $\mathcal{TBO}^1 = \mathcal{TBO}^2 = \{TBO^{min}, \ldots, TBO^{max}\}$

Phase 1 – TBO estimation phase

for each PU $i \in \{1, 2\}$ **do**
    for all $TBO^i \in \mathcal{TBO}^i$ **do**
        Mediator: $S^i_{TBO^i} \leftarrow$ `GenerateFirstContract(`TBO^i`)` ▷ see Section 6.2.1
        Mediator: Send contract $S^i_{TBO^i}$ to PU i
    end for
end for
Each PU $i \in \{1, 2\}$: $score^i_{TBO^i} \leftarrow$ `Evaluate(`$S^i_{TBO^i}$`, ` i`)` ▷ see Section 6.3
Mediator: Update $\mathcal{TBO}^i = \{TBO^i \in \{TBO, ..., TBO^*\} | score^i_{TBO} = score^i_{TBO^*} = 1\}$

Phase 2 – Monte Carlo sampling phase

while stopping condition not met **do**
    Mediator: $S \leftarrow$ `GenerateContract(`$\mathcal{TBO}^1, \mathcal{TBO}^2$`)` ▷ see Section 6.2.2
    Mediator: Send contract S
    Each PU $i \in \{1, 2\}$: $score^i \leftarrow$ `Evaluate(`S`, ` i`)` ▷ see Section 6.3
    $\mathcal{S} = \mathcal{S} \cup \{S\}$
end while
Mediator: $S_{best} \leftarrow$ `ChooseBestContract(`$\mathcal{S}$`)` ▷ see Section 6.4

6.2. Contract generation by the mediator

The generation of a contract S is performed by solving the following mixed-integer linear program, aiming to estimate the best total cost by fixing all unknown cost parameters as described below.

$$C(S) = \min \sum_{t=1}^{T} (f^1 Y^1_t + h^1 I^1_t + \hat{h} J_t + f^2 Y^2_t + h^2 I^2_t - (g - b^1 + q - b^2) W_t) \quad (36)$$

$$\text{s.t. } (3) - (14) \quad (37)$$

$$I^1_T = 0 \quad (38)$$

$$I^2_T = 0 \quad (39)$$

6.2.1. Function `GenerateFirstContract(•)`

The goal of the first step is to estimate the true values of TBO^1 and TBO^2. Recall that each PU has to rate each proposed contract, and provides a score up to a maximal value, fixed to 1. Contracts being expressed as an exchanged plan of by-products, the idea of the mediator is to fix values of the unknown parameters and solve two independent models, one for each PU. In this way, the exchange plan proposed to each PU corresponds to its production plan and the mediator is able to guess TBO^1 and TBO^2 by varying only them and looking at their impact on the scores provided by PUs. To this end, the inventory holding cost related to the by-product is set to a constant non-zero value, i.e., $\hat{h} = 1$. The gains are set to a constant non-zero value to encourage the exchanges between production units, i.e., $g - b^1 = q - b^2 = 1$. Inventory holding costs are set to non-zero values to avoid $f^1 = 0$ and/or $f^2 = 0$, e.g. $h^1 = h^2 = 1$. Then, setup costs are computed using formula (1) and they are the only non-constant parameters of the first set of contracts.

For PU1, the model to solve is the following:

$$\min \sum_{t=1}^{T} (f^1 Y_t^1 + I_t^1 + J_t - W_t)$$

$$\text{s.t. } (3) - (5), (9) - (11), (13) - (14)$$

For PU2, the model to solve is the following:

$$\min \sum_{t=1}^{T} (f^2 Y_t^2 + I_t^2 - W_t)$$

$$\text{s.t. } (6) - (8), (12) - (14)$$

Note that solving these models leads to contracts, which are not necessarily beneficial for each PU. The timing and quantity of the proposed exchange of by-products do not necessarily match the production plans of PUs. For each PU, the contract built using its true setup cost, corresponds to perfect settings (not necessarily feasible at a network level) when there is no by-product disposal and no external purchasing of raw materials while all demands are satisfied. This contract will lead to a higher score. The collaboration policy corresponding to these perfect settings is called `zeroWaste` policy. In that way, the definition domains of TBO^1 and TBO^2 are reduced to accelerate the Monte Carlo sampling procedure.

6.2.2. Function `GenerateContract(•)`

This function generates randomly the unknown parameters and solves a mathematical problem for TBO^1 and TBO^2 in given sets $\mathcal{TBO}^1$ and $\mathcal{TBO}^2$, respectively. Recall that setup costs f^i are calculated based on h^i and TBO^i. The mediator solves the model (36)-(39) with the given values of the parameters and returns a contract (Function `SolveModel(•)`). A contract is expressed as an exchange plan of by-products. Algorithm 2 provides the main steps of the proposed procedure: **(i)** generate randomly the unknown parameters, **(ii)** compute the setup costs f^1 and f^2, and **(iii)** solve the problem to derive the exchange plan. Note that TBO^1 and TBO^2 are fixed within the domains defined in Phase 1 of Algorithm 1.

Algorithm 2 `GenerateContract(`$\mathcal{TBO}^1, \mathcal{TBO}^2$`)`

1: $(h^1, h^2, \hat{h}, g-b^1, q-b^2) \leftarrow$ Randomly select values of the parameters within their definition domains
2: $(TBO^1, TBO^2) \leftarrow$ Randomly select values in $(\mathcal{TBO}^1, \mathcal{TBO}^2)$
3: **for all** $i \in \{1,2\}$ **do**
4: $\quad f^i \leftarrow \frac{1}{2} h^i \left(TBO^i\right)^2 \bar{d}^i$
5: **end for**
6: $S \leftarrow$ `SolveModel(`$f^1, f^2, h^1, h^2, \hat{h}, g-b^1, q-b^2$`)`
7: **return** contract S

6.3. Contract evaluation

At this step, each production unit has to evaluate the contracts provided by the mediator. Function `Evaluate(`S, i`)` provides a score calculated based on the local costs of production unit $i \in \{1,2\}$ when implementing contract S. Contract S provides the quantity $w_t(S)$ of by-products exchanged in each period $t \in \mathcal{T}$.

Each PU $i \in \{1,2\}$ computes its internal optimal cost $C^i_{eval}(S)$ for each contract S, and transforms it into a score, denoted by $score^i$. $C^i_{eval}(S)$ is obtained by solving a lot-sizing problem associated with PU i given the exchange plan $(w_1(S), \ldots, w_T(S))$ induced by contract S. The mathematical model that PU1 (resp. PU2) has to solve is given by expressions (40)-(44) (resp. (45)-(49)).

Evaluation in PU1.

$$C^1_{eval}(S) = \min \sum_{t=1}^{T} (f^1 Y^1_t + h^1 I^1_t + \hat{h} J_t - (g - b^1) W_t) \tag{40}$$

$$\begin{aligned}
\text{s.t. } & (3)-(5), (9)-(11) & & (41)\\
& W_t = w_t(S), & \forall t \in \mathcal{T} \quad & (42)\\
& X^1_t, I^1_t, W_t, J_t, L_t \geq 0, & \forall t \in \mathcal{T} \quad & (43)\\
& Y^1_t \in \{0,1\}, & \forall t \in \mathcal{T} \quad & (44)
\end{aligned}$$

Evaluation in PU2.

$$C^2_{eval}(S) = \min \sum_{t=1}^{T} (f^2 Y^2_t + h^2 I^2_t - (q - b^2) W_t) \tag{45}$$

$$\begin{aligned}
\text{s.t. } & (6)-(8), (12) & & (46)\\
& W_t = w_t(S), & \forall t \in \mathcal{T} \quad & (47)\\
& X^2_t, I^2_t, W_t, Z_t \geq 0, & \forall t \in \mathcal{T} \quad & (48)\\
& Y^2_t \in \{0,1\}, & \forall t \in \mathcal{T} \quad & (49)
\end{aligned}$$

Evaluation of a contract (i.e., an exchange plan). Each PU calculates its own internal optimal cost, but this cost is not provided to the mediator for privacy reasons. Instead of providing the true estimated cost, each PU $i \in \{1,2\}$ provides a score, which is calculated based on two internal costs:

- $C^i_{nominal}$: No collaboration, i.e., no symbiotic partnership is considered between production units. The by-products generated by PU1 are disposed of, and raw materials used by PU2 are purchased from an external supplier. Let the costs obtained in the framework of this policy be called nominal costs as in (Suzanne et al., 2021), and be denoted by `No_Co`.

- $C^i_{zeroWaste}$: This local cost is calculated under perfect settings (not necessarily feasible at a network level) when there is no by-product disposal and no external purchasing of raw materials. Note that $C^i_{zeroWaste} \leq C^i_{nominal}$, $\forall i \in \{1, 2\}$.

The interval of each score is $]-\infty, 1]$, where 1 is the maximal score. Each PU $i \in \{1, 2\}$ gives back a score ($score^i$) to contract S, calculated as follows:

$$score^i(S) = \frac{C^i_{nominal} - C^i_{eval}(S)}{C^i_{nominal} - C^i_{zeroWaste}} \tag{50}$$

Note that:

- If $C^i_{eval}(S) = C^i_{zeroWaste}$, then $score^i(S) = 1$.
- If $C^i_{zeroWaste} \leq C^i_{eval}(S) \leq C^i_{nominal}$, then $0 \leq score^i(S) \leq 1$.
- If $C^i_{eval}(S) \geq C^i_{nominal}$, then $score^i(S) \leq 0$.

In addition to the scores provided by each PU, the mediator computes an environmental score (denoted by $score^{exc}$) corresponding to the quantity of exchanged by-products. It is computed as follows, for each contract S:

$$score^{exc}(S) = \frac{\sum_{t=1}^{T} W_t(S)}{\min\{d^1_{1T}, d^2_{1T}\}} \tag{51}$$

If there is no collaboration between PU1 and PU2, i.e., $\sum_{t=1}^{T} W_t = 0$, then $score^{exc} = 0$. On the contrary, if $\sum_{t=1}^{T} W_t = \min\{d^1_{1T}, d^2_{1T}\}$, then $score^{exc} = 1$.

6.4. Selecting the best contract

When Algorithm 1 terminates, the mediator has to choose the final contract to implement among a set $\mathcal{S}$ of proposed contracts. We propose to sort the generated contracts according to three criteria: **(i)** a satisfaction criterion based on the scores returned by PUs (Sat), **(ii)** an environmentally-related score based on $score^{exc}$ (Env), and **(iii)** an economic-related score, calculated based on the estimated unknown parameters (Eco).

Satisfaction criterion. This criterion is based on the global satisfaction of PUs. To satisfy both PUs, the chosen contract must induce the highest cumulative scores. To do this, we compute the satisfaction criterion ($Sat(S)$) as follows, $\forall S \in \mathcal{S}$:

$$Sat(S) = \frac{score^1(S) + score^2(S)}{2} \tag{52}$$

Environmental impact. Industrial symbiosis allows PUs to reduce waste and raw material extraction, having thus a positive environmental impact. In our study, the environmental implication of a given contract is expressed by the quantity of exchanged by-products. To do this, the environmental impact ($Env(S)$) can be defined as follows, $\forall S \in \mathcal{S}$:

$$Env(S) = score^{exc}(S)$$

Economic impact. From a purely economic point of view, the centralized collaboration policy serves as a baseline by virtue of its definition. When selecting a final contract, the mediator aims, among other criteria, to minimize the global cost without taking into account the cost-sharing between production units. Recall that the mediator does not know the true values of local costs associated with contracts, but can use the estimated parameters to determine the contract, which has a minimal global cost. Let $C(S)$ be the global cost of contract S computed by the mediator when solving Model (36)-(39). To properly determine the value of a given contract in terms of the aforementioned criteria, the estimated global cost has to be normalized to scale the range $[0, 1]$. To do this, as the mediator does not know `zeroWaste` and `nominal` costs, the range of the estimated global cost $C(S)$ is computed based on the best and worst costs of the proposed contracts. Let C_{min} and C_{max} be the extreme values of the range in which the estimated global costs vary:

$$C_{min} = \min_{S \in \mathcal{S}} C(S) \text{ and } C_{max} = \max_{S \in \mathcal{S}} C(S)$$

The value of the economic criterion $Eco(S)$ is evaluated as follows, $\forall S \in \mathcal{S}$:

$$Eco(S) = \left(1 - \frac{C(S) - C_{min}}{C_{max} - C_{min}}\right)$$

A weight is given to each criterion by the mediator. Hence, the final contract proposed by the mediator maximizes the weighted sum of three criteria, determined via:

$$S^* = \underset{S \in \{K \in \mathcal{S} | score^i(K) \geq 0,\, i \in \{1,2\}\}}{\operatorname{argmax}} \Big(\mu_{Sat} Sat(S) + \mu_{Env} Env(S) + \mu_{Eco} Eco(S)\Big) \tag{53}$$

where μ_{Sat}, μ_{Env} and μ_{Eco} are the weights describing the importance given to the three considered criteria (namely, satisfaction, environmental, and economic), such that $\mu_{Sat} + \mu_{Env} + \mu_{Eco} = 1$. Function `ChooseBestContract(•)` in Algorithm 1 returns a single resulting contract S^*. Note that, in case of multiple optimal contracts, the mediator only proposes one of them, arbitrarily chosen, in order to avoid conflicts of preference between the two actors.

7. Managerial implications

In this section, let us discuss the economic and environmental opportunities induced by the exchange of by-products between two production units, by examining the following three collaboration policies for partial information sharing against the baseline collaboration policies introduced by Suzanne et al. (2021) and described in Section 7.1:

- Two game-theoretic collaboration policies for one-sided asymmetric information sharing: **(i)** `PU1_Leader`: the collaboration policy introduced in Section 5.1, i.e., the supplier is the leader and the receiver is the follower, **(ii)** `PU2_Leader`: the collaboration policy introduced in Section 5.2, i.e., the receiver is the leader and the supplier is the follower. Only one parameter is assumed unknown and is described via a probability distribution.

- `Nego`: The contractual-based collaboration policy obtained via the proposed negotiation-based scheme managed by a blind mediator for bilateral asymmetric information sharing, introduced in Section 6.

More precisely, after having described the framework of computational experiments in Section 7.1, let us:

- Investigate the domain of the estimated parameter on game theory-based collaboration policies (see Appendix A.1).
- Discuss the game theory-based collaboration policies with respect to baseline collaboration policies (see Section 7.2).
- Validate our negotiation procedure (see Appendix A.2).
- Discuss the final choice of the contract by the mediator during the negotiation process (see Section 7.3).
- Discuss the negotiation-based collaboration policy with respect to baseline collaboration policies (see Section 7.4).
- Position the different collaboration policies on a 3D score-based system of coordinates on a specific instance (see Section 7.5).

7.1. Design of experiments

Computational experiments have been conducted on 7,290 benchmark instances generated in Suzanne et al. (2021), on a computer with Intel Xeon e5-2620 2.1GHz CPU with 32GB RAM. These instances satisfy Assumptions 1-8 and are defined by: **(i)** a planning horizon length $T = 24$, **(ii)** a link between PU1 and PU2 $\Delta = \frac{h^2}{h^1} \in \{0.75, 1, 1.25\}$, **(iii)** Time Between Order of production units $TBO^1, TBO^2 \in \{3, 4, 5\}$, **(iv)** demands d_t^1 and d_t^2 generated following a normal distribution with an average in set $\{50, 100, 200\}$ and a standard deviation of 20%, $\forall t \in \mathcal{T}$ and, **(v)** a by-product inventory capacity $B \in \{0, 1.2\bar{d}^1, 3\bar{d}^2\}$. For each PU $i \in \{1, 2\}$, given TBO^i, holding cost h^i and average demand $\bar{d}^i$, setup cost f^i is approximated via the economic order quantity formula (1).

Mathematical programs are solved using IBM ILOG CPLEX 12.6. The characteristics of the negotiation procedure are set as follows: The stopping condition corresponds to a fixed number of contracts to generate. We fixed the following values: $TBO^{min} = 1$, $TBO^{max} = 7$, $h^1 = 1$, $h^{min} = 0.75$ and $h^{max} = 1.25$.

Regarding the policy `PU2_Leader` (resp. `PU1_Leader`), the following situations have been investigated:

- The inventory holding cost of the main product of PU1 (resp. PU2) is unknown, i.e., $h^1(\theta) \in \{\underline{\theta}, \ldots, \overline{\theta}\}$, $f^1(\theta) = \mathtt{f}(h^1(\theta), TBO^1, \bar{d}^1)$, $(g - b^1)(\theta) = g - b^1$ and $\hat{h}(\theta) = \hat{h}$, $\forall \theta \in \Theta$ (resp. $h^2(\theta) \in \{\underline{\theta}, \ldots, \overline{\theta}\}$, $f^2(\theta) = \mathtt{f}(h^2(\theta), TBO^2, \bar{d}^2)$ and $(q - b^2)(\theta) = q - b^2$, $\forall \theta \in \Theta$),
- The TBO of PU1 (resp. PU2) is unknown, i.e., $h^1(\theta) = h^1$, $f^1(\theta) = \mathtt{f}(h^1, TBO^1(\theta), \bar{d}^1)$ with $TBO^1(\theta) \in \{\underline{\theta}, \ldots, \overline{\theta}\}$, $(g - b^1)(\theta) = g - b^1$ and $\hat{h}(\theta) = \hat{h}$, $\forall \theta \in \Theta$ (resp. $h^2(\theta) = h^2$, $f^2(\theta) = \mathtt{f}(h^2, TBO^2(\theta), \bar{d}^2)$ with $TBO^2(\theta) \in \{\underline{\theta}, \ldots, \overline{\theta}\}$ and $(q - b^2)(\theta) = q - b^2$, $\forall \theta \in \Theta$),
- The gain for PU1 (resp. PU2) of reusing the by-product of PU1 instead of purchasing raw materials from an external supplier is unknown, i.e., $h^1(\theta) = h^1$, $f^1(\theta) = f^1$, $(g - b^1)(\theta) \in \{\underline{\theta}, \ldots, \overline{\theta}\}$ and $\hat{h}(\theta) = \hat{h}$, $\forall \theta \in \Theta$ (resp. $h^2(\theta) = h^2$, $f^2(\theta) = f^2$ and $(q - b^2)(\theta) \in \{\underline{\theta}, \ldots, \overline{\theta}\}$, $\forall \theta \in \Theta$),
- The inventory holding cost of the by-product of PU1 is unknown, i.e., $h^1(\theta) = h^1$, $f^1(\theta) = f^1$, $(g - b^1)(\theta) = g - b^1$ and $\hat{h}(\theta) \in \{\underline{\theta}, \ldots, \overline{\theta}\}$, $\forall \theta \in \Theta$.

To make possible the evaluation and discussion of the collaboration policies for partial information sharing provided in this paper, let us take a look at the baseline collaboration policies for none and full information sharing introduced by Suzanne et al. (2021), namely:

- `Full_Co`: *Full collaboration*, i.e., the exchange of by-products is planned in the framework of a centralized collaboration policy. No other policy can provide better system-wide gains. The costs obtained under this policy are called *centralized costs*.

- `Opp_Co`: *Opportunistic collaboration*, i.e., the exchange of by-products is being done by taking advantage of a fortunate matching between the production plans of the supplier (PU1) and the receiver (PU2). More precisely, each production unit makes its own production plan and they collaborate only when there is an availability and a requirement of by-product at the same time.

- Two sequential decentralized collaboration policies: **(i)** `PU1_First`: PU1 makes its production plan first and then PU2 makes its production plan within the limits of the available quantities of by-products in PU1, **(ii)** `PU2_First`: PU2 makes its production plan first, and then PU1 makes its production plan within the limits of quantities of by-products required by PU2.

Table 3: Collaboration policies against `No_Co`: *Economic and environmental gains (in %)*

Gains	B	`Opp_Co`			`PU1_First`			`PU2_First`			`Full_Co`		
		min	avg	max	min	avg	max	min	avg	max	min	avg	max
PU1	$= 0$	0.1	2.8	9.5	0.3	6.4	21.7	0.1	5.5	20.6	-0.9	7.0	21.6
	> 0	0.1	3.0	10.1	0.2	7.1	21.0	0.1	6.7	18.9	-1.0	7.9	22.0
PU2	$= 0$	0.1	2.4	9.6	0.1	5.0	22.6	0.2	5.5	23.1	-0.7	5.9	23.0
	> 0	0.0	2.4	8.7	0.0	5.2	21.4	0.4	6.2	22.0	0.2	6.4	21.8
Env.	$= 0$	11.3	35.5	61.0	20.5	82.3	100	11.3	82.7	100	65.1	97.5	100
	> 0	3.2	34.3	61.0	7.0	82.0	100	15.3	88.8	100	70.4	98.7	100

avg: Average, Env.: Environmental

In the following, the individual gain of each PU i is calculated with respect to its nominal cost c^i obtained outside any symbiotic partnership, as follows $(1 - c^i_p/c^i) \times 100$, where c^i_p is the cost of production unit i obtained in the framework of a collaboration policy denoted by p, $p \in \{$`Full_Co`, `Opp_Co`, `PU1_First`, `PU2_First`, `PU1_Leader`, `PU2_Leader`, `Nego`$\}$, $i \in \{1, 2\}$. The environmental gain represents the proportion of the reused by-products compared to the total quantity of reusable by-products. Note that the environmental gain is directly related to the local costs associated with the management of production residues and raw materials. For this reason, economic and environmental benefits are correlated. Note that, Table 3 has been largely discussed in Suzanne et al. (2021) and the associated findings are not given again in the current paper. Table 3 is only used for the purposes of positioning the collaboration policies for partial information sharing according to the baseline collaboration policies.

7.2. Discussions on collaboration policies managed by a leader

The gains obtained for each of the aforementioned policies and for each estimated parameter $\{TBO^2,\ TBO^1,\ h^1,\ h^2,\ q - b^2,\ g - b^1,\ \hat{h}\}$ are provided in Tables 4 and 5. The environmental gain represents the proportion of reused by-products compared to the total quantity of generated by-products reusable by PU2. The side payment is computed as a proportion of the cost of the leader conceded to the follower.

Table 4: `PU1_Leader` versus `No_Co`: *Economic and environmental gains (in %)*

Collaboration policy `PU1_Leader`										
Gains	B	h^2			TBO^2			$q-b^2$		
		min	avg	max	min	avg	max	min	avg	max
PU1	$=0$	0.4	7.3	22.0	0.5	7.4	22.0	0.4	7.2	22.0
	>0	0.6	8.6	23.3	0.6	8.6	23.3	0.6	8.5	23.3
PU2	$=0$	0.1	2.1	11.0	0	2.1	20.0	0.1	3.2	18.5
	>0	0.1	2.2	10.8	0	2.2	18.6	0.1	3.4	17.6
Env.	$=0$	39.3	97.7	100	55.9	98.4	100	37.6	97.4	100
	>0	54.2	98.6	100	66.7	99.0	100	39.3	98.5	100
Pay.	$=0$	0	≈0	1.8	0	≈0	2.2	0	≈0	4.7
	>0	0	≈0	1.6	0	≈0	1.9	0	≈0	4.0

avg: Average, Env.: Environmental, Pay.: Side payment

Table 5: `PU2_Leader` versus `No_Co`: *Economic and environmental gains (in %)*

Collaboration policy `PU2_Leader`													
Gains	B	h^1			TBO^1			$g-b^1$			$\hat{h}$		
		min	avg	max	min	avg	max	min	avg	max	min	avg	max
PU1	$=0$	0.1	2.4	15.4	0	2.4	18.2	0.1	3.6	17.6			
	>0	0	1.7	9.9	0	1.2	15.6	0.1	4.2	18.6	0	0.5	6.4
PU2	$=0$	0.4	6.3	23.1	0.9	6.4	23.1	0.4	6.3	23.1			
	>0	0.6	6.8	22.3	0.6	6.8	22.3	0.4	6.8	22.3	0.6	6.8	22.3
Env.	$=0$	41.0	97.3	100	52.7	98.1	100	37.1	97.1	100			
	>0	81.9	99.9	100	87.5	99.9	100	65.6	99.3	100	75.7	99.9	100
Pay.	$=0$	0	≈0	1.7	0	≈0	0.9	0	≈0	4.2			
	>0	0	≈0	1.3	0	≈0	1.0	0	≈0	4.6	0	≈0	1.4

avg: Average, Env.: Environmental, Pay.: Side payment

First, we perform a comparison with the collaboration policies based on leadership, `PU1_First` and `PU2_First`, summarized in Table 3. Environmentally speaking, note that policies `PU1_Leader` and `PU2_Leader` are comparable to the full collaboration policy `Full_Co` with a gain of 97.1% on average in the worst case (see Table 5).

Focusing on the economic indicators, we can notice that in collaboration policies based on game theory, the leader has a profit higher than those obtained via other collaboration policies. For instance, under collaboration policy `PU1_Leader`, the gain of PU1 is at least 0.2% better than `Full_Co`, when the by-product is unstorable, and 0.6% better than `Full_Co`, when the by-product is storable with a limited capacity, on average. As the leader has control of the proposed contracts, it is intuitive to think that it is economically profitable, but the gain is limited due to the fact that the follower can reject the contract if he/she judges it not enough fair.

Under collaboration policy `PU2_Leader`, PU2 increases its economic gains by 0.4% on average. Collaboration policy `PU2_Leader` provides better gains for the follower when the by-product is unstorable than when the by-product is storable with a limited capacity.

As regards the incomes of the follower, we obtain gains below those obtained with collaboration policy `Opp_Co` despite the side payments (e.g., 2.2% on average for PU2 when she/he is the follower, according to Table 4, versus 2.4% under the collaboration policy `Opp_Co` in Table 3), when the TBO or the inventory holding costs are unknown.

When $q-b^2$ or $g-b^1$ are unknown, the gains of the follower are higher than those obtained under opportunistic collaboration policy `Opp_Co`, but below the gains provided by the sequential collaboration policy, where the follower makes its production plan in the second place (e.g., under collaboration policy `PU1_Leader`, the gain of PU2 is 3.3% on average versus 2.4% for `Opp_Co` and 5.1% for `PU1_First`).

Recall that side payments are allowed in collaboration policies `PU1_Leader` and `PU2_Leader`. However, in the majority of the cases, no side payment is given (see Tables 4 and 5). It is mainly due to the definition of the problem that requires the leader to propose to the follower contracts at least as good as collaboration policy `No_Co`. Reusing the by-product is sufficient to reduce the costs of the follower. Then, a side payment is not required to meet constraint (17) contrary to problems studied in the literature. As the leader wants to minimize his/her own costs, she/he will not give money to the follower if it is not a necessary condition to collaborate.

In brief, we can conclude from Tables 4 and 5 that:

- Collaboration policies `PU1_Leader` and `PU2_Leader` are environmentally interesting. Globally, they provide gains, summarized in Tables 4 and 5, clearly better than those obtained under sequential collaboration policies `PU1_First` and `PU2_First`.
- The presented game theoretic-based collaboration policies do not provide an equitable distribution of the costs, but allow both the leader and follower to obtain financial savings.

7.3. Focus on the choice of the final contract in collaboration policy `Nego`

Recall that, at the end of Algorithm 1, a set of contracts is obtained. The mediator has no access to the internal evaluation of contracts made by PUs, and can only operate with the scores they provided, as explained in Section 6.4. In this paragraph, we discuss the properties of contracts chosen based on an aggregation of scores against the contract obtained under `Full_Co`. To perform this comparison, we compute the scores associated with `Full_Co` for each PU $i \in \{1, 2\}$ via Formula (50) ($score^1_{Full\_Co}$, $score^2_{Full\_Co}$) and the score expressing the quantity of exchanged by-products via Formula (51) ($score^{exc}_{Full\_Co}$).

The final contract selected by the mediator depends on the importance given to each criterion. Denote by `All` the special case, where all criteria have the same importance and the same domain ($[0, 1]$) when choosing the final contract, i.e., $\mu_{Sat} = \mu_{Env} = \mu_{Eco} = 1/3$ in Equation (53). Let `u_v` be the final contract lexicographically chosen by using firstly criterion $\mathtt{u} \in \{\mathtt{Sat},\mathtt{Env},\mathtt{Eco}\}$ (i.e., $\mu_u = 1$ in Equation (53)) and then criterion $\mathtt{v} \in \{\mathtt{Sat},\mathtt{Env},\mathtt{Eco}\}$ with $\mathtt{v} \neq \mathtt{u}$. For each combination of criteria, `u_v` used to select the final contract and for each type of score $i \in \{1, 2, exc\}$, the gap to `Full_Co` is computed as follows:

$$Gap = 100 \times \frac{score^i_{Full\_Co} - score^i_{\mathtt{u\_v}}}{score^i_{Full\_Co}}$$

Figure 2 provides the average gaps to `Full_Co` for seven combinations of criteria. It shows that favoring the economic criterion instead of the satisfaction of PUs (`Eco_Sat`) or the environmental impact (`Eco_Env`) leads to high gaps, that are positive for all scores. Even when the economic criterion is applied in the second place to discriminate equivalent contracts on other criteria (`Sat_Eco`, `Env_Eco`), the economic criterion is not competitive: `Sat_Eco` provides gaps equivalent to `Sat_Env` on average. `Env_Eco` highly decreases the quality of the contract for PU2 (i.e., $score^2$ becomes high) for very small gains of PU1 compared to `Env_Sat`. These uncompetitive outputs provided by `Eco` can be explained by the fact that a low economic score assigned to a contract does not mean that this contract is bad, it is just less good than the other ones. Moreover, the true global cost can be different from the global cost estimated by the mediator. Indeed, the mediator proposes contracts only composed of exchange plans, while PUs are free to make their own production plans. The true production plan of each PU is not necessarily the same

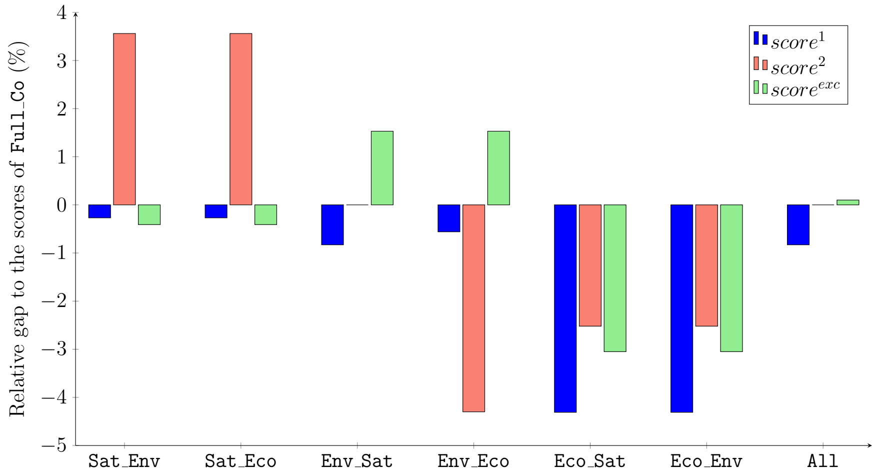

Figure 2: `Full_Co` versus `u_v`, $\mathtt{u}, \mathtt{v} \in \{\mathtt{Sat}, \mathtt{Env}, \mathtt{Eco}\}$: Average gaps

as the one computed by the mediator. Consequently, choosing the contract based on the best economic score is not always relevant, as illustrated in Figure 2.

Being driven by the economic criterion, the contract selected by formula (53) with $\mu_{Sat} = \mu_{Env} = \mu_{Eco}$ does not allow us to keep the environmental gain and the satisfaction of PU2 as interesting as when $\mu_{Eco} = 0$. In the following, combination `All` is changed by setting $\mu_{Eco} = 0$, and $\mu_{Sat} = \mu_{Env} = 1/2$, in Equation (53). This new setting for combination `All` is denoted `All(Sat,Env)`.

7.4. Discussions on collaboration policy managed by a mediator (IT platform)

Table 6 summarizes the average economic and environmental gains calculated for different final contracts. These gains are computed in the same way as for the baseline policies provided in Table 3. The obtained results are compared to collaboration policy `Full_Co`. As previously discussed, the main difference between these two collaboration policies relies on the level of information sharing.

Globally, gains obtained with `Sat_Env`, `Env_Sat` and `All(Sat,Env)` are very close to those obtained by `Full_Co` both economically and environmentally: +/-0.2% for the economic gains, and -0.9% in the worst case for the environmental benefits. It appears that the lack of information sharing is not prohibitive. This empirical fact can be explained by the context and market knowledge of the mediator about the private cost parameters and its poor estimation of the TBO (the most impactful parameter).

Table 6: `Nego` versus `No_Co`: *Economic and environmental gains (in %)*

Gains	B	`Sat_Env`			`Env_Sat`			`All`		
		min	avg	max	min	avg	max	min	avg	max
PU1	$= 0$	0.1	7.0	21.6	0.1	7.0	21.6	0.1	7.0	21.6
	> 0	0.1	7.8	22.0	0.1	7.7	22.0	0.2	7.8	22.0
PU2	$= 0$	0.3	5.9	23.0	0.0	5.7	23.0	0.1	5.8	23.0
	> 0	0.3	6.5	21.8	0.2	6.4	21.8	0.4	6.5	21.8
Env.	$= 0$	51.7	96.6	100.0	65.5	99.3	100.0	65.5	≈100	100.0
	> 0	67.5	98.6	100.0	86.5	≈100.0	100.0	86.4	≈100	100.0

avg: Average, Env.: Environmental

Note that, whatever the criterion used for the choice of the final contract is, the environmental gain is high, 96.6% of the total reusable quantity of by-products are reused on average in the worst case. Sat_Env is economically attractive especially when the by-product is unstorable because it costs nothing to both production units. The economic gain of Sat_Env is equal to the one obtained under Full_Co (7.0% for PU1 and 5.9% for PU2), but the distribution of gains is better as the lowest gain is positive (0.1% for PU1 and 0.3% for PU2 instead of -0.9% and -0.7% respectively for PU1 and PU2 under Full_Co). When the by-product is storable with a limited capacity, Sat_Env still provides positive economic benefits (7.8% for PU1 and 6.5% on average for PU2 against 7.9% for PU1 and 6.4% on average for PU2 under Full_Co), due to the fact that the mediator cannot propose a contract economically disadvantageous for at least one PU. These economic gains imply that the environmental gains are penalized. On the contrary, Env_Sat increases the percentage of reused by-products by at least 1.2% compared to Full_Co, by implying an economic loss of only 0.2% for PU1 and PU2 in the worst case. Combination Env_Sat can be very interesting when the by-product is forbidden in landfills.

Generally speaking, combination All(Sat,Env) represents a sound trade-off between economic and environmental gains, which leads to a percentage of the reused quantity of by-products close to 100% on average. The economic gains involved by All(Sat,Env) are similar to Sat_Env and slightly below Full_Co (0.2% below).

To summarize, we can conclude from Table 6 that collaboration policy Nego: **(i)** outperforms other policies, environmentally speaking, **(ii)** provides results very close to the centralized collaboration policy while keeping information private, and **(iii)** allows to choose between several contracts to favor the economic gain or the environmental impact.

7.5. Value of collaboration policies within an industrial symbiosis network

Let us take focus on a specific instance, characterized by unbalanced demands and TBOs, which make the setup costs of PU2 much higher than the setup costs of PU1. The contracts obtained under different policies are represented in a 3D score-based system of coordinates in Figures 3 and 4.

Collaboration policy Nego. Figure 3 represents the contracts obtained for a specific instance with the negotiation procedure (denoted by Nego) with respect to other studied collaboration policies on three score-based coordinates: score of PU1, score of PU2, and global environmental score. The contracts are mostly located in the center of the figure. We notice that the deviation between the scores of production units is relatively low.

The contract obtained under Full_Co is found by Nego, but it is never selected by the proposed criteria allowing to choose the final contract, discussed in Section 6.4. The chosen contract by Nego is indicated by an arrow in Figure 3. This contract is the best one from a twofold point of view of the environment and satisfaction of PUs. Contract Nego dominates collaboration policies PU1_First and Opp_Co. Contract Nego is also more attractive than Full_Co, as it is situated in the quarter $[0.5, 1] \times [0, 0.5]$ corresponding to high-rated contracts by PU1 and PU2. However, Nego cannot provide a better contract than Full_Co, because Full_Co provides the minimal global cost by definition. High gains for PU1 do not thus compensate for a low loss of gains for PU2. As illustrated via this particular instance, the scale of a 3D coordinate system based on scores can be misleading on the global quality of the solution, according to the magnitude of the associated costs (an increase of 1 unit in a score does not have the same importance on the whole network if it leads to an increase of 1,000 or 10,000 in terms of cost). Being blind, the mediator cannot know the true costs.

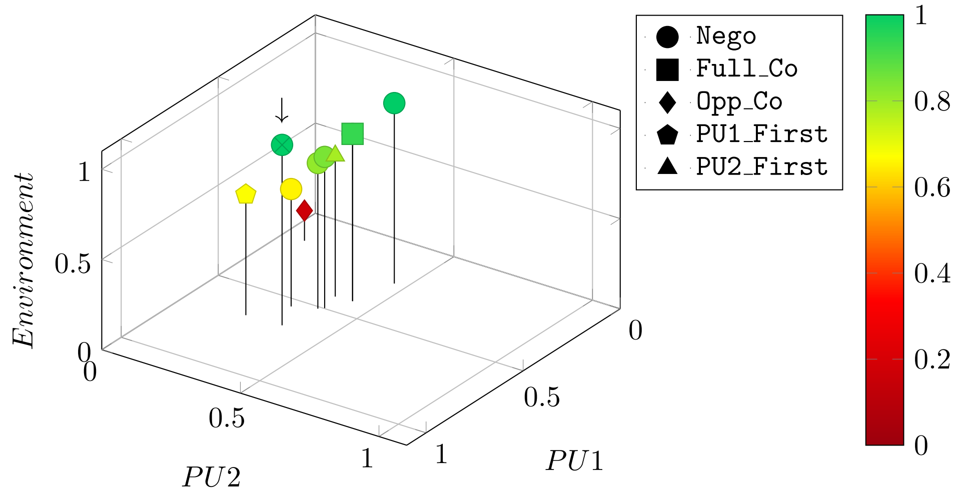

Figure 3: Mapping of contracts for a given instance with $TBO^1 = 3$, $TBO^2 = 5$, $\bar{d}^1 = 50$, $\bar{d}^2 = 100$, $\delta = 1$ and $B = 0$ in a three-dimensional system of coordinates (x *-axis*: score of PU1, y *-axis*: score of PU2, and z *-axis* and color bar: environmental global score)

Collaboration policies `PU1_Leader` *and* `PU2_Leader`. Let us now position contracts obtained in the case of one-sided asymmetric information sharing using game-theoretic collaboration policies `PU1_Leader` and `PU2_Leader`. Figure 4 represents the obtained contracts for a specific instance under policies `PU1_Leader` and `PU2_Leader` for different scenarios (i.e., realizations) of the unknown parameter against other baseline collaboration policies on three score-based coordinates: score of PU1, score of PU2, and global environmental score.

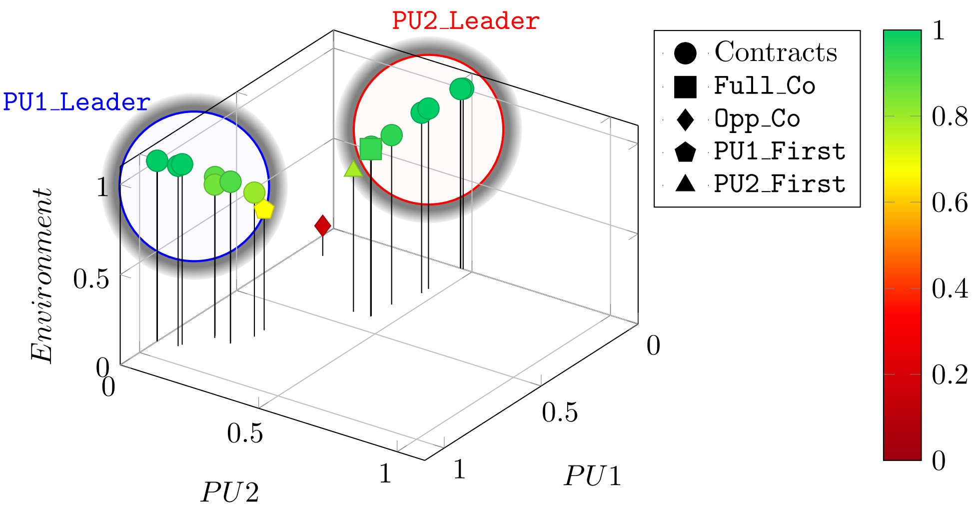

Figure 4: Contracts positioning for a given instance with $TBO^1 = 3$, $TBO^2 = 5$, $\bar{d}^1 = 50$, $\bar{d}^2 = 100$, $\Delta = 1$ and $B = 0$ in a three-dimensional system of coordinates (x *-axis*: score of PU1, y *-axis*: score of PU2, and z *-axis* and color bar: environmental global score)

The contracts corresponding to each unknown parameter form a Pareto front with contracts of policy `PU1_Leader` on one side, and contracts of policy `PU2_Leader` on the other side. Note that there is no contract in the middle due to a bad distribution of costs despite the side payments. Figure 4 highlights that the lower the score of the follower is, the more the environmental score is good. Generally speaking, the collaboration policies based on the game theory are environmentally better than the sequential collaboration policies.

Regarding the economic gains, the score of the follower is between 0 and the score obtained under the sequential collaboration policy when she/he starts first. On the contrary, the score of the leader is higher than the score obtained with the sequential collaboration policy. It can be explained by the fact that, whatever the unknown parameter, the leader allows himself/herself to move production periods, and increases the storage of the main product to reduce in return the quantities of disposed of by-products and purchased raw materials. The gains related to the reuse of the by-product can allow the follower to compensate for losses due to the storage of the main product without creating particular benefits.

8. Conclusion and perspectives

This paper investigates collaboration policies for partial information sharing within an industrial symbiosis network composed of two actors: a supplier and a receiver. For one-sided asymmetric information sharing, two collaboration policies based on game theory are studied, and for bilateral asymmetric information sharing, a negotiation scheme managed by a blind impartial mediator is proposed. The economic and environmental impacts associated with these collaboration policies are discussed with respect to several baseline industrial symbiosis-based collaboration policies introduced by Suzanne et al. (2021).

We show that the negotiation scheme is a good alternative to the centralized collaboration policy. The conducted numerical experiments prove that the proposed negotiation scheme provides very close results compared to the centralized collaboration policy while keeping information private. Regarding the game theory-based collaboration policies, their gains are clearly better than those provided under sequential collaboration policies.

To close the existing industrial symbiosis applications described by Evans et al. (2017), this work could be extended in several directions:

- **Sustainable impact:** In our approaches, the collaboration policies are based on economic benefits, while the environmental aspect is only considered as an additional Key Performance Indicator (KPI) to assess the quality of proposed contracts. Future works could be dedicated to integrating the environmental and social aspects in all the collaboration policies as part of the negotiation process (i.e., as an optimization criterion). Moreover, the actors in the network may have different hierarchies of KPIs, which could be explicitly considered. Another perspective of this paper is to integrate incentives intra- (generated by actors within the industrial symbiosis network) and extra- (environment-regulatory instances) networks.

- **Complex unitary production systems:** One perspective consists in extending the problem, by considering several co-products produced at the same time as the main product and/or several by-products with different characteristics. Several by-products, like steam or waste food, can be stored before their reuse for a limited time. Such a constraint could be considered. In the same way, different by-products can be used to produce the same product with different quality levels and after different treatment processes. Moreover, multiple transportation modes can be used depending on the quantity of by-products and the distance between production units.

- **Industrial symbiosis network:** The current paper studies an industrial symbiosis network composed of two actors, but real-life industrial symbiosis often involves more than two actors. In terms of the morphology of relationships between actors, several extensions of the industrial symbiosis network studied in this paper could be considered: with a third party, one-to-many relationships, multiple suppliers-multiple receivers, and cycle of industrial symbiosis.

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Appendix A. Numerical experiments

In this section, we: **(i)** discuss the definition of the domain of estimation used for game theory-based collaboration policies and, **(ii)** validate our approach for the negotiation procedure. These numerical experiments are conducted in the framework given in Section 7.1 and the obtained results are analyzed on average and discussed in Sections Appendix A.1-Appendix A.2.

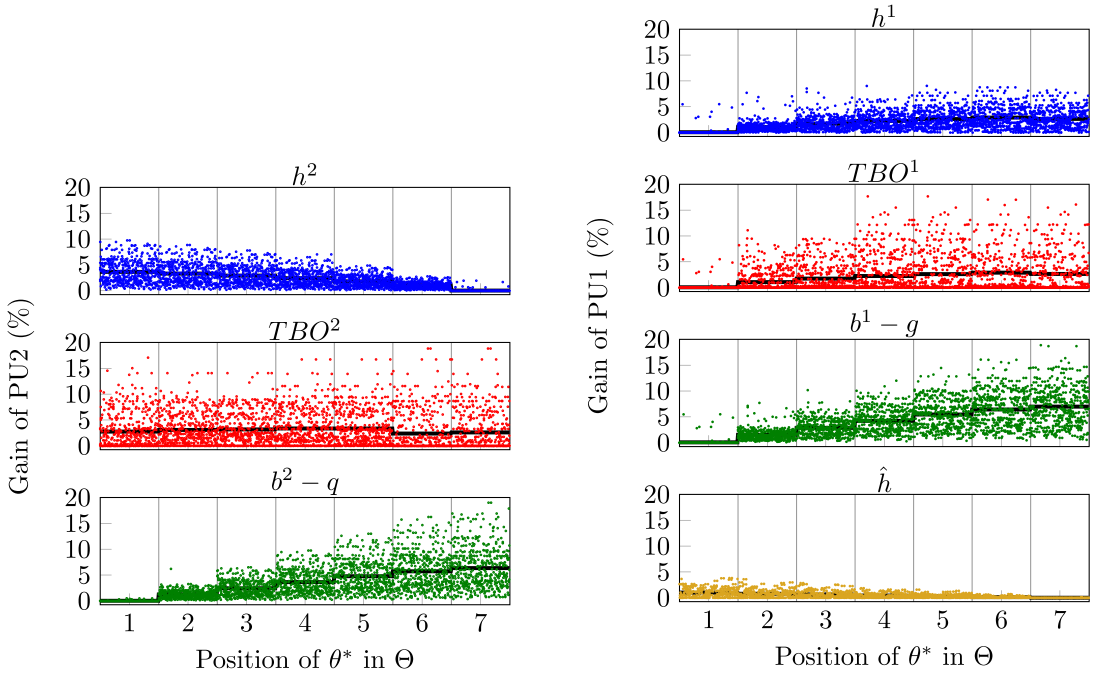
Appendix A.1. Game theory-based collaboration policies

In this section, we first discuss the quality of the estimated data within the game theory-based collaboration policies. Then, we analyze the gains of each PU applying these collaboration policies in view of baseline policies. Recall that in the conducted experiments, we consider only one unknown parameter at a time, namely: TBO^2, TBO^1, h^1, h^2, $q-b^2$, $g-b^1$, or $\hat{h}$.

Let us now place the emphasis on the impact of key characteristics of the domain of estimation of the unknown parameter $\{\underline{\theta}, \dots, \overline{\theta}\}$ on the problem output. It is computed based on its true value θ^* and three following features:

- ***Range:*** $2\Delta_\theta\theta^*$ is the range of the definition domain of unknown parameters, i.e., $\underline{\theta} = \theta^* - \Delta_\theta\theta^*$, $\overline{\theta} = \theta^* + \Delta_\theta\theta^*$. We fixed Δ_θ in interval $\{0.2, 0.5, 0.8\}$.

- ***Cardinality of the interval:*** $|\Theta|$ corresponds to the number of scenarios (i.e., realizations). Let $|\Theta|$ take its values in $\{3, 7\}$.



(a) Relative gains of PU2 under `PU1_Leader` for unknown parameters h^2, TBO^2, $q - b^2$ against `No_Co`

(b) Relative gains of PU1 under `PU2_Leader` for unknown parameters h^1, TBO^1, $g - b^1$, $\hat{h}$ against `No_Co`

Figure A.5: Policy `PU`i `_Leader` ($i \in \{1, 2\}$) versus policy `No_Co` ($|\Theta| = 7$): *Follower gains* (in %)

- ***Position of*** θ^* ***in set*** $\{\underline{\theta}, \dots, \overline{\theta}\}$***:*** p_{θ^*} is the position of θ^* in set $\{\underline{\theta}, \dots, \overline{\theta}\}$. Let p_{θ^*} take its values in $\{1, 2, \dots, |\{\underline{\theta}, \dots, \overline{\theta}\}|\}$. Range $\{\underline{\theta}, \dots, \overline{\theta}\}$ is shift to the right or the left depending on p_{θ^*}. For example, when $|\Theta| = 3$ and $p_{\theta^*} = 2$, the range of the unknown parameter is $\{\underline{\theta}, \theta^*, \overline{\theta}\}$.

  Note that, the cost functions estimating the unknown parameters are strictly increasing, thus the position of θ^* in set $\{\underline{\theta}, \dots, \overline{\theta}\}$ corresponds to the scenario number and $|\Theta| = |\{\underline{\theta}, \dots, \overline{\theta}\}|$.

Position of true value θ^* *within the domain of estimation* $\{\underline{\theta}, \dots, \overline{\theta}\}$*:* . Extensive numerical experiments have shown that the economic benefit of the leader, the environmental impact, and the side payment are not impacted by the position of the true value of the unknown parameter within $\{\underline{\theta}, \dots, \overline{\theta}\}$. Figure A.5 highlights the economic benefit of the follower in collaboration policies `PU1_Leader` and `PU2_Leader` against `No_Co` as a function of the position of the true value within $\{\underline{\theta}, \dots, \overline{\theta}\}$ for all possible unknown parameters. Due to the high computational time needed to conduct experiments, results are provided for a subset of 733 randomly selected instances and $|\{\underline{\theta}, \dots, \overline{\theta}\}|$ is fixed to 7.

Figure A.5 shows that both collaboration policies for one-sided asymmetric information sharing do not react in the same way to the position of θ^* within set $\{\underline{\theta}, \dots, \overline{\theta}\}$. Hence, let us discuss the results under the lens of the estimated parameter:

- **Inventory holding cost of the main product** (h^2, h^1)**:** The more the leader underestimates h^2, the more the gain of the follower decreases. On the contrary, when the leader overestimates the gains, she/he leads to an increase of the gain of the follower, up to 20% when $\theta^* = \underline{\theta}$. The behavior of the gain of the follower is symmetric when the unknown is h^1 and when `PU2_Leader`.

- **Time between orders (TBO^1,TBO^2):** When `PU1_Leader`, the position of θ^* in set Θ has no impact on the gain of the follower. It is distributed between 0% and 20%. Under `PU2_Leader`, the distribution of the gain of the follower is almost the same except when closing $\underline{\theta}$. When $\theta^* = \underline{\theta}$, the average gain of PU1 is slightly higher than 0.

- **Gain of reusing the by-product ($q - b^2$, $g - b^1$):** The more the leader underestimates $q - b^2$ and $g - b^1$, the more the gain increases, up to almost 20%. Note that, in case of underestimation, the gain of reusing the by-product is almost always strictly greater than 0. On the contrary, when the leader overestimates the gains, she/he leads to a decrease in the gain of the follower, which is always between 0% and 1% when $\theta^* = \bar{\theta}$.

- **Inventory holding cost of the by-product when `PU2_Leader` ($\hat{h}$):** When PU2 underestimates $\hat{h}$ (especially when $\theta^* = \bar{\theta}$), the average gain of the follower is negligible and this last one is legitimatized to questions the well-founded of the collaboration for him/her. On the contrary, when PU2 overestimates the by-product inventory holding cost, the economic benefit increases for PU1 but it never exceeds 5%.

In view of these findings and to avoid any bias, let us position θ^* in the middle of the range of estimation in what follows.

Range and cardinality of estimation domain $\{\underline{\theta}, \dots, \overline{\theta}\}$: . Table A.7 summarizes the gaps between `PU1_Leader` (resp. `PU2_Leader`) and collaboration policy `No_Co` for all unknown parameters $\{TBO^2,\ TBO^1,\ h^1,\ h^2,\ q - b^2,\ g - b^1,\ \hat{h}\}$, and for different ranges and cardinalities of estimation domains. This gap is calculated as follows: $100 \times (c^1 + c^2 - c^1_p - c^2_p)/(c^1 + c^2)$, where $p \in \{\texttt{PU1\_Leader}, \texttt{PU2\_Leader}\}$ and c^i is the nominal cost of PU $i \in \{1, 2\}$.

Table A.7 shows a number of empirical pieces of evidence of the impact of the range and cardinality of estimation domains on game theory-based collaboration policies. When the unknown parameter is the gain of reusing the by-product (i.e., $g - b^1$ or $q - b^2$), the gap to `No_Co` is the highest. We can also notice that the number of values in the estimation domain has a low impact on the global economic gain of the industrial symbiosis network. On the contrary, a large range leads to a significant difference in terms of gain. The more the range is tight, the lower the economic gain is.

Table A.7: Gaps between game theory based collaboration policies and `No_Co` (in %) for unknown parameters $\{TBO^2,\ TBO^1,\ h^1,\ h^2,\ q - b^2,\ g - b^1,\ \hat{h}\}$ (θ^* in the middle of the estimation range)

	θ	h^2			TBO^2			$q - b^2$					
`PU1_Leader`	Δ_θ / $\lvert\Theta\rvert$	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8			
	3	4.2	4.6	4.9	4.2	4.6	4.4	4.3	5.1	5.8			
	7	4.2	4.6	4.9	4.1	5.0	5.1	4.3	5.1	5.8			
	θ	h^1			TBO^1			$g - b^1$			$\hat{h}$		
`PU2_Leader`	Δ_θ / $\lvert\Theta\rvert$	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8
	3	3.5	3.9	4.2	3.4	3.8	3.5	3.8	4.8	5.7	3.2	3.3	3.4
	7	3.5	4.0	4.2	3.5	4.2	4.2	3.8	4.8	5.7	3.2	3.3	3.4

To summarize, even if the position of θ^* within $\{\underline{\theta}, \dots, \overline{\theta}\}$ does not have an impact on the cost of the leader, gains of the follower are more or less high when θ^* is an endpoint of the estimation domain depending on the unknown parameter. The leader has to be careful when defining the domain of estimation because additional values can have a negative global effect.

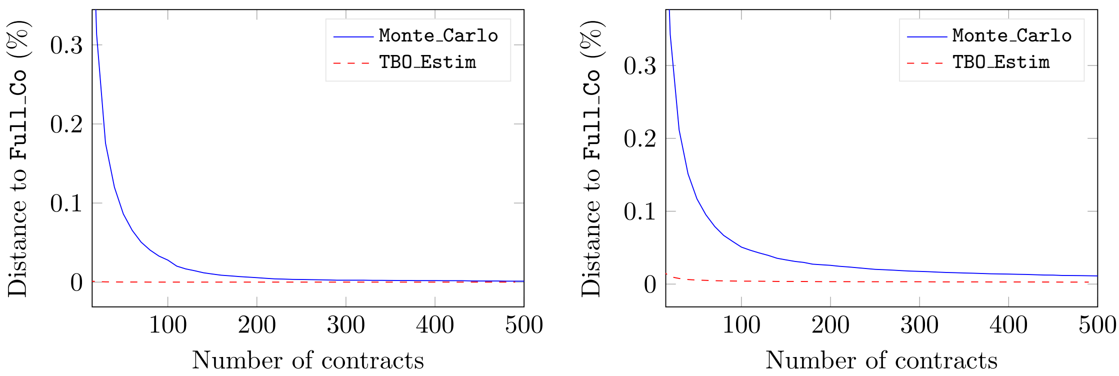
Appendix A.2. Negotiation procedure managed by a mediator (Algorithm 1)

In this section, we perform several experiments to analyze the contracts negotiated by a mediator in the case of bilateral asymmetric information sharing. The goal of this section is to evaluate the impact of the critical parameters and to show the industrial soundness of the proposed approach. To do this, we carry out the comparison between the following approaches:

- `TBO_Estim`: Negotiation procedure (Algorithm 1).
- `Monte_Carlo`: Negotiation procedure (Algorithm 1) without Phase 1, i.e., based only on the Monte Carlo sampling procedure. It is a crude approach, which consists in randomly generating a large number of contracts to converge towards efficient and attractive contracts by virtue of the law of large numbers.
- `Full_Co`: Centralized collaboration policy (i.e., Model (2)-(14)) of the ULS-IS problem.

By definition, `Full_Co` provides the best contract that the mediator can propose in terms of economic benefits. Given that, to discuss the competitiveness of `TBO_Estim` against `Monte_Carlo`, we compute for each contract its gap to `Full_Co` using the following formula: $(c_p^1 + c_p^2 - c) \times 100/c$, where c_p^1 (resp. c_p^2) is the cost of PU1 (resp. PU2) for approach $p \in \{\texttt{TBO\_Estim}, \texttt{Monte\_Carlo}\}$, and c is the global cost associated with `Full_Co`. For each problem instance, only the minimal gap to `Full_Co` is kept.

Note that the computational time needed to solve both versions of the negotiation scheme depends linearly on the number of proposed contracts. For instance, the computational time of `TBO_Estim` and `Monte_Carlo` for 50 contracts is about 6 seconds on average.



(a) $B/\bar{d}^1 = 0$ (b) $B/\bar{d}^1 > 0$

Figure A.6: Average gap to `Full_Co` (in %)

TBO_Estim versus Monte_Carlo. Figure A.6 provides the average gap between Full_Co and both versions of the negotiation scheme (TBO_Estim and Monte_Carlo) for different numbers of proposed contracts for two cases: when the by-product is unstorable ($B = 0$), and when it is storable with a limited capacity ($B/\bar{d}^1 > 0$).

When the by-product is unstorable, the gap of TBO_Estim to Full_Co converges quickly to very small values (below 0.001%). When the by-product is storable with a limited capacity, the gap of TBO_Estim to Full_Co starts from 0.019%, and converges to values below 0.003% after 200 contracts. The gap of TBO_Estim at the first iteration is better than the results obtained with Monte_Carlo after 500 contracts, which reaches 0.011%. To summarize, TBO_Estim allows us to reduce the number of proposed contracts, and consequently the computational time. TBO_Estim provides better results than Monte_Carlo, whatever the storability of the by-product is. Due to the stabilization of the results for TBO_Estim after 200 contracts, the number of contracts is fixed at 200 in the rest of the experiments.

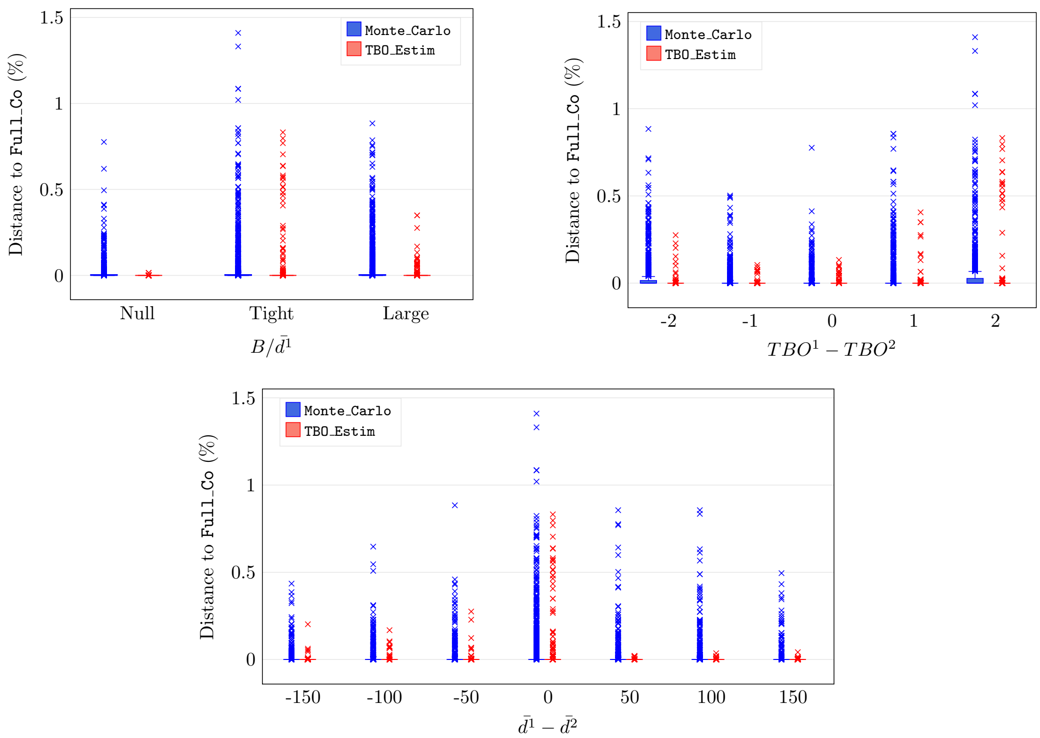


Figure A.7: Average gap to Full_Co (in %) depending on $B/\bar{d}^1$, $TBO^1 - TBO^2$ and $\bar{d}^1 - \bar{d}^2$ for 200 contracts

Focus on the critical parameters. As illustrated in Figure A.6, the negotiation procedure described in Algorithm 1 is very competitive (the average gap to Full_Co is very close to 0%). Figure A.7 provides the boxplots representing the distribution of the minimal gap of TBO_Estim and Monte_Carlo to Full_Co of each instance, depending on the critical parameters of instances $\{B/\bar{d}^1, TBO^1 - TBO^2, \bar{d}^1 - \bar{d}^2\}$. Note that the results are not discussed with respect to Δ since its impact on the results is very low, as shown in (Suzanne et al., 2021).

Let us mention, first, that all the gaps to Full_Co are below 0.9% for TBO_Estim and 1.5% for Monte_Carlo. Figure A.7 shows that TBO_Estim is persistently better than Monte_Carlo, and the gap of at least 75% of instances is at 0% for both TBO_Estim and

Monte_Carlo. In the following, we focus on the distribution of gaps depending on the critical parameters:

- **By-product inventory capacity:** The more the by-product inventory capacity is close to the average demand of PU1 ($\bar{d^1}$), the higher the gap of TBO_Estim to Full_Co is. For the particular case, where the by-product is unstorable, the gap is below 0.05% for all instances. Note that in this configuration, the difference between the gap of Monte_Carlo and the gap of TBO_Estim is the highest.
- **Setup cost-holding cost ratios TBO^1 and TBO^2:** The more TBO^1 and TBO^2 are different, the higher the gaps to Full_Co can be. We can also notice when $TBO^1 > TBO^2$ the gaps are more significant than when $TBO^1 < TBO^2$.
- **Average demands $\bar{d^1}$ and $\bar{d^2}$:** When $\bar{d^1} > \bar{d^2}$, the gap of TBO_Estim to Full_Co is very close to 0% for all instances. When $\bar{d^1} < \bar{d^2}$, gaps are below 0.3%. Finally, for instances having $\bar{d^1} = \bar{d^2}$, TBO_Estim provides the worst results even if the gaps of TBO_Estim are better than those provided by Monte_Carlo.