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## 1 Introduction

This paper investigates the integration of the employee timetabling and production scheduling problems. At the first level, we have to manage a classical employee timetabling problem. At the second level, we aim at supplying a feasible production schedule for a job-shop scheduling problem (NP-hard problem). Instead of using a hierarchical approach as in the current practice, we here integrate the two decision stages and propose two exact methods for solving the resulting problem. The former is similar to the cut generation algorithm proposed in (Guyon O. *et. al.* 2010) for a problem integrating a classical employee timetabling problem and a polynomially solvable production scheduling problem. The latter is based on a Branch-And-Cut process that exploits the same feasibility cuts than the first approach. Preliminar experimental results on instances proposed in (Artigues C. *et. al.* 2009) reveal a real interest for the approaches described here.

## 2 A model for an integrated employee timetabling and job-shop scheduling problem

### 2.1 Problem description

We consider the production of a set $J$ of $n$ jobs on a set $M$ of $m$ machines. Each job $i$ is made of a chain of operations $\{O_{ij}\}_{j=1,...,m}$. Operation $O_{ij}$ is defined by its assigned machine $m_{ij} \in M$ and its duration $p_{ij} \in \mathbb{N}^*$. The duration of the operation of job $i$ on machine $k$ is denoted $\rho_{ik}$. Operations are not interruptible and require during their processing one employee $e$ qualified to use machine $k$. $E$ denotes the set of the $\mu$ employees and $A_e$ the set of machines employee $e$ is able to control.

Employees work under a three-shift system. The timetabling horizon $H = \sigma \times \pi$ is thus defined over a set $S$ of $\sigma$ consecutive shifts $s$ which have the same duration time $\pi$.

Employee $e$ is assumed to be available for a subset of shifts $T_e$. Each employee is furthermore assumed to work at most one shift during each gliding window of three shifts (regulation constraints). The cost of assigning employee $e$ to machine $k$ during shift $s$ is denoted $c_{eks}$.

Solving the problem lies in assigning at minimum cost employees to both machines and shifts in order to be able to provide a feasible production plan, i.e. a schedule for which all operations are completed before a given scheduling completion time $C_{max} \leq H$.

## 2.2 MIP formulation

$x_{eks}$ and $y_{ikt}$ are binary decision variables respectively equal to 1 if employee $e$ is assigned to machine $k$ during shift $s$ and if job $i$ starts to be processed on machine $k$ at time instant $t$.

Using variables and notations mentioned above, a MIP formulation $[P]$ of the problem can hence be proposed as follows:

**Objective function:**

$$[P] : \min \Theta = \sum_{e \in E} \sum_{k \in A_e} \sum_{s \in T_e} c_{eks} \cdot x_{eks} \tag{1}$$

**Employee timetabling specific constraints:**

$$\sum_{k \notin A_e} \sum_{s=0}^{\sigma} x_{eks} = 0 \qquad e = 1, \ldots, \mu \tag{2}$$

$$\sum_{k \in A_e} \sum_{s \notin T_e} x_{eks} = 0 \qquad e = 1, \ldots, \mu \tag{3}$$

$$\sum_{k \in A_e} (x_{eks} + x_{ek(s+1)} + x_{ek(s+2)}) \leq 1 \; e = 1, \ldots, \mu \quad s = 0, \ldots, \sigma - 2 \tag{4}$$

$$x_{eks} \in \{0,1\} \qquad e = 1, \ldots, \mu \quad k = 1, \ldots, m \quad s = 0, \ldots, \sigma \tag{5}$$

For the employee timetabling part, constraints (2) and (3) respectively fix assignment variables $x$ to 0 because of the lack of a skill for an employee and his unavailability during some shifts. Following constraints (4) are regulation constraints that state that each employee can work at most one shift during each gliding window of three shifts.

**Job-shop specific constraints:**

$$\sum_{t=0}^{d_{ik} - \rho_{ik}} t \cdot y_{ikt} + \rho_{ik} \leq C_{max} \quad i = 1, \ldots, n \quad k = m_{im} \tag{6}$$

$$\sum_{t=r_{ik}}^{d_{ik} - \rho_{ik}} y_{ikt} = 1 \qquad i = 1, \ldots, n \quad k = 1, \ldots, m \tag{7}$$

$$\sum_{t=0}^{r_{ik}} y_{ikt} + \sum_{t=d_{ik} - \rho_{ik} + 1}^{C_{max}} y_{ikt} = 0 \; i = 1, \ldots, n \quad k = 1, \ldots, m \tag{8}$$

$$\sum_{u=r_{ik} + \rho_{ik}}^{t} y_{ilu} - \sum_{u=r_{ik}}^{t - \rho_{ik}} y_{iku} \leq 0 \; i = 1, \ldots, n \quad j = 1, \ldots, m - 1$$

$$k = m_{ij} \quad l = m_{i(j+1)}$$

$$t = \rho_{ik} + p_{ik}, \ldots, d_{il} - \rho_{il} \tag{9}$$

$$\sum_{i=1}^{n} \sum_{u=\max(r_{ik}, t-\rho_{ik}+1)}^{\min(d_{ik}-\rho_{ik}, t)} y_{iku} \leq 1 \; k = 1, \ldots, m \quad t = 0, \ldots, C_{max} \tag{10}$$

$$y_{ikt} \in \{0,1\} \qquad i = 1, \ldots, n \quad k = 1, \ldots, m \quad t = 0, \ldots, C_{max} \tag{11}$$

where $r_{ik}$ and $d_{ik}$ are the respective earliest starting time and the due date of operation of job $i$ on machine $k$. They are computed in a pre-processing stage with the recursive equations (12) and (13).

$$\begin{cases} r_{ik} = 0 & i = 1, \ldots, n & k = m_{i1} \\ r_{il} = r_{ik} + \rho_{ik} & i = 1, \ldots, n & j = 1, \ldots, m-1 & k = m_{ij} & l = m_{i(j+1)} \end{cases} \quad (12)$$

$$\begin{cases} d_{ik} = C_{max} & i = 1, \ldots, n & k = m_{im} \\ d_{ik} = d_{il} - \rho_{il} & i = 1, \ldots, n & j = 1, \ldots, m-1 & k = m_{ij} & l = m_{i(j+1)} \end{cases} \quad (13)$$

For the job-shop part, constraints (6) ensure all jobs to be completed before the scheduling completion time $C_{max}$. Each operation has to be processed within its time window (7)-(8) and cannot start before the completion of its job predecessor (9). At most one operation can be processed on a given machine at each time instant (10).

**Coupling constraints:**

$$\sum_{i=1}^{n} \sum_{u=\max(r_{ik}, t-\rho_{ik}+1)}^{\min(d_{ik}-\rho_{ik}, t)} y_{iku} \leq \sum_{e \in E} x_{eks} \quad k = 1, \ldots, m \quad t = 0, \ldots, C_{max} \quad s = \lfloor t/\pi \rceil \quad (14)$$

Coupling constraints (14) compel that an employee is working on any machine and for any instant an operation is being executed.

## 3   Cut generation process

Due to its intrinsic two-decision-stage structure, it seems quite natural to investigate a decomposition method for solving problem $[P]$. We propose here to adapt the cut generation algorithm proposed in (Guyon O. *et. al.* 2010) in order to solve $[P]$. In their paper, the authors propose a specific cut decomposition and cut generation process to solve a problem integrating a classical employee timetabling problem and a polynomially solvable production scheduling problem.

We thus also exploit here the splitting of the overall problem into two sub-problems. A master problem $[ETP]$ first finds a solution for the employee timetabling part of the problem. Using this solution as an entry, a satellite sub-problem $[JobShop]$ then checks if a feasible job-shop schedule verifying the human resources defined by the current solution exists. If no schedule can be found, a valid cut is generated in order to invalide the current solution for $[ETP]$. The process then iterates until the minimum cost solution for the master problem $[ETP]$ leads to get a feasible production schedule for $[JobShop]$.

Master problem $[ETP]$ can be formalized as follows:

$$[ETP] : \min \Theta = \sum_{e \in E} \sum_{k \in A_e} \sum_{s \in T_e} c_{eks} \cdot x_{eks}$$
$$\text{s.t.} \qquad (2)\text{-}(5)$$
$$Cut$$

where $Cut$ is the set of feasibility cuts iteratively added to the model. They invalidate solutions that are not feasible according to the whole set of constraints of $[P]$.

Let us assume a fixed vector $\bar{x}$ as an optimal solution for $[ETP]$. We have to check whether $\bar{x}$ is feasible with regards to the other constraints of $[P]$. We thus introduce the satellite sub-problem $[JobShop_{\bar{x}}]$:

$[JobShop_{\bar{x}}]$ *Does a feasible schedule exist satisfying:*

$$(6)\text{-}(11)$$

$$\sum_{i=1}^{n} \sum_{u=\max(r_{ik}, t-\rho_{ik}+1)}^{\min(d_{ik}-\rho_{ik}, t)} y_{iku} \leq \sum_{e \in E} \bar{x}_{eks} \quad k = 1, \ldots, m \quad t = 0, \ldots, C_{max} \quad s = \lfloor t/\pi \rceil$$

$[JobShop_{\bar{x}}]$ can be rewritten as a classical job-shop scheduling problem and solved by the branch-and-bound algorithm described in (Carlier J. *et. al.* 2004). If $[JobShop_{\bar{x}}]$ is feasible, $\bar{x}$ is a feasible and optimal solution for $[P]$. Otherwise, the valid cut (15) is added to the set $Cut$ of $[ETP]$

$$\sum_{e=1}^{\mu} \sum_{k \in A_e} \sum_{s \in T_e} \alpha_{ks} \cdot x_{eks} \geq 1 \qquad (15)$$

where $\alpha_{ks} = 1$ if $\sum_{e=1}^{\mu} \bar{x}_{eks} = 0$, 0 otherwise.

Process iterates, in a finite number of steps, until the optimal solution $\bar{x}$ for $[ETP]$ is proved to be a feasible solution for $[P]$ by $[JobShop_{\bar{x}}]$.

## 4  Branch-And-Cut algorithm

An alternative exact approach for solving problem $[P]$ has been experimented. As the cut generation process defined above, it exploits the decomposition of $[P]$ into the two sub-problems $[ETP]$ and $[JobShop]$. The way of finding feasible solutions for the employee timetabling part of the problem however differs here. Instead of defining a complete assignment of employees to machine and shift, we indeed propose here to use a branch-and-bound procedure. Each node of the search tree thus defines a partial assignment of work on machine and shift. $[ETP]$ and $[JobShop]$ are then solved in order to check if such a partial assignment is feasible for both sub-problems. If at least one of the two sub-problems fails, the feasibility cut (15) is added to the set $Cut$ of $[ETP]$ and the current node is pruned. Process iterates until each node of the search tree has been explored or pruned.

## 5  Experimental results

The two exact approaches described here have been experimented on the instances proposed in (Artigues C. *et. al.* 2009) and on generated instances. We compared the results of both methods with an ILP solver (ILOG Cplex) applied to the direct model $[P]$. Preliminary computational experiments reveal a real interest for the two methods proposed here. The ILP solver indeed fails to find any solution for most of the instances whereas the two exact approaches both have an interesting rate of success. We can specially mention the real effectiveness of the Branch-And-Cut process which benefits from the advantages of the direct cut generation process (i.e. the decomposition of the problem and the use of generated feasibility cuts) without undergoing its disadvantages (i.e. a long computational time to solve up to optimality $[ETP]$ at each iteration).

## References

Artigues C., Gendreau M., Rousseau L.-M. and Vergnaud A., 2009, "Solving an integrated employee timetabling and job-shop scheduling problem via hybrid branch-and-bound", *Computers and Operations Research*, Vol. 36, Issue 8 (August 2009), pp. 2330-2340

Carlier J., Péridy L., Pinson É. and Rivreau D., 2004, "Handbook of scheduling: algorithms, models and performances analysis", Chapter: Elimination Rules for the job-shop, *Chapman & Hall / CRC*, pp. 1-19

Guyon O., Lemaire P., Pinson É and Rivreau D., 2010, "Cut generation for an integrated employee timetabling and production scheduling problem", *European Journal of Operational Research*, Vol. 201, Issue 2 (1 March 2010), pp. 557-567