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Production planning for industrial symbiosis

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Chapter 1

General introduction

Within the umbrella term *circular economy*, the increasing lines of actions around waste and resource managements aim at fostering circular alternatives to linear practices (produce, consume and dispose) in actual production supply chains. Throughout the world, a growing number of waste prevention and management policies were implemented over the last two decades, to promote and support the environmentally-friendly operations related to the reuse of end-of-life products and recovery of waste materials.

Industrial symbiosis is one of the sustainable ways that aims to convert production residues into useful and added-value products. It is a collaborative form between companies based on the exchange of physical flows, such as production residues or other secondary resources (e.g. water, and energy), and/or the sharing of services like knowledge, logistics, expertise. The particular configuration of an industrial symbiosis system, where the production residues generated by a production unit are used as raw materials by another production unit, is called *by-product synergy*. As described in Chapters 2-3, a by-product is a lawful production residue, obtained unavoidably as an integral part of a production process, ready for a certain use without further transformation. The by-product exchange can take place either within a single parent entity, or between two or several different autonomous companies. The resulting network includes at least two actors: **(i)** a *supplier*, which generates by-products, and **(ii)** a *receiver*, which uses them. In the absence of a single parent entity, the intervention of a third party can be required to ensure the coordination between the supplier and the receiver of a by-product, by means of collaboration policies.

In this context, this thesis addresses lot-sizing problems, including the management of production residues, with an ultimate goal of investigating an industrial symbiosis and its contribution to the sustainable development. More precisely, we study two extensions of the classical lot-sizing problem dealing explicitly with by-products generated during the production of a main product. The first extension is a single-item lot-sizing problem with a by-product and inventory capacities. The second one is a lot-sizing problem modeling an industrial symbiosis network composed of two actors: one supplier and one receiver of by-products. The complexity of the studied problems lies on by-product inventory capacities and the inter-relatedness between the flows of by-products and main products of production units.

The thesis starts with an overview of the industrial symbiosis within the circular economy, followed by a discussion of the role of production planning in managing direct and reverse production flows. In Chapter 2, the concepts related to the circular economy are cross-analyzed and their definitions are clarified. This chapter discusses the major issues posed in the framework of an industrial symbiosis, and motivates the lot-sizing problems studied in this thesis. Finally, basics on lot-sizing problems are recalled.

In line with the taxonomy of the processes related to the circular economy, Chapter 3 surveys the production planning problems encountered in the literature, with respect to the main recovery operations, namely: **(i)** disassembly for recycling, **(ii)** from product to raw materials recycling, and **(iii)** by-products versus co-products. A special attention is paid to greenhouse gas emissions and energy consumption. Gaps and avenues for future research are emphasized. In particular, the literature review puts the spotlight on the lack of studies on production planning problems encountered in industrial symbiosis networks. The remainder of the thesis focuses on the by-product synergy.

With an ultimate objective of addressing a production planning problem at the industrial symbiosis level, we first study, in Chapter 4, the problem encountered by a supplier of by-products, i.e. a single-item lot-sizing problem including the management of a by-product (ULS-B problem). We deal with the general case, where the by-product is storable with a limited capacity. A complexity analysis of the problem is performed for two types of capacity: time-dependent and stationary. To do this, we identify important structural properties of the optimal solutions to derive dynamic programming algorithms based on block decomposition of the planning horizon. We show that with a time-dependent capacity the problem is weakly *NP*-Hard, and with a stationary capacity the problem can be polynomially solved. This problem is a first step towards understanding and solving an industrial symbiosis related problem.

To continue in this direction, we extend the ULS-B problem in Chapter 5, by considering a basic network composed of one supplier of by-products and its receiver, which leads to a two-level lot-sizing problem. Inspired from the existing industrial symbiosis networks, we take interest of two cases of this novel problem in the literature: **(i)** *general case*: the by-product is storable with a limited capacity and, **(ii)** *particular case*: the context makes the generated by-product unstorable, i.e. the by-product has to be immediately either disposed of or reused by the receiver. In Chapter 5, we show that two-level lot-sizing problem is *NP*-Hard. This problem is solved using an efficient algorithm based on the Lagrangian decomposition and Lagrangian relaxation.

In Chapter 5, we solve a centralized problem, where all the information are supposed to be known. In real-life contexts, an actor may have sensitive information, that she/he wants to keep private. As a result, a centralized collaboration policy is rarely possible. Hence, we introduce four decentralized collaboration policies with no information sharing: **(i)** one for no collaboration, **(ii)** one for an opportunistic collaboration, and **(iii)** two for sequential symbiotic collaborations. These decentralized policies are discussed with respect to the centralized collaboration policy based on full information sharing.

The decentralized collaboration policies studied in Chapter 5 do not require information sharing among production units. In less restrictive industrial environments, only some sensitive information are kept private. In the complementarity of the baseline collaboration policies introduced in Chapter 5, we investigate two collaboration policies for partial information sharing based on multiple contract offers in Chapter 6: **(i)** a game-theoretic collaboration policy for asymmetric information sharing, and **(ii)** a contractual-based collaboration policy obtained via a negotiation-based scheme managed by a blinded mediator for symmetric feedback sharing. To assess the sustainable impact of an industrial symbiosis network, the contracts obtained in the framework of the studied collaboration policies are analyzed with respect to three criteria: economic, environmental and based on the satisfaction of both the supplier and the receiver of by-products.

Finally, Chapter 7 summarizes the main achievements and conclusions that have been obtained as a result of this thesis and provides an outline of avenues for future research. The perspectives are derived into three axis: **(i)** by-products in production planning problems, **(ii)** the variants of an industrial symbiosis networks and, **(iii)** the implications of an industrial symbiosis network on the three pillars of the sustainable development (economic, social, environmental).

Chapter 2

Circular economy and production planning

In the actual era of the international trade, global warming and depletion of Earth natural resources, the willingness to generate sustainable and competitive benefits determines us to stop thinking *linearly* and to shift towards a *circular* approach by closing material loops. The circular way of thinking falls within the concept of *circular economy*. This chapter describes the background of the thesis by: **(i)** motivating and defining the circular economy, **(ii)** positioning the circular economy in history and with respect to other environmentally-friendly concepts, **(iii)** providing a description of waste management options and discussing its sustainable implications.

At the heart of this thesis, a special focus is put on the *industrial symbiosis* defined as a sustainable option to convert production residues into high-added valued products or to share other collateral resources like services or facilities. This chapter puts the spotlight on the importance of production planning when regarding industrial symbiosis.

Contents

2.1 Introduction

During the last decade, the expression *Circular Economy* (CE) experiences an increasing interest, particularly with the advent of environmental regulations around the world, including: in Europe, the Waste Framework Directive[1] (2008); in USA, the Enactment of the Resource Conservation and Recovery Act[2] (1984) and the Pollution Prevention Act[3] (amended in 2002); in China, the Circular Economy Promotion Law[4] (2008); in Japan, the Law for establishing a Material Cycles Society[5]; in Vietnam, the Environmental Protection Law[6] (2005); in Korea, the Waste Control Act[7] (amended in 2007) and the Act on Promotion of Resources Saving and Recycling[8] (amended in 2008). A side effect of the rising popularity of the concept of circular economy among political, industrial and academic communities, is the lack of consistency around its definition and scope of action. Research streams originating from different scientific disciplines gave rise to various schools of thoughts of the circular economy. Among those adopted in production and operations management, let us mention e.g.: cradle-to-cradle [Kumar and Putnam, 2008, Baki et al., 2014], industrial ecology [Genovese et al., 2017].

Several academic efforts have been specially dedicated to clarifying and conceptualizing the term of *circular economy* [Kirchherr et al., 2017, Reike et al., 2018, Homrich et al., 2018]. Although the notion *circular economy* still remains wide, various definitions coexist and are consolidated as follows:

Definition 1 (Homrich et al. [2018], Reike et al. [2018])**.** *The* ***circular economy*** *(CE) is an economic system that emerges to oppose the linear open-ended system (produce, consume, dispose), with the aim to accomplish sustainable development, simultaneously creating environmental quality, economic prosperity and social equity to the benefit of current and future generations.*

Aware of the business opportunities that the circular economy can procure, the European Commission makes significant efforts to support the transition to a more sustainable, low carbon, resource efficient and competitive economy. In this spirit, institutions such as the Scottish Institute for Remanufacture[9] (United Kingdom) and the Institut de l'Économie Circulaire[10] (France), have been created to help industrial actors adopting this concept in their production and supply chains.

The goal of the current chapter is to introduce the context and delineate the framework of the thesis. More precisely, after having clarified the definition of circular economy, this chapter positions it in time and in relation to other environmentally-friendly concepts, in Section 2.2. According to the differences between these concepts and the *five step waste hierarchy* introduced by the European Commission, Section 2.3 provides a classification and a taxonomy of the processes inherent to the circular economy. A focus on the industrial symbiosis is provided in Section 2.4. Finally, the role of production planning in circular economy and more precisely in industrial symbiosis is discussed and basics of production planning problems (more specifically, lot-sizing problems) are recalled in Section 2.5.

2.2 Brief historical development

Since the 2000s, the literature on circular economy (CE) has been reviewed in the framework of various academic disciplines, such as environmental science, energy, engineering, or resource management. The definition and the contextualization of CE given in Chapter 3 are based on the findings discussed in previous general-purpose reviews on the topic (see e.g. Kirchherr et al. [2017], Homrich et al. [2018], Reike et al. [2018]). To provide an overarching picture of the CE emergence and its actual context, let us take a historical look and examine the literature reviews conducted on other notable CE-related concepts (see Figure 2.1).

Falling within the CE, the circular way of thinking derives from the concept of *reverse logistics* (or reverse supply chain) introduced in the late 1970's and early 1980's, which is defined by the American Reverse Logistics Executive Council as: *"the process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal"* [Rogers et al., 1999]. The network including simultaneously

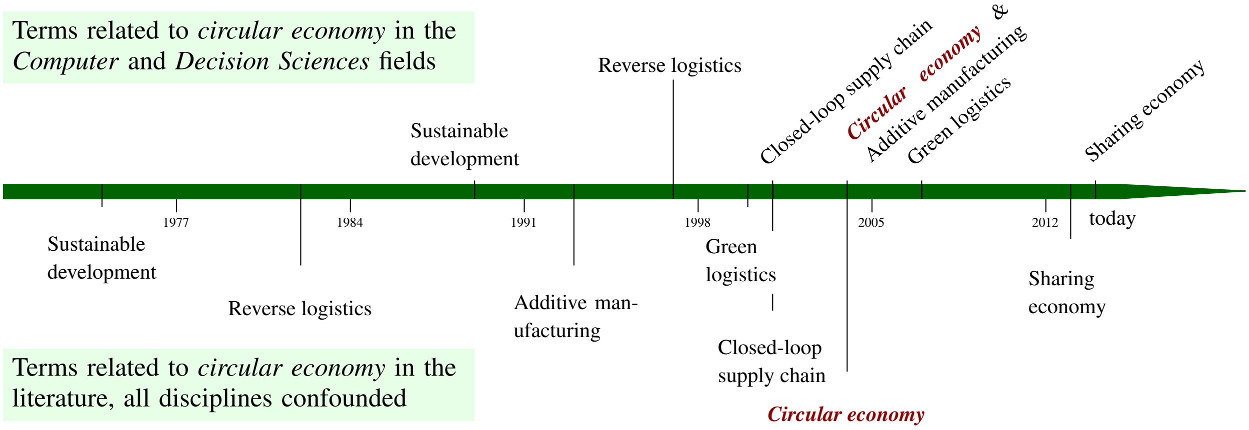


Figure 2.1: The emergence of *circular economy* and its precursors in the literature: *A brief chronology based on the database Scopus (in Title, Abstract and Keywords)*

both forward and reverse supply chains is known in the literature under the umbrella term *closed-loop supply chain* (CLSC) [Dekker et al., 2013, Govindan et al., 2015], and starts to be evoked in earnest in the 2000s. Since then, a number of surveys have been conducted on reverse logistics and CLSCs. The interested reader can refer to the review of Govindan et al. [2015] to take note of review studies published until 2015. For years after 2015, Singh et al. [2016] evaluate the impact of reverse logistics on apparel industry in their industry-specific survey. More generally, Kazemi et al. [2019] review the articles on reverse logistics and CLSCs published only in the International Journal of Production Research.

To foster the generation of sustainable and competitive benefits, governments definitely want to stop thinking linearly for shifting towards a circular approach by: **(i)** eco-designing products, **(ii)** waste preventing, reusing and recovering, **(iii)** exploiting renewable energy resources. Built on the three pillars of the sustainable development (namely, economic, environmental and societal), the ultimate goal of the adopted series of measures aims at reaching zero waste and extracting zero raw materials, by ranging from legislation to financial levers. Even if the sustainable development finds its roots much earlier, its formal introduction dates from 1987 by the World Commission on Environment and Development with the publication of the so-called *Our Common Future* or *Bruntland Report* [Keeble, 1988]. Since this milestone, multiple events and research efforts continue to trace the evolution of the sustainable development [Eustachio et al., 2019]. For example, let us mention the additive manufacturing (also known as 3D printing) developed in the 1980's, whose potential to support the sustainable product design is actively promoted in the last decade [Sauerwein et al., 2019]. Further on the subject of sustainability, sustainable consumption practices are, in recent years, propagandized in the framework of the sharing economy, a broad concept still in maturation [Curtis and Lehner, 2019].

Since 2003, more than a hundred of reviews are published, preponderantly in environment, social and energy fields. In the field of *Computer* and *Decision Sciences*, the literature on sustainability is mainly reviewed to highlight and quantify the effect of its consideration in the conventional organization of different activity sectors, such as urban systems [Bibri, 2019], transportation [Nenni et al., 2019], supply chain [Moreno-Camacho et al., 2019, Barbosa-Póvoa et al., 2018, Marshall et al., 2015], and production [Thies et al., 2019].

With regard to the environmental dimension of sustainability, the prefix *green* is reserved, and commonly used to underscore the explicit consideration of environmental aspects in a given industrial activity. In *Computer* and *Decision Sciences* literature, the greenness of different logistics activities has been reviewed [Sbihi and Eglese, 2007] e.g.: manufacturing [Mathiyazhagan et al., 2019], freight transportation [Demir et al., 2014, Bektaş et al., 2019], supply chain [Waltho et al., 2019, Kaur and Awasthi, 2018, Fahimnia et al., 2015, Sarkis et al., 2011]. Note that green logistics activities are not necessarily reverse logistics [Rogers et al., 1999].

2.3 Waste management and its implications

Since production processes have a high impact throughout a product life on supply, resource use and waste generation, let us distinguish four main topics dealing with the circular economy and production management:

- **Reverse logistics** and **waste management** refer to all environmentally-friendly operations related to the reuse of products and raw materials. For example, the European Commission established an order of priority of recovery operations (so-called *five step waste hierarchy*), starting with the preferred option of waste prevention, followed by preparing waste for reuse, recycling and other recovery (i.e. backfilling), with disposal (i.e. landfilling) as the last resort (see Figure 2.2).

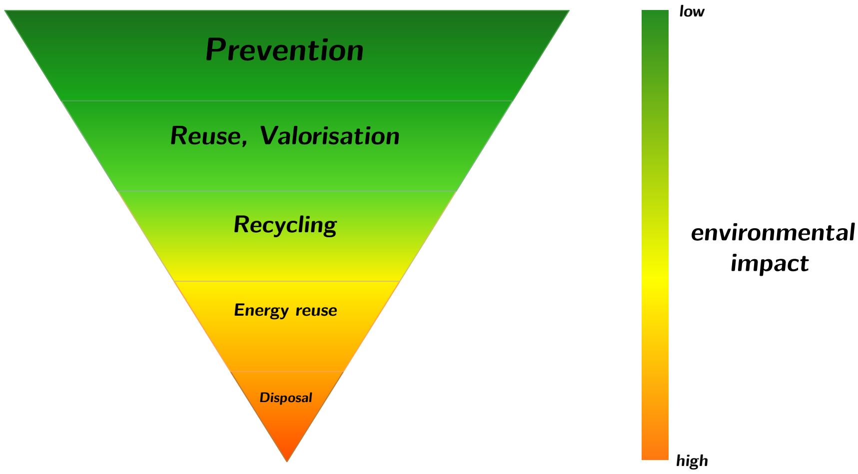


Figure 2.2: Five step waste hierarchy (from Waste Framework Directive[1])

  After its prevention, the reuse of waste is the next most desirable option in the hierarchy of waste management options, specified in the framework of the European Commission legislation. Reuse represents the using again without any structural changes of products that are not waste for the original purpose. This operation may require collection, but negligible or no processing. Reused products are generally sold in peer-to-peer, without any repairs or tests.

- **From product to raw material recycling:** Under Article 3 of the Waste Framework Directive[1], *recycling* means: *"any recovery operation by which waste materials are reprocessed into products, materials or substances whether for the original or other purposes".* In the literature, recycling options can be encountered under different terms, like reconditioning, repurposing, refurbishment, remanufacturing. By tending to superpose each other in their meaning, the definitions of these trending concepts are blurring and blending.

  In production planning literature, two recycling terms stand mainly out, namely refurbishment and remanufacturing. *Refurbishment* emerges as a recovery process, by which waste are collected, tested, repaired, cleaned and resold as used products in working order, without having been disassembled. Refurbished products are often put back under warranty.

  Meanwhile, *remanufacturing* is most frequently identified as a recovery operation of used products, including collection, repairing, disassembly and replacing of worn components for rebuilding products to the quality level of a newly manufactured ones. The main particularity of remanufacturing resides in product disassembly, the first and most important step in the markets for spare parts or re-processing operations in production.

- **Co- and by-products:** The notions of by-products and co-products recently emerge in supply chain optimization problems. Being of similar importance as a main product, *co-products* are generated together with a main product and have their own demand, whereas *by-products* are usually unexpected products issued from a manufacturing process and have less economic value than controllable production outputs.

- **Sustainability and its three pillars (economic, environmental and social):** Generally speaking, the consideration of sustainability issues in operations management problems is attracting an increasing attention in the research field after the mid-1995s (see Figure 2.1). Acquired by inheritance from linear production schemes, the economic dimension continues to enjoy all the attention in the production planning literature. Until not long ago, some researchers start to take a serious interest in reducing greenhouse gases, by integrating the carbon emissions generated during the production, transportation and remanufacturing operations in their models.

  To the best of our knowledge, the social criterion has been neglected in production planning problems. As concluded by other closely-related studies, this is also the case for other operations management problems [Moreno-Camacho et al., 2019]. In the production planning literature, the only found social-aware studies operate in the framework of traditional (continuous-time) economic order quantity models (see e.g. Battini et al. [2014], Andriolo et al. [2016]).

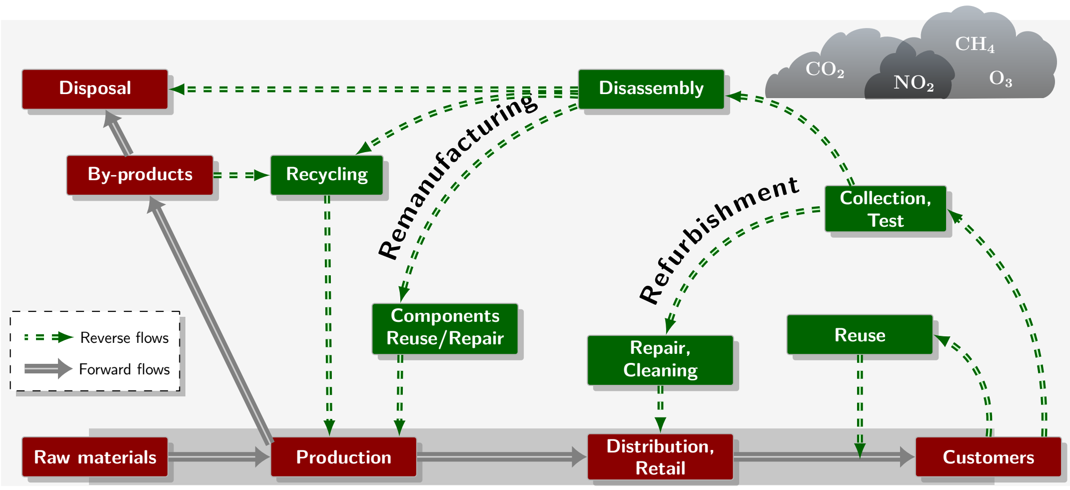


Figure 2.3: Circular economy: *An overview*

To highlight how the circular economy redraws the classical linear production approach, Figure 2.3 maps the aforementioned five waste management options into production systems. Dashed backward arcs correspond to the production activities related to the circular economy, and bring out the relationships between them. This figure is inspired by some literature reviews related to the circular economy (see e.g. Govindan and Soleimani [2017], Kumar and Putnam [2008], Soleimani and Govindan [2014], Srivastava [2008], Thierry et al. [1995]).

2.4 Industrial symbiosis

Consistent with global environmentally-friendly trends, the industrial symbiosis (also called eco-industrial parks) seeks to take sustainable competitive advantage of binding traditionally separate industrial processes in a joint production approach involving *exchange of physical flows*, such as production residues or other secondary resources (e.g. water, and energy), and/or the *sharing of services* like knowledge, logistics, expertise [Chertow, 2000, Lombardi and Laybourn, 2012]. In particular, the European Commission promotes the industrial symbiosis as one of the ways to shift towards a circular economy through its programs and action plans[11] and by funding projects such as MAESTRI project[12] (under grant No 680570) and SCALER project[13] (under grant No 768748), which aim to deliver practical tools and guidelines for industry actors engaging in resource efficiency, reuse and sharing.

Inter-firm symbiotic linkages can take different forms. The three most important symbiotic relationships are [Chertow et al., 2008]:

Utility sharing. It consists in sharing material, utilities or infrastructures for pooled use and management of commonly used resources such as steam, electricity, water, and wastewater. The main feature of this form of symbiosis is that a group of firms jointly assumes the responsibility for providing utility services or infrastructure, such as water, energy/heat provision systems (e.g. co-generation plants), and wastewater treatment plants (see Figure 2.4). Utility sharing leads to a reduction of the transportation costs and the associated greenhouse gas emissions. In addition to the economic benefits, the proximity between the infrastructures promotes the treatment and the recycling of waste that would otherwise be disposed of.

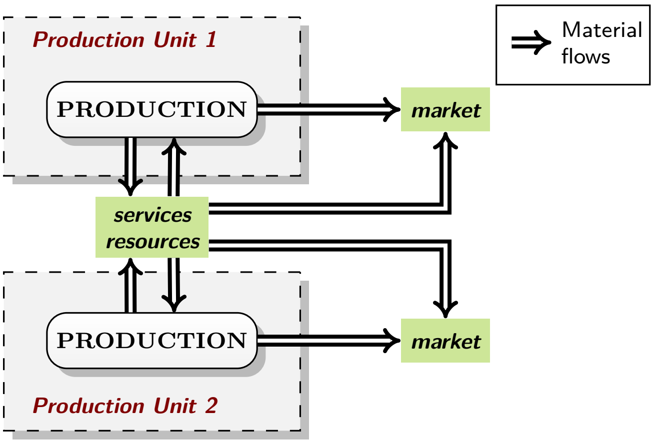


Figure 2.4: Utility sharing

Joint provision of services. Industries can exchange information to increase the collective efficiency of their operations, and to coordinate planning, project management and regulatory permits. Fire suppression, security, cleaning, catering, and waste management are examples of ancillary services that have positive environmental implications. This inter-firm symbiotic linkage differs from the utility sharing by the fact that it does not refer to the production process, but to the auxiliary services.

By-product synergy. First, let us define and position clearly the notion of by-product in relation to other closely related terms. Based on the Waste Framework Directive[1], let us consider a *product* as all lawful material mainly aimed resulting from a production process. This term includes *co-products*, which have the same importance as main products and have their own demand. According to the European Interpretative Communication[14] on waste and by-products, production residues are classified into: **(i)** *by-products*, i.e. lawful production residues unavoidably obtained as an integral part of the production process, ready for use without further transformation, whose use is certain, and **(ii)** *wastes*, i.e. production residues, which are not by-products. Note that by-products are, by definition, lawful production outputs, whose further use is economically and environmentally sustainable (see Figure 2.5). The definitions of the inherent terms will be more largely discussed in Section 3.5. The use by a company of a by-product of another company as raw material forms a by-product synergy. The resulting network includes two main types of actors: **(i)** a *supplier*, which generates the by-product and, **(ii)** a *receiver*, which uses it. The intervention of a third party can be needed to ensure the connection between the supplier and the receiver. The topic of this thesis focuses on production planning in the framework of a by-product synergy, In the rest of this thesis the term *industrial symbiosis* is used to designate *by-product synergy*.

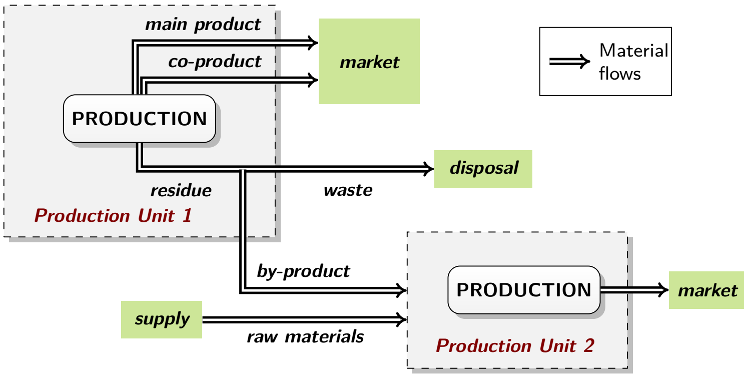


Figure 2.5: By-product synergy

Industrial symbiosis offers opportunities to improve the three dimensions of the sustainable development:

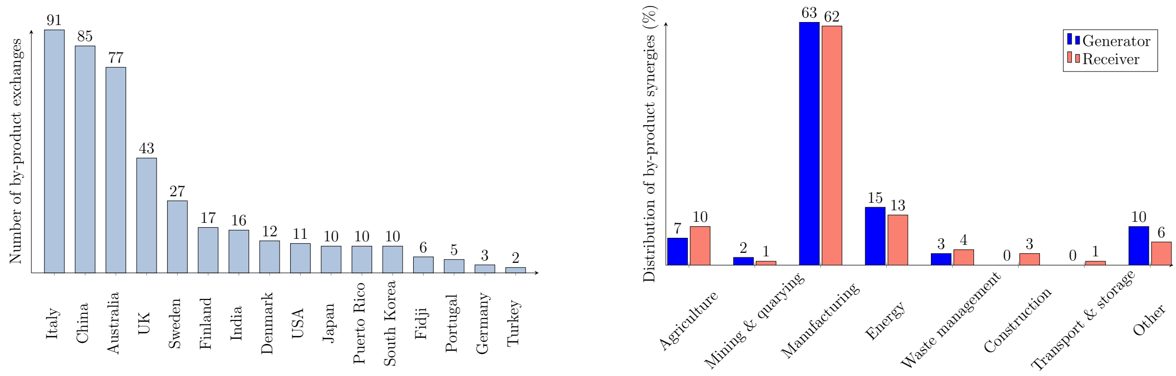
- **Economic:** by the reduction of disposal costs of the production residues for the supplier and of purchasing of raw materials for the receiver. Both including transportation costs, they are reduced since companies forming an industrial symbiosis are close to each other. For instance, in an eco-industrial park in North Carolina in the USA, an acetone partnership was found to reduce the costs of both the supplier and the receiver of acetone. The supplier, a gem manufacturer, saves $11,000 in treatment and disposal costs per year and the receiver of acetone, a plastic company, saves $18,000 per year [Chertow, 2000].

- **Environmental:** by the reduction of waste accumulation and raw material extraction. For instance, in Gladstone, the industrial symbiosis between a power station and a cement producer allows to save 40 kilotons of fly ash per year [Golev et al., 2014]. Moreover, the industrial symbiosis allows to limit the greenhouse gas emission related to the transportation of the production residues and promotes the reuse of the carbon dioxide emitted during manufacturing processes. As an example, 42 million of tonnes of CO2 equivalent emissions have been diverted from the atmosphere in the UK over eight years thanks to the different industrial symbiosis existing in the country [Earley, 2015]. In the same way, energy and resources like heat, steam and water are saved [Chertow, 2000].

- **Social:** Generally, companies forming the industrial symbiosis are close to each other, this may support the regional economic development. For example, in the UK, 80 jobs have been created thanks to a nitrogen producer, which captures its steam and CO2, and uses them to heat greenhouses and support the growth of fruits and vegetables [Earley, 2015]. Contrary to the two previous dimensions, the impact of industrial symbiosis on the social sustainability are much more difficult to be quantified because the effects are not always direct.

The noticeable economic, environmental and social gains related to the implementation of an industrial symbiosis push governments and companies to turn to a more sustainable economy and to adopt new restrictive directives. For instance, some chemical by-products are not allowed in landfills anymore, due to Directive 1999/31/EC[30], ratified by the European Commission in 1999. Consequently, a number of industrial symbiosis networks (also called eco-industrial parks) emerge all around the world. Based on a large list of existing or in project of by-product synergies provided by Evans et al. [2017], Figure 2.6a shows the distribution of by-product synergies in the world. Countries of almost all continents appear. Europe is more largely represented. Among the 425 listed by-product synergies, 144 are either planned or under feasibility study. This shows the topicality and the growing interest in industrial symbiosis.

The main industries concerned with the generation or the use of the by-product is the manufacturing industry (see Figure 2.6b). The implementation of the industrial symbiosis is facilitated by the predictability of the quantities of by-product generated and required. Note that for some other industries like construction and agriculture industries, the lack of long-term view of their quantities of generated production residue makes the reuse of their by-product difficult to manage. These industries are conducive to the reuse of the by-products of other industries.



(a) Per country

(b) Per industrial sector

Figure 2.6: Distribution of the by-product synergies per country and industrial sector in 2017, based on Evans et al. [2017]

As long as at least two production units are involved, practical questions arise and make the implementation of an industrial symbiosis difficult: Can the by-product be stored or do we have to used it immediately? What transportation mode should be used? How to distribute the related costs or benefits between the different actors? What collaboration policy should be implemented?, etc. This thesis will try to provide some answers to all these questions, from the perspective of managerial and organizational aspects.

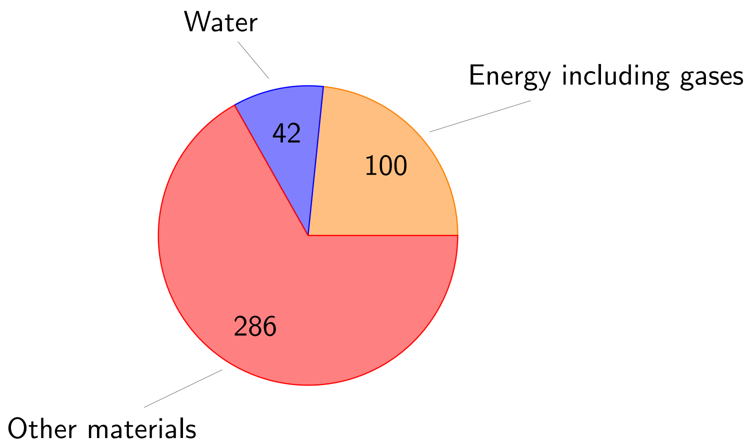


Figure 2.7: Nature of the by-products exchanged in 2017, based on Evans et al. [2017]

Being industry-dependent, the industrial symbiosis networks can also include specific intermediate facilities required by the by-product handling system:

- **Transportation modes:** Depending on the distance between the actors of the industrial symbiosis network, on the nature of the production residue and on the frequency and the size of the exchanges, the actors can opt for a continuous transportation ways like pipeline (e.g. waste water, gases) or a intermittent transportation way like trucks and tanks (e.g. sludge, end-of-life semiconductor).

- **Treatment processes:** By definition, a by-product is not a virgin material because it comes from a production process. As such, a treatment process can be necessary before its reuse. For instance, waste water needs to be purified, some foodstuffs have to be dried. In particular, hazardous production residues cannot be reused as specified by law, but some of them are useful (e.g. ammonia and sulphur can be extracted from sour gas). To sum up, the treatment processes lead to additional costs related to the use of different transportation modes and potential intermediate storage facilities, in particular, if the treatment is performed by a third party.

- **Storage facility:** Figure 2.7 shows that production residues of different nature are represented in existing by-product synergies: **(i)** unstorable like gases, **(ii)** storable during a short time slot (e.g. waste food, heat) and **(iii)** storable (e.g. sludge, water). Depending on the nature of the by-product, the chosen transportation mode and the required treatment processes, an intermediate storage facility can be necessary in some cases (e.g. warehouse, shed). Note that the storable quantity of by-products can be limited for questions of space or available material, like tanks, silos, lagoons, and of treatment capacity (see e.g. the sludge storability in Kalundborg explained by Larsen et al. [1991]). The storability of the by-product will be more largely discussed in Chapter 4.

These intermediate facilities can belong to production units, or provided by third parties, complicating thereby the logistics related to the by-product handling. An extra fee is sometimes paid by one of the actors or both of them to compensate the generated extra costs incurred by the owner of these facilities to encourage its participation in the industrial symbiosis.

The collaboration policies can differ from one industrial symbiosis to another. There exist cases, where the involved actors develop a mutual trust between themselves and share all their information, enabling thus possible a centralized decision-making related to the by-product exchange. The full information sharing is usually encountered when both actors belong to the same parent company or when a third party manages the industrial symbiosis network. In general, the full information sharing between actors may be difficult to be set up for different reasons such as the requirements to keep sensitive information private or not to reveal the risks related to production disruptions or production recipes of products [Vladimirova et al., 2018, Fraccascia and Yazan, 2018]. Partial information sharing is commonly addressed via decentralized collaboration policies. Let us distinguish two main types of decentralized collaboration policies with respect to their time frames:

- **Opportunistic (Short-term):** In the decentralized collaboration policy applied to regulate spontaneous exchange of by-products, we assume that both the supplier and the receiver make their own production plans independently. The production plans are then brought together to put the generated by-products into value. This kind of decentralized collaboration policy is specific to short-term horizontal collaboration in classical supply chains.

- **Symbiotic (Long-term):** A long-term collaboration may be expressed in the framework of sequential decentralized collaboration policies, made either from the supplier to the receiver or from the receiver to the supplier. Commonly encountered in the vertical collaboration in classical supply chains, this kind of decentralized collaboration policies can also take place in an industrial symbiosis framework when a long-term collaboration is committed to promoting the by-product recovery. They are based on the leadership (i.e. on primacy in decision-making) of one actor compared to another one: one actor makes its decisions first, then the second one makes its production plan accordingly.

2.5 Production planning in industrial symbiosis

Fallen within the new challenges brought by the industrial symbiosis (technical, managerial, legislative, environmental, etc.), new optimization problems emerge at all the decision levels:

- The *operational* optimization problems refer to short-term decisions, such as line balancing, scheduling or vehicle routing. Once the mid-term needs are determined, these decision-making problems occur at the entity level.

- The *tactical* optimization problems seek to assist managers/firms in making their mid-term decisions related to the production. The flows of main products and production residues are managed at this level, over a planning horizon. This level is crucial as it makes the link between the flows of the different actors of the supply chain and the mid-term decisions impact each actor at its operational level. In case of industrial symbiosis, optimization problems aim at deciding the quantities of production residues to dispose of or to reuse and the quantities of raw materials to purchase, while managing the flows of the main product of each actor in its own supply chain. Decisions related to the production can be made separately by each production unit but decision related to the transportation and/or pre-processing of the by-product have to be made together.

- More broadly, *strategic* optimization problems operate at the supply chain level, by integrating the choice of the collaborative policy used between the different entities. This can bring a supplier selection problem for the by-product receiver production unit. Facility location problems can also arise due to the need of an intermediate facility or a particular transportation mode depending on the by-product characteristics (storability, hazardousness, etc.). These decisions being of crucial importance and investment, their posing and making are less frequent and take place upstream the industrial symbiosis implementation, in accordance with all the involved production units.

New sustainability-oriented directives, and by-product flows in particular, break the structure of the well-studied tactical decision-making problems based on a linear production approach. Given the undeniable challenges the industrial symbiosis triggers related to tactical production planning problems, this thesis aims at dealing with production planning problems posed in the framework of a by-product synergy.

Table 2.1: Parameters and decision variables of the LS-U problem

Parameters	
T	Number of time periods
d_t	Demand for the product in period t
p_t	Production cost in period t
f_t	Fixed setup cost in period t
h_t	Holding cost for the product in period t
Decision variables	
X_t	Production quantity in period t
Y_t	Binary setup indicator for period t
I_t	Inventory level of the product at the end of period t

One of the main production planning problems, which will serve as a basis for the rest of the thesis, is the uncapacitated single-item lot-sizing problem (LS-U), introduced by Wagner and Whitin [1958]. This problem, dealing with a single product and unlimited production and inventory capacities, aims to determine over a planning horizon of T periods when and how much to produce in order to satisfy a deterministic demand d_t $\forall t \in \{1,2,...,T\}$, while minimizing the total resulting cost. The production system involves a fixed setup cost f_t and a unitary production cost p_t per period of time. The surplus quantities of the product can be kept in inventory at a unitary cost h_t from period t to period $t+1$.

The two main decisions posed by the LS-U problem are: when and how much to produce. Accordingly, decisions related to the inventory levels of the product are implied. Hence, let X_t be the decision variable that represents the quantity of product to be produced at period t. Denote by Y_t the decision variable that indicates if production occurs in period t. The inventory level of the product at the end of period t is represented by I_t.

By making use of the notation summarized in Table 2.1, the LS-U problem can be formulated as a mixed-integer linear program as follows:

$$\text{minimize} \sum_{t=1}^{T}(p_t X_t + f_t Y_t + h_t I_t) \tag{2.1}$$

subject to:

$$I_{t-1} + X_t - I_t = d_t, \qquad \forall t \in \{1,2,...,T\} \tag{2.2}$$

$$I_0 = 0, \tag{2.3}$$

$$X_t \leq \sum_{i=t}^{T} d_i Y_t, \qquad \forall t \in \{1,2,...,T\} \tag{2.4}$$

$$X_t, I_t \geq 0, \qquad \forall t \in \{1,2,...,T\} \tag{2.5}$$

$$Y_t \in \{0,1\}, \qquad \forall t \in \{1,2,...,T\} \tag{2.6}$$

The objective function (2.1) minimizes the sum of fixed and variable production and holding costs. Constraints (2.2) model the flow conservation of the product. The initial inventory is set to zero by Constraint 2.3. Constraints (2.4) ensure that there is a setup if there is production. Constraints (2.5) and (2.6) are the nonnegativity and binary requirement constraints.

The formulation introduced above is straightforward. Other formulations have been proposed in the literature, like a facility location formulation, a shortest path formulation, etc. For more details on existing formulations, the reader is referred to Pochet and Wolsey [2006].

The LS-U is an *easy* problem, which can be solved in a polynomial time. It was first solved by a forward dynamic programming algorithm proposed by Wagner and Whitin [1958]. Dynamic programming is an implicit enumeration method, which explores the solution space of a problem to find its optimum. The dynamic programming algorithm proposed by Wagner and Whitin [1958] is based on the following property:

Property 1. *There exists an optimal solution of the LS-U in which $I_{t-1}X_t = 0$ for all t.*

This property reduces the search space to at most T^2 states. Indeed, the quantity of product, needed to fulfill the demand of each time period, is produced in one period. Let us suppose all the demands of each period are different from zero. The cost of producing at t to satisfy the demands from period t to period u is denoted by C_{tu}:

$$C_{tu} = f_t + p_t d_{tu} + \sum_{k=t}^{u-1} h_k d_{k+1u}, \qquad \forall t,u \in \{1,2,...,T\}. \tag{2.7}$$

H_t provides the optimal solution from period t to period T when the algorithm works backward. It is given by:

$$H_t = \min_{t \leq k \leq T} C_{tk} + H_{k+1}, \qquad \forall t \in \{1,2,...,T\} \tag{2.8}$$

with $H_{T+1} = 0$.

The dynamic programming algorithm based on (2.7) and (2.8) runs in $\mathcal{O}(T^2)$ due to the computation of all the C_{tu}. In the early nineties, the complexity of the dynamic programming algorithm for the LS-U was improved to $\mathcal{O}(T \log T)$. This result was found independently by Wagelmans et al. [1992], Aggarwal and Park [1993], Federgruen and Tzur [1991].

There exists a lot of variants of the single-item lot-sizing problem. The most well known ones are: with a time-dependent production capacity (LS-C), when this capacity is constant (LS-CC) and, with inventory capacities (LS-IC).

Table 2.2: Summary of the complexity of the main variants of the lot-sizing problem

Problem	Complexity	Authors
LS-U	$\mathcal{O}(T \log T)$	Wagner and Whitin [1958], Federgruen and Tzur [1991], Wagelmans et al. [1992] Aggarwal and Park [1993], Van Hoesel et al. [1994]
LS-CC	$\mathcal{O}(T^3)$	Florian and Klein [1971], van Hoesel and Wagelmans [1996]
LS-C	NP-Hard	Florian et al. [1980], Bitran and Yanasse [1982]
LS-IC	$\mathcal{O}(T^2)$	Love [1973]

LS-CC: LS with constant capacity, LS-C: Capacitated LS, LS-IC: LS with inventory capacities

All these variants are well-studied in the literature and their complexity is provided in Table 2.2. To summarize, a time-dependent production capacity is sufficient to make a basic lot-sizing problem initially polynomially solvable NP-Hard.

In this thesis, extensions of the single-item lot-sizing problem are studied for the management of a by-product. More precisely, we will introduce several extensions of the LS-IC problem where a by-product is generated in the same time and in the same quantity as the main product. The inventory capacities are on the by-product. There is no demand to meet for the by-product so it has to be transported outside the production unit. First, the by-product is transported with a fixed cost and we show that this extension of the LS-IC stays polynomially solvable when the inventory capacity is constant over the time but it becomes NP-Hard when the inventory capacity is time-dependent.

Second, the problem encountered by the supplier of by-product becomes a lot-sizing problem with two options to get rid of the by-product. It can be disposed of or send to another production unit which will use it as raw materials for the production of its own product. The problem encountered by the receiver of by-product is a type of single-item lot-sizing problem with two options to obtain the raw materials required for the production: using the by-product of the supplier or purchasing raw materials to an external supplier. The resulting problem that is studied in this thesis is a two-level lot-sizing problem where the first level corresponds to the problem encountered by the supplier and the second level corresponds to the problem encountered by the receiver. The studied problem is composed by an extension of a LS-IC at the first level and an extension of the single-item lot-sizing problem at the second level. The two levels are linked by the by-product. This particularity, typical of an industrial symbiosis, makes this problem NP-Hard and new in the lot-sizing literature.

2.6 Conclusion

Motivated by the topicality of the environmental concerns, this chapter provides a brief historical development of the circular economy. The waste management options are classified and their implications are discussed. New environmentally-friendly optimization problems emerge from processes arising from the circular economy. More specifically, the interference between the reverse and forward flows affects the production planning and creates a number of new mid-term production planning problems. They are described and reviewed in the next chapter.

Among all these new issues, the industrial symbiosis catches our attention as it influences by itself the three pillars of the sustainable development (economic, environmental, societal). As described above, the challenges related to the implementation of an industrial symbiosis are numerous, technical, organizational, environmental, economic and legislative, and a number of these issues will explicitly or implicitly be handle over this thesis.

Chapter 3

Literature review

Given the actual environmental context, and based on the taxonomy of the environmentally-friendly processes defined in Chapter 2, this chapter proposes a comprehensive state-of-the-art around the topic of *circular economy* and *reverse logistics* with a particular emphasis on *mid-term production planning* under discrete time settings. The broad spectrum of reviewed publications is categorized and discussed with respect to the main recovery operations, namely: **(i)** disassembly for recycling, **(ii)** from product to raw material recycling, and **(iii)** by-products and co-production. For each of aforementioned recovery options, this chapter elucidates the related definitions, reviews the mathematical formulations jointly with a structured overview of the solution methods, and discusses their industrial implications. Given the legislative pressure to mitigate environmental impacts caused by production processes, a special attention is paid to the greenhouse gas emissions and energy consumption. Gaps in the literature are identified and future research opportunities are suggested.

Contents

3.1 Introduction

The willingness to generate sustainable and competitive benefits pushes us to stop thinking linearly (produce, consume and dispose) and to shift towards a circular approach by: **(i)** eco-designing products, **(ii)** remanufacturing, recycling and reusing both products and byproducts, **(iii)** exploiting renewable energy resources. This last way of thinking falls within the concept of circular economy that, in turn, derives from reverse logistics. Built on the three pillars of the sustainable development (namely economic, environmental and societal), the main objectives of the circular economy aim at reaching zero waste and extracting zero raw materials.

Aware of the business opportunities that the circular economy can procure, both companies and academics start to consider sustainability in the supply chains. In this context, this literature review puts the spotlight on mid-term production planning viewed through the prism of the circular economy and more precisely based on the taxonomy of the inherent processes provided in Section 2.3. The current chapter aims to: **(i)** offer a representative overview of the research and industrial efforts made towards a circular economy in production planning, **(ii)** provide definitions of inherent CE-oriented industrial processes, and **(iii)** derive new avenues for both academics and practitioners as to how recovery options can be suitably integrated within traditional production environments to converge towards an environmentally-friendly economy.

To the best of our knowledge and based on the Scopus database, only two review studies, including in their title the keyword *circular economy*, have been conducted in the field of *Computer* and *Decision Sciences*, namely: **(i)** Nascimento et al. [2019] surveyed the integration of industry 4.0 technologies with CE practices in a manufacturing context, from an operations management point of view, and **(ii)** Pinheiro et al. [2019] touched upon the new product development and life cycle within the CE. More widely, our review seeks to map the existing literature on production planning integrating CE practices, while taking into account earlier review studies, discussed in Chapter 2.2, from a range of disciplines (see Table 3.1).

Content structure. Being written in the complementarity of the existing related surveys, this chapter is structured as follows. The reviewed publications are categorized and discussed with respect to the taxonomy of the recovery processes provided by Chapter 2.3, namely: **(i)** disassembly for recycling in Section 3.3, **(ii)** from product to material recovery in Section 3.4, and **(iii)** by-products and co-production in Section 3.5. To go further than these reverse logistics processes, Section 3.6 takes interest in the quantitative implications resulting from the consideration of greenhouse gas emissions and energy consumption in the production planning process. Section 3.7 consolidates the findings of this state-of-the-art review and derives a number of opportunities for future research based on the identified gaps. Finally, Section 3.8 concludes this review chapter.

Content analysis. For the sake of rigor, this literature review follows a systematic scheme to present and to analyze the content of the collected papers:

- **Material collection:** see Section 3.2.
- **Material description:** This step aims: **(i)** to present the context of the topic under focus, **(ii)** to elucidate the related definitions, and **(iii)** to review the mathematical formulations jointly with a structured overview of the existing solution methods, and to discuss the industrial implications of addressed problems.
- **Material analysis:** Each of Sections 3.3-3.6 complies with a unique common thread. For each of the topics addressed in Sections 3.3-3.6, we identify the key classification parameters and features based on the characteristics of the problems under study. In accordance with the proposed topic-specific taxonomy, the papers are characterized in tables and graphics, and their content is cross-analyzed and discussed.

Table 3.1: Previous review studies

Reference	Field		Focus	Scope	Covered period
	GP	OR			
Sbihi and Eglese [2007]		✓	GL	quantitative models dealing with GL issues	until 2007
Sarkis et al. [2011]		✓	GL	organizational theory of green supply chain management	until 2010
Dekker et al. [2013]		✓	CLSC	quantitative models for closed-loop supply chains	until 2003
Demir et al. [2014]		✓	GL	quantitative models for transportation	until 2013
Fahimnia et al. [2015]		✓	GL	bibliometric and network analysis of green supply chain management	until 2013
Govindan et al. [2015]		✓	RL, CLSC	content analysis of RL and CLSC	2007 - 2013
Marshall et al. [2015]		✓	SD	environmental and social pillars of SD in the supply chain management	until 2015
Singh et al. [2016]		✓	RL	RL in the apparel industry	until 2016
Kirchherr et al. [2017]	✓		CE	definitions and content analysis related to CE	until 2017
Barbosa-Póvoa et al. [2018]		✓	SD	OR methods to support SD in the supply chain management	until 2016
Kaur and Awasthi [2018]		✓	GL	challenges in green supply chain management	until 2015
Homrich et al. [2018]	✓		CE	definitions; semantic, bibliometric and content analysis related to CE	until 2017
Reike et al. [2018]	✓		CE	historical development of CE, resource value retention options	until 2017
Bektaş et al. [2019]		✓	GL	OR methods in green freight transportation	until 2018
Bibri [2019]	✓		SD	sustainability of smart cities in the era of big data	until 2018
Curtis and Lehner [2019]	✓		SD	sharing economy for sustainability	until 2019
Kazemi et al. [2019]	✓		RL, CLSC	bibliometric and content analysis of CLSC	until 2017
Mathiyazhagan et al. [2019]	✓		GL	environmental pillar in sustainable manufacturing	2002 - 2017
Moreno-Camacho et al. [2019]	✓		SD	sustainability metrics of the supply chain network design	2015 - 2018
Nascimento et al. [2019]		✓	CE	industry 4.0 technologies for CE in manufacturing	until 2018
Nenni et al. [2019]		✓	SD	OR methods for sustainable urban freight transportation	2009 - 2018
Pinheiro et al. [2019]		✓	CE	new product design and development with CE	until 2018
Sauerwein et al. [2019]	✓		CE	additive manufacturing for the product design in CE	until 2019
Thies et al. [2019]		✓	SD	OR methods for sustainability assessment of products	until 2018
Waltho et al. [2019]		✓	GL	carbon emissions and environmental policies in the supply chain design	2010 - 2017
This literature review		✓	CE	CE in production planning	until 2019

GP: General-Purpose, OR: Operations Research, GL: Green Logistics, RL: Reverse Logistics, CLSC: Closed-Loop Supply Chain, SD: Sustainable Development, CE: Circular Economy

3.2 Material collection

This literature review covers 160 papers, which reveal together the circular economy challenges and opportunities in mid-term production planning. Relevant material collection has been performed in several steps:

- In accordance with the purpose and scope of this review, we defined two sets of keywords, namely:
  - *Production planning:* Besides the keyword *production planning*, this set includes also the keyword *lot-sizing*. Note that the production planning aiming at determining the size of production lots and the time of production, is known in the literature as the *lot-sizing* problem.
  - *Circular economy:* This set includes the terms corresponding to all industrial CE-oriented processes illustrated in Figure 2.3: *recycling*, *disassembly*, *remanufacturing*, *refurbishment*, *repair*, *reuse*, (*by-product* or *byproduct*), (*co-production*, *co-product* or *coproduct*). In addition, 4 keyword strings have been considered to capture the sustainable property of CE: (*sustainability* or *sustainable*), *zero waste*, (*human* or *social* or *ergonomics*), (*carbon emissions* or *greenhouse gas emissions* or *energy*).
- All of 16 possible combinations of keywords from the first and second sets have been applied to query Scopus, one of the largest online databases of peer-reviewed literature. Interestingly, no documents were found in Scopus by using the string of keywords: [*zero waste* and (*production planning* or *lot sizing*)]. We only considered journal articles published or "in press" and book chapters, without time limit.
- In the next stage, we proceeded to a backward reference search, by checking the complete reference lists of all previously retrieved papers to identify other relevant articles that cited them.
- After having removed articles deemed out of scope and duplicates, we reached a total of 103 papers distributed per topic as follows: 29 papers on *disassembly for recycling*, 50 papers on *recovery options like remanufacturing and refurbishment*, 10 papers on *by-products* and/or *co-production*, 14 papers about *greenhouse gas emissions* and *carbon emissions*, and 7 papers on *energy*.
- In order to suitably contextualize the findings derived from the collection of 103 papers in the field of operations research, we added 56 transdisciplinary general-purpose and review papers.
- A particular attention has been paid to industrial applications found in the reviewed articles. To feed into the discussions on the CE development, the industrial dimension of this state-of-the-art review has been strengthened by surveying: **(i)** the library of industrial symbiosis case studies proposed by Evans et al. [2017], **(ii)** the research and innovation projects relevant to the circular economy strategy funded by the European Commission[11].

3.3 Disassembly for recycling

The first crucial step in most processing operations of end-of-life/use products is disassembly. Allowing a selective retrieving of desired parts or components, it truly belongs to the area of environmentally conscious manufacturing and product recovery [Ilgin and Gupta, 2010]. Disassembly appears in different recycling options (from product to raw material recycling), which results in planning problems with particular specifications.

From an engineering point of view, *disassembly* can be defined as a systematic and selective process of separating an item into components, subassemblies or other groupings [Ilgin and Gupta, 2010]. Within the realm of operations management, quantitative disassembly problems can be classified into four generic types of problems:

- **Disassembly-to-order** (also called *disassembly leveling*): Determine the lot size of a mix of different types of end-of-life/use products to be disassembled for satisfying the demand of parts or components (see e.g. Kim et al. [2009, 2018]). Two optimization criteria are mainly considered in the literature, either minimizing the number of products to be disassembled, or the sum of costs related to the disassembly process. End-of-life/use products can have parts in common. The *parts commonality* means that products or subassemblies share their parts or components.

- **Disassembly lot-sizing** (also called *disassembly scheduling*): For a given disassembly structure, schedule the quantity of disassembling end-of-life/use products and their components in each period of a planning horizon in order to meet the demand of their parts or components (see e.g. Barba-Gutiérrez et al. [2008], Kim et al. [2009]). The considered optimization criterion seeks commonly to minimize the sum of a combination of costs: setup, penalty, overload, lost sales and inventory holding. Note that disassembly scheduling includes a timing of disassembling, unlike disassembly-to-order.

- **Disassembly sequencing:** Find the best order of disassembly operations, while optimizing the costs related to the life-cycle of the end-of-life/use products (see e.g. Han et al. [2013a], Lambert [2003]).

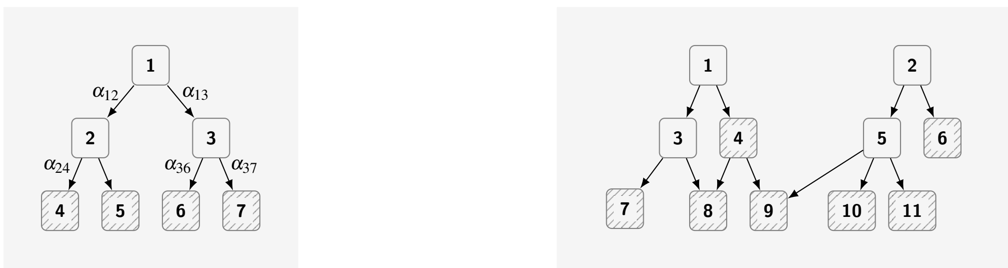
- **Disassembly line balancing:** Assign disassembly tasks to qualified workstations, while respecting the precedence relations. The objective usually aims at minimizing the number of workstations, the idle time of workstations, the cycle time, etc. or a combination of these parameters (see e.g. Kalaycılar et al. [2016]).

For comprehensive reviews on disassembly systems in their broad application, the interested reader is referred to Lambert [2003], Kim et al. [2007], Ilgin and Gupta [2010].

3.3.1 Mathematical formulation

Two classes of problems revolve around production planning in disassembly systems, namely disassembly-to-order and disassembly lot-sizing. Table 3.2 analyzes the identified papers focusing particularly on the class of disassembly scheduling problems, which can allow:

Distinct structure and number of product types. Both cases of single and multiple product types have been addressed in the literature (see Table 3.2). Note that it is not so much the number of product types, but rather their structure which increases the problem complexity. Two product structures can be distinguished: **(i)** *assembly type*, where each child item has at most one parent, i.e. a given product type does not allow parts commonality (see Figure 3.1a), and **(ii)** *general type*, otherwise (see Figure 3.1b).



(a) Assembly structure (b) General structure

Product part Product part with demand

Figure 3.1: Disassembly Bill Of Materials (d-BOM), where α_{ij} is the quantity of part j obtained from one unit of its parent i, $i \in \{1,2,3\}$, $j \in \{2,3,\ldots,11\}$.

Table 3.2: Disassembly lot-sizing

Reference		Partial disassembly	Multiple products	Common parts	Multi-level demand	Cap	Type		Resolution			Instance		Application area
							D	ND	Ex	App	Sol	R/B	I	
Gupta and Taleb [1994]	Reviewed in Kim et al. [2007]	✓					✓		✓			✓		G
Taleb and Gupta [1997]		✓	✓	✓			✓			✓		✓		G
Taleb et al. [1997]		✓		✓			✓			✓		✓		G
Neuendorf et al. [2001]		✓		✓			✓			✓		✓		G
Lee et al. [2002]		✓				✓	✓				✓		✓	inkjet printers
Kim et al. [2003]		✓	✓	✓			✓			✓		✓		G
Lee and Xirouchakis [2004]		✓		✓			✓			✓		✓		G
Lee et al. [2004]		✓	✓	✓			✓				✓	✓		G
Kim et al. [2006a]		✓				✓	✓			✓		✓		G
Kim et al. [2006b]		✓	✓	✓			✓			✓		✓		G
Kim et al. [2006c]		✓				✓	✓			✓		✓		G
Langella [2007]		✓	✓	✓			✓			✓		✓		G
Qu and Williams [2008]			✓			✓	✓				✓		✓	automotive
Barba-Gutiérrez and Adenso-Díaz [2009]		✓						✓		✓		✓		G
Kim et al. [2009]		✓					✓		✓			✓		G
Xanthopoulos and Iakovou [2009]			✓	✓		✓	✓				✓	✓		electric appliance
Kim and Xirouchakis [2010]			✓			✓		✓		✓		✓		G
Ahn et al. [2011]							✓			✓		✓		G
Prakash et al. [2012]		✓		✓			✓			✓		✓		G
Han et al. [2013b]						✓	✓				✓	✓		G
Wang and Huang [2013]		✓	✓	✓		✓		✓			✓	✓		G
Sung and Jeong [2014]		✓				✓	✓			✓		✓		G
Ji et al. [2016]			✓	✓		✓	✓			✓			✓	valve factory
Fang et al. [2017a]			✓	✓		✓		✓		✓			✓	iron and stell
Habibi et al. [2017]						✓	✓				✓	✓		G
Kim et al. [2018]		✓	✓	✓	✓		✓			✓		✓		G
Liu and Zhang [2018]						✓		✓		✓			✓	valve factory
Tian and Zhang [2019]			✓			✓	✓			✓		✓		G

Cap: Capacitated, D: Deterministic, ND: Nondeterministic, Ex: Exact, App: Approximate, Sol: Solver, R/B: Random or Benchmark, I: Industrial, G: Generic

With or without parts commonality. Owing to interdependencies among different parts or components of end-of-life/use products, disassembly lot-sizing problems with parts commonality become more complex. Both versions with or without parts commonality have been addressed in the literature for a single or multiple product types (see Table 3.2).

Multi-level demand. Relatively little research addresses problems which integrate both disassembly-to-order and scheduling decisions [Kang et al., 2012, Kim et al., 2018]. These two interrelated problems are separately treated in the literature.

Partial or complete disassembly. No information about parent-child matching between items is required in complete disassembly setting, the root-leaf relationship being sufficient (see e.g. Kim et al. [2007], Habibi et al. [2017], Liu and Zhang [2018]). Against the complete disassembly planning, partial disassembly setting involves mainly two questions: **(i)** To what depth the products have to be disassembled in each period of time horizon? and **(ii)** In the case with parts commonality, which disassembly sequence has to be performed?

Capacitated or uncapacitated. Similar to production planning problems in assembly systems, the resource capacity constraint is an important consideration due to its industrial soundness. As Table 3.2 witnesses, both uncapacitated and capacitated disassembly lot-sizing problems are treated in the recent literature.

Consider a given disassembly Bill Of Materials (d-BOM) with an assembly structure as illustrated in Figure 3.1a. All items are numbered level by level: $1, 2, \dots, \ell, \ell+1, \dots, N$, where 1 represents the root index and ℓ is the index of the first leaf item. All indices greater or equal to ℓ correspond to leaf parts. The disassembly of one unit of parent i results in α_{ij} units of part j. Denote in parentheses (i) the parent of a part i.

For this basic d-BOM, Kim et al. [2007] formalized a generic version of the disassembly lot-sizing problem which aims to determine the disassembly quantity and timing X_{it} of all parents i $(\forall i < \ell)$ in order to meet the demand of leaf parts d_j $(\forall j \geq \ell)$ over a planning time horizon $\{1, 2, \dots, T\}$. Let the objective function be cost-based and include two costs unrelated to disassembly timing. A fixed setup cost f_i is required if any disassembly operation of part $i < \ell$ is performed in period t. This condition is verified via the indicator variables Y_{it}, $\forall t \in [\![1, T]\!], \forall i \in [\![1, \ell-1]\!]$.

Table 3.3: Disassembly for recycling: *Notations*

Indexes and parameters:	
T	Index of the last time period
N	Index of the last item
ℓ	Index of the first leaf item
(i)	Parent of item $i > 1$
α_{ij}	Proportion of item j obtained after disassembly of one unit of parent i
f_i	Fixed setup cost for the disassembly of the parent item $i < \ell$ in each period
h_i	Unit holding cost of the item $i > 1$ in each period
d_{it}	Demand of the leaf item $i \geq \ell$ in period t
M	Arbitrary large number
Decision variables:	
X_{it}	Disassembly quantity of the parent item $i < \ell$ in period t
Y_{it}	Binary setup indicator for disassembly of the parent item $i < \ell$ in period t
I_{it}	Inventory level of the item $i > 1$ at the end of period t

In order to satisfy the demand of leaf-items, partial disassembly is allowed during the planning horizon. An inventory holding cost h_i is thus incurred, when I_{it} parts of type i are stored from period t to period $t+1$ to meet future demands, $\forall t \in [\![1,T]\!], \forall i \in [\![2,N]\!]$. The available quantity of the root-item is supposed unlimited. The parameters and decision variables are given in Table 3.3. A generic version of the disassembly lot-sizing problem is given below:

$$\text{minimize} \sum_{t=1}^{T} \left[\sum_{i=1}^{\ell-1} f_i Y_{it} + \sum_{i=2}^{N} h_i I_{it} \right] \tag{3.1}$$

subject to:

$$I_{i,t-1} + \alpha_{(i),i} X_{(i),t} = I_{it} + d_{it} \qquad \forall t \in [\![1,T]\!], \forall i \in [\![\ell,N]\!] \tag{3.2}$$

$$I_{i,t-1} + \alpha_{(i),i} X_{(i),t} = I_{it} + X_{it} \qquad \forall t \in [\![1,T]\!], \forall i \in [\![2,\ell-1]\!] \tag{3.3}$$

$$I_{i,0} = 0 \qquad \forall i \in [\![2,N]\!] \tag{3.4}$$

$$X_{it} \leq M Y_{it} \qquad \forall t \in [\![1,T]\!], \forall i \in [\![1,\ell-1]\!] \tag{3.5}$$

$$I_{it} \geq 0 \qquad \forall t \in [\![1,T]\!], \forall i \in [\![2,N]\!] \tag{3.6}$$

$$X_{it} \geq 0 \text{ and integer} \qquad \forall t \in [\![1,T]\!], \forall i \in [\![1,\ell-1]\!] \tag{3.7}$$

$$Y_{it} \in \{0,1\} \qquad \forall t \in [\![1,T]\!], \forall i \in [\![1,\ell-1]\!] \tag{3.8}$$

The set of equalities (3.2)-(3.3) expresses the flow conservation constraints. As constraints (3.4) specify, the initial inventory level of each part is null. Constraints (3.5) involve a setup cost in each period if any disassembly operation is realized in that period. The definition domains of all variables are stated in constraints (3.6)-(3.8). Note that besides the cost-based objective function (3.1), another optimization criterion considered in the literature seeks to minimize the number of products to be disassembled i.e. $\sum_{i=1}^{T} X_{1i}$ (see e.g. Gupta and Taleb [1994], Taleb et al. [1997]). Even if rare, of importance to mention is the explicit consideration in the mathematical models of CE issues, other than those related to the disassembly process per se. For example, the legislative and environmental requirements, imposed by the Directive 2002/96/EC, appear in the constraints of a mathematical model proposed by Xanthopoulos and Iakovou [2009] for a case study encountered in a manufacturing enterprise of electric heating appliances.

3.3.2 Complexity and solution approaches

As much emphasized in the literature, disassembly planning cannot be assimilated as a reverse production planning problem. By design, the assembly process converges to a single demand source (final product), while the disassembly process diverges to multiple demand sources (parts or components). Due to the divergent disassembly structure, the complexity of related problems grows drastically with the number of product types to be disassembled [Prakash et al., 2012]. Note that the well-known zero-inventory property [Wagner and Whitin, 1958] does not hold in the case of disassembly scheduling, and the classical lot-sizing algorithms cannot be directly applied to solve the disassembly scheduling problem [Kim et al., 2007].

Kim et al. [2009] proved that the uncapacitated disassembly lot-sizing problem with assembly product type (3.1)-(3.8) is NP-hard. Together with this complexity result, Kim et al. [2009] are the only authors who propose an exact branch-and-bound approach for the cost-based disassembly lot-sizing problem. Before this work, Gupta and Taleb [1994] developed an exact algorithm based on reverse materials requirement planning (MRP), which minimizes the number of disassembled products.

Due to the combinatorial nature of decisions involved by partial disassembly or parts commonality, various heuristic solution methods have been developed to tackle the different extensions of the basic problem (3.1)-(3.8) pointed out in Table 3.2, namely: partial disassembly, bounded capacity, multi-level demand and parts commonality. Among these methods, general and special-purpose heuristic approaches can be found in the literature: hybridized MIP combined with Lagrangian relaxation [Ji et al., 2016], constructive heuristics [Barba-Gutiérrez et al., 2008, Sung and Jeong, 2014, Kim et al., 2018], metaheuristics [Prakash et al., 2012, Tian and Zhang, 2019].

Facing uncertainties, little but varied approaches can be found in the literature to deal with them: chance-constrained programming [Liu and Zhang, 2018], stochastic programming without recourse [Kim and Xirouchakis, 2010] and fuzzy reverse MRP [Barba-Gutiérrez and Adenso-Díaz, 2009].

3.3.3 Industrial implications and discussion

With respect to disassembly scheduling, Kim et al. [2007] provided a literature review on the generic problem (3.1)-(3.8) and its generalizations. Based on this prior work, let us review the advances achieved since then, in terms of the research directions identified by Kim et al. [2007]:

- *Problem extensions:* Among all problem extensions suggested by Kim et al. [2007], only the capacitated problem with general product structure and complete disassembly has been addressed in the literature by Ji et al. [2016]. Note that the following problems remain still open: capacitated problem with general product structure and partial disassembly, problems with setup time, problems with storage capacity, problems with defective returns and problems with backlogging.

- *Consideration of non-deterministic parameters:* Since the review of Kim et al. [2007], several studies dealt with nondeterministic demand and yield of the reusable product parts [Barba-Gutiérrez and Adenso-Díaz, 2009, Kim and Xirouchakis, 2010, Liu and Zhang, 2018]. Two formats of uncertain data representations have been used: fuzzy sets [Barba-Gutiérrez and Adenso-Díaz, 2009] and probability distributions [Kim and Xirouchakis, 2010, Liu and Zhang, 2018]. Further research efforts deserve to be devoted to handling related uncertainties, e.g.: product quality, quantity of defective parts, setup time. Given the high prevalence of uncertainty in upstream and downstream flows of the disassembly operations, it could be interesting to supply the planning decisions with valuable knowledge derived by data mining of some available raw data.

- *Embedding of other related decisions:* Without a special established taxonomy, there also exist generalized disassembly problems, which integrate various disassembly aspects, such as routing and lot-sizing [Habibi et al., 2017], lot-sizing and pricing [Tian and Zhang, 2019, Qu and Williams, 2008], leveling and scheduling [Kang et al., 2012].

- *Integration with other activities in recovery systems:* Roughly speaking, a typical remanufacturing system covers three stages: disassembly, reprocessing, and reassembly. The process interconnections and information flows between these core stages make recovery systems complex to be managed as a whole, and pose multiple research problems, earlier identified by Guide Jr [2000]. From the 2000s onward, several research studies have been undertaken, e.g.: Ahn et al. [2011] studied a three-stage lot-sizing model integrating disassembly, reprocessing and reassembly processes; Hashemi et al. [2014] addressed an integrated system of manufacturing and remanufacturing, including disassembly for the aerospace industry. As also underscored in Section 3.4, additional efforts remain to be carried out to harmonize the coordination between disassembly release with reassembly, while sustainably closing the products use cycle.

From an industrial perspective, several real-life applications have been discussed in the literature (see Table 3.2). These case studies notably underline the importance and the relevance to combine optimization approaches with sensitivity and/or what-if analysis for supporting the decision-making process in complex industrial frameworks [Qu and Williams, 2008, Ji et al., 2016, Fang et al., 2017a, Liu and Zhang, 2018].

3.4 From product to raw material recycling

In this section, let us discuss the production planning systems including the following recycling operations: **(i)** the conversion of worn-out goods into new or as good as new ones, and **(ii)** the flow back of material obtained during disassembly into production as valuable material. These recovery operations fall within the concept of *remanufacturing*. Various terms confused with remanufacturing can be enumerated, such as restoring, reconditioning, repurposing, refurbishment. No clear-cut definitions and distinctions between these recovery options exist in the literature. One thing is certain, *remanufacturing* becomes a standard term for an industrial recovery process of returned products, which requires several processing operations including often the disassembly operation.

Manufacturing and remanufacturing are two alternative and competing production ways, which share the same industrial environment and often lead to the same serviceable products. Accordingly, production planning systems for remanufacturing raise new questions for production and inventory management, the well-posedness of which heavily depends on the systems settings. In the production planning literature, the classic lot-sizing problem has been extended with a remanufacturing option under different settings with or without final disposal options, as depicted in Figure 3.2. A significant part of the identified articles operates on production systems, where manufactured and remanufactured products are identical, and assimilated as *serviceable products* (see e.g. Teunter et al. [2006]). Another part distinguishes the newly produced from remanufactured products in customer demand (see e.g. Fang et al. [2017b], Baki et al. [2014]).

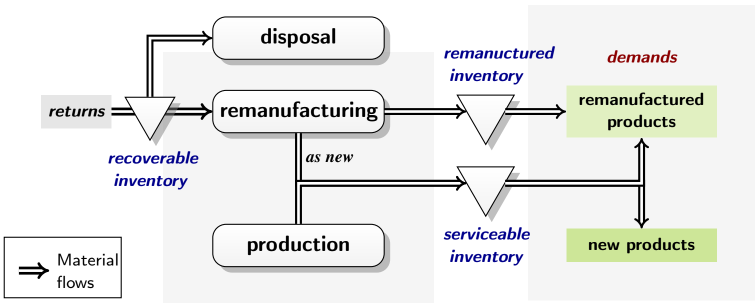


Figure 3.2: Production system including remanufacturing

3.4.1 Mathematical formulation

In seeking to better define industrial contexts, the academic community investigated different variants of the lot-sizing problem with remanufacturing options (LSR). Apart from the classical capacitated and uncapacitated cases of the lot-sizing problem, let us review the main remanufacturing-oriented characteristics of this problem, which tend to define a nomenclature within the scientific community:

Joint or separated setups. Both configurations have been studied in the literature (see e.g. Sahling [2013]), namely when:

- Manufacturing and remanufacturing are performed in two separate processes, each having its own setup costs. This problem is commonly called *lot-sizing with remanufacturing and separate setups*.
- Manufacturing and remanufacturing share the same production routes and have one joint setup cost. Defined on this assumption, the problem is known as *lot-sizing with remanufacturing and joint setups*.

As Tables 3.4-3.5 clearly show, academic problem definitions favor production configurations with separated setup and tend thereby to be close to real-life settings.

Stationary or time-dependent parameters. Special cases of LSR with stationary parameters have been not neglected and a number of useful analytical results have been derived for them. For example, Teunter et al. [2006] proposed a polynomial-time dynamic programming algorithm for the LSR with joint setups and stationary costs.

Table 3.4: Lot-sizing with remanufacturing options

Reference	Separated demands	Time-dep parameters	Setups		Cap	Type		Resolution			Instance		Application area
			Joint	Separated		D	ND	Ex	App	Sol	R/B	I	
Richter and Sombrutzki [2000]		✓		✓		✓		✓	✓		✓		G
Golany et al. [2001]		✓				✓		✓			-	-	G
Richter and Weber [2001]		✓		✓		✓		-	-	-	-	-	G
Yang et al. [2005]		✓				✓		✓	✓		✓		G
Jayaraman [2006]		✓			✓	✓		✓				✓	cellular telephones
Li et al. [2006]		✓		✓		✓		✓	✓		✓		G
Teunter et al. [2006]			✓	✓		✓		✓	✓		✓		G
Li et al. [2007]		✓		✓	✓	✓			✓		✓		G
Li et al. [2009]							✓	✓			✓		G
Pan et al. [2009]		✓			✓	✓		✓			✓		G
Piñeyro and Viera [2009]		✓		✓		✓			✓		✓		G
Xanthopoulos and Iakovou [2009]	✓			✓	✓	✓				✓	✓		G
Denizel et al. [2010]		✓			✓		✓		✓		✓		G
Piñeyro and Viera [2010]	✓	✓		✓		✓			✓		✓		G
Ahn et al. [2011]		✓		✓		✓			✓		✓		G
Schulz [2011]				✓		✓			✓		✓		G
Zhang et al. [2011]		✓		✓	✓	✓			✓			✓	steel
Tao et al. [2012]							✓		✓		✓		G
Zhang et al. [2012b]	✓	✓		✓	✓	✓			✓		✓		G
Han et al. [2013b]	✓	✓			✓	✓				✓	✓		G
Naeem et al. [2013]				✓	✓	✓	✓	✓			✓		G
Sahling [2013]			✓	✓	✓	✓				✓	✓		G
Wang and Huang [2013]				✓	✓		✓			✓	✓		G

Time-dep: Time-dependent, Cap: Capacitated, D: Deterministic, ND: Nondeterministic, Ex: Exact, App: Approximate, Sol: Solver, R/B: Random or Benchmark, I: Industrial, G: Generic, -: None

Table 3.5: Lot-sizing with remanufacturing options

Reference	Separated demands	Time-dep parameters	Setups		Cap	Type		Resolution			Instance		Application area
			Joint	Separated		D	ND	Ex	App	Sol	R/B	I	
Baki et al. [2014]				✓		✓			✓		✓		G
Cai et al. [2014]							✓			✓	✓		G
Chen and Abrishami [2014]	✓	✓		✓		✓			✓		✓		G
Hashemi et al. [2014]				✓	✓	✓				✓	✓		aerospace
Li et al. [2014]				✓		✓			✓		✓		G
Mehdizadeh and Fatehi Kivi [2014]		✓		✓	✓	✓			✓		✓		G
Retel Helmrich et al. [2014]		✓	✓	✓		✓				✓	✓		G
Parsopoulos et al. [2015]				✓		✓			✓		✓		G
Piñeyro and Viera [2015]				✓		✓			✓		✓		G
Sifaleras and Konstantaras [2015]				✓		✓			✓		✓		G
Sifaleras et al. [2015]				✓		✓			✓		✓		G
Cunha and Melo [2016]		✓	✓	✓		✓				✓	✓		G
Hilger et al. [2016]				✓	✓		✓		✓		✓		G
Macedo et al. [2016]		✓	✓	✓		✓	✓			✓	✓		G
Cunha et al. [2017]		✓		✓		✓				✓	✓		G
Fang et al. [2017a]	✓	✓		✓			✓		✓			✓	iron and steel
Giglio et al. [2017]		✓		✓	✓	✓			✓		✓		G
Sifaleras and Konstantaras [2017]				✓		✓			✓		✓		G
Ali et al. [2018]		✓		✓		✓				✓	✓		G
Kilic et al. [2018]				✓			✓		✓		✓		G
Koken et al. [2018a]	✓				✓	✓			✓		✓		G
Koken et al. [2018b]	✓				✓	✓			✓		✓		G
Piñeyro and Viera [2018]				✓		✓			✓		✓		G
Zouadi et al. [2018]				✓		✓			✓		✓		G
Cunha et al. [2019]		✓		✓	✓	✓			✓		✓		G
Kilic and Tunc [2019]				✓			✓		✓		✓		G
Kilic and van den Heuvel [2019]		✓		✓		✓			✓		✓		G

Time-dep: Time-dependent, Cap: Capacitated, D: Deterministic, ND: Nondeterministic, Ex: Exact, App: Approximate, Sol: Solver, R/B: Random or Benchmark, I: Industrial, G: Generic

Inventory management. As illustrated in Figure 3.2, the integration of products returns and remanufacturing-related goods into production environment affects the traditional inventory management [Ilgin and Gupta, 2010]. In this respect, decisions related to the recoverable (of products return), serviceable (of identical manufactured and remanufactured products) and remanufactured inventories are inherent to LSR problems for a suitable coordination between the regular policies of procurement and remanufacturing.

With or without products substitution. In contrast to the classical lot-sizing problem, one of the main specificities of LSR lies on the demand, that can be fulfilled from a single stream of serviceable products or be fitted into two categories of newly produced and remanufactured ones. Even if no distinction is commonly made between manufactured and remanufactured products, there exist some studies that considered the market divided into new and remanufactured segments [Koken et al., 2018a,b, Zhang et al., 2011, Chen and Abrishami, 2014]. To go further, Piñeyro and Viera [2010] allowed the substitution of remanufactured products by the new ones for absorbing the fluctuations in the quantity of product returns.

The general form of the lot-sizing problem with remanufacturing, time-dependent parameters and separated setups can be formally defined as done in [Retel Helmrich et al., 2014]. Let the planning horizon be spread over T periods. Denote by d_t the demand for serviceable products and r_t the quantity of returns, $\forall t \in [\![1,T]\!]$. The related industrial process involves the following costs: unit production costs for manufacturing (remanufacturing) p_t ($\hat{p}_t$), setup costs for manufacturing (remanufacturing) f_t ($\hat{f}_t$), unit holding costs for serviceable products h_t and returns $\hat{h}_t$, $\forall t \in [\![1,T]\!]$.

Table 3.6: From product to raw material recycling: *Notations*

Indexes and parameters:	
T	Number of time periods
p_t	Unit production cost for manufacturing in period t
$\hat{p}_t$	Unit production cost for remanufacturing in period t
f_t	Fixed setup cost for manufacturing in period t
$\hat{f}_t$	Fixed setup cost for remanufacturing in period t
h_t	Unit holding cost for serviceable products in period t
$\hat{h}_t$	Unit holding cost for returns in period t
d_t	Demand for serviceable products in period t
r_t	Quantity of returns in period t
Decision variables:	
X_t	Production quantity for manufacturing in period t
$\hat{X}_t$	Production quantity for remanufacturing in period t
Y_t	Binary setup indicator of manufacturing in period t
$\hat{Y}_t$	Binary setup indicator of remanufacturing in period t
I_t	Inventory level of serviceable products in period t
$\hat{I}_t$	Inventory level of returns in period t

Let X_t ($\hat{X}_t$) be the quantity of manufactured (remanufactured) products and Y_t be ($\hat{Y}_t$) the binary indicator for manufacturing (remanufacturing) in period $t \in [\![1,T]\!]$. Variables I_t and $\hat{I}_t$ are used to express the inventory levels of serviceable products and returns, respectively. Making use of the above notations summarized in Table 3.6, the LSR problem with time-dependent parameters and separated setups can be formulated as follows:

$$\text{minimize} \sum_{t=1}^{T} \left(p_t X_t + \hat{p}_t \hat{X}_t + f_t Y_t + \hat{f}_t \hat{Y}_t + h_t I_t + \hat{h}_t \hat{I}_t \right) \tag{3.9}$$

subject to:

$$\hat{I}_{t-1} + r_t = \hat{I}_t + \hat{X}_t \qquad \forall t \in [\![1,T]\!] \tag{3.10}$$

$$I_{t-1} + \hat{X}_t + X_t = I_t + d_t \qquad \forall t \in [\![1,T]\!] \tag{3.11}$$

$$I_0 = \hat{I}_0 = 0 \tag{3.12}$$

$$\hat{X}_t \leq \sum_{i=t}^{T} d_i \hat{Y}_t \qquad \forall t \in [\![1,T]\!] \tag{3.13}$$

$$X_t \leq \sum_{i=t}^{T} d_i Y_t \qquad \forall t \in [\![1,T]\!] \tag{3.14}$$

$$\hat{X}_t, X_t, \hat{I}_t, I_t \geq 0 \qquad \forall t \in [\![1,T]\!] \tag{3.15}$$

$$\hat{Y}_t, Y_t \in \{0,1\} \qquad \forall t \in [\![1,T]\!] \tag{3.16}$$

The objective function (3.9) minimizes the sum of production, setup and holding costs associated to manufacturing and remanufacturing processes. The sets of equalities (3.10)-(3.11) express the flow conservation constraints. Both serviceable and returns inventories are initialized via constraints (3.12). Constraints (3.13)-(3.14) track the manufacturing and remanufacturing setups. Binary and nonnegative requirements are imposed in constraints (3.15)-(3.16).

3.4.2 Complexity and solution approaches

The uncapacitated lot-sizing problem with remanufacturing, joint setups and stationary parameters is polynomial and can be solved by a dynamic programming algorithm running in $O(T^4)$ [Teunter et al., 2006]. Some special cases of the LSR problem with joint and separate setup costs preserve the zero-inventory property and can be solved in polynomial time [Golany et al., 2001, Richter and Sombrutzki, 2000, Richter and Weber, 2001].

The following uncapacitated LSR problems have been proven to be NP-hard: **(i)** LSR with separate setups for stationary cost parameters [Retel Helmrich et al., 2014], **(ii)** LSR with joint setups in general [Retel Helmrich et al., 2014], **(iii)** LSR with general and stationary concave-cost structures [Golany et al., 2001, Yang et al., 2005]. As the capacitated lot-sizing problem, the LSR problem with bounded capacity remains NP-hard. A number of interesting complexity results and useful properties for the capacitated LSR problem with different cost structures have been found by Pan et al. [2009].

Very few studies have been dedicated to developing exact methods to solve the difficult variants of the deterministic LSR problem. In this regard, let us mention the work of Li et al. [2006], who developed an exact dynamic programming algorithm for the LSR problem with separated setups and product substitution. Sahling [2013] combined the column generation with a truncated branch-and-bound method to obtain high-quality solutions. Ali et al. [2018] conducted a polyhedral analysis of the LSR with separated setups based on two reformulations and derived important properties related to their strength. On the flip side, an abundance of literature proposes heuristic solution methods such as: constructive [Baki et al., 2014], based on relaxations [Chen and Abrishami, 2014], adaptation of the part period balancing and heuristics [Zouadi et al., 2018]), metaheuristics (see e.g. Li et al. [2014], Koken et al. [2018a,b]).

Despite the predominance of deterministic models, several research efforts have been undertaken to cope with non-deterministic problem settings. Various sources of uncertainties are separately or jointly considered in: **(i)** setup costs, **(ii)** demands, **(iii)** quantity, quality or yield of products returns. To deal with them, classical stochastic programming paradigms have been used: approaches based on the expected value [Hilger et al., 2016, Denizel et al., 2010], two-stage programming [Macedo et al., 2016], stochastic dynamic programming [Naeem et al., 2013, Cai et al., 2014, Tao et al., 2012].

The competitiveness of heuristic solution approaches is commonly measured in an empirical way via numerical experiments. Instances reproducibility and performance comparability are thus key aspects for the field of operations research. In this respect, note that Sifaleras et al. [2015] and Sifaleras and Konstantaras [2017] created benchmark data for two lot-sizing problems: **(i)** multi-product case with remanufacturing (MDLSRP), and **(ii)** with product returns and recovery (ELSRP). These instances are available on `http://users.uom.gr/~sifalera/benchmarks.html` and described in [Sifaleras et al., 2015, Sifaleras and Konstantaras, 2017].

3.4.3 Industrial implications and discussion

The management of production planning with remanufacturing differs from management activities in traditional production systems in several aspects: **(i)** the integration of return flows, whose quantity, quality and timing are difficult to predict, **(ii)** the coordination between manufacturing and remanufacturing routes and multiple simultaneous inventory management and, **(iii)** the production streams down-flow. As appears from Tables 3.4-3.5, lot-sizing with remanufacturing is an increasingly active area of research since the 2000s.

Prior to this work, Guide Jr [2000] has identified multiple complex characteristics of remanufacturing. Considerable progress has been made since then, notably in terms of: **(i)** models/methods to aggregate production planning models that consider returned products and balance returns with demand, while managing the associated inventories (see Tables 3.4-3.5), **(ii)** models/methods to help in planning what parts and components to recover in disassembly (see Section 3.3), **(iii)** investigations related to traditional purchasing activities versus purchasing for remanufacturing [Cai et al., 2014]. Moreover, research efforts have been also spent to integrate into LSR problems, non-conventional but inherent aspects including: acquisition pricing [Cai et al., 2014], carbon emission constraints [Zouadi et al., 2018], supplier selection [Zouadi et al., 2018].

Several research topics on production planning with remanufacturing are open for further analysis and studies:

- There is an evident lack of exact solution methods for the LSR problem and its variants.

- As highlighted in Section 3.3, the coordination between disassembly, remanufacturing and reassembly deserves more theoretical and practical attention. The evaluation of different management policies between these strongly correlated industrial processes could help to identify opportunities to improve the overall performance of such integrated systems.

- From an academic point of view, production planning with remanufacturing has received a lot of interest. Meanwhile, a weak link with industrial applications may be perceived. Only a few real-life applications can be found in the literature, e.g. in the steel industry [Zhang et al., 2011, Fang et al., 2017a], aerospace manufacturing [Hashemi et al., 2014], or cellular telephone industry [Jayaraman, 2006]. The coming into effect of the European Directive Waste Electrical and Electronic Equipment[15] (WEEE) and other similar legislative instruments all around the world, has given birth to many companies specialized in recovery operations and the resale of WEEE, like ARC[16] (USA), Recommerce[17] (France), Refurb Phone[18] (UK). The proliferation of such companies makes us expect new innovative applications in the electrical and electronic industry, playing one of the leading roles in the (re-)manufacturing sector.

3.5 By-products versus co-products

A wide range of industrial production processes generates several products in a single production run with different quality levels, economic values, environmental impacts, and waste or non-waste statuses. This phenomenon is known in the literature as *co-production* and can be: **(i)** deliberated or non-deliberated; **(ii)** controlled or uncontrolled (see Table 3.7).

The depletion of scarce natural resources and the abundance of waste accumulation in landfills lead our and future generations to seek pathways for converting unavoidable production outputs into useful and high added-value products. Besides the technological feasibility, making industry processes less wasteful raises new legislative, economic, environmental and management questions.

Table 3.7: By-products versus co-products

Reference	Type of product(s) under study	Type		Resolution			Instance		Application area
		D	ND	Ex	App	Sol	R/B	I	
Bitran and Leong [1992]	"… multiple products produced simultaneously or products with *by-product*."		✓		✓		✓		G
Bitran and Gilbert [1994]	"… a *co-production* process is one in which a family of several different products is produced simultaneously."	✓			✓			✓	semiconductor
Bitran and Leong [1995]	"Units not meeting the specifications of a target product are commonly called *byproducts* [...] difficult to differentiate the main product from the byproducts since the products are all equally important."		✓		✓		✓		G
Spengler et al. [1997]	"the problem structure of *by-product* management [...] [includes] collection of valuable residues, handling and recycling of production residues, car and electronic scrap recycling …"	✓				✓		✓	iron and steel
Taşkın and Ünal [2009]	"This is how *co-production* is encountered in float glass production: non-controllable errors in process result in simultaneous production of several products."	✓				✓		✓	float glass
Lu and Qi [2011]	"… the production of some products will generate some other *by-products* that can also be sold, and the production quantity of each by-product is linearly proportional to the quantity of the main product."	✓		✓	✓		✓		G
Ağralı [2012]	"… products [co-products] that have to be produced simultaneously and producing one item of a product requires producing <u>exactly one</u> item of other products [co-products] [...] when a certain product is produced some <u>known percentage</u> of other products are also produced as *by-products* …"	✓		✓			✓		G
Santos and Almada-Lobo [2012]	"During the cooking of wood chips in the digester, two *by-products* are produced …"	✓			✓			✓	pulp and paper mill
Sridhar et al. [2014]	"… the production process creates a mixture of desirable products and undesirable *byproducts*."	✓			✓		✓		G
Rowshannahad et al. [2018]	"The used raw material (considered as a kind of *by-product*) …"	✓				✓		✓	semiconductor

D: Deterministic, ND: Nondeterministic, Ex: Exact, App: Approximate, Sol: Solver, R/B: Random or Benchmark, I: Industrial, G: Generic

Given the industrial specificities varying across sectors and continuously evolving production technology, no clear and universal distinction between non-waste and waste products can be drawn. Apart from the rigorous definition of waste, the texts of environmental and waste legislations use notions such as by-products or secondary material, while explaining their meaning in a more or less detailed manner. For example, the Circular Economy Promotion Law[4] (China) mentions the by-products without giving an explicit definition. Under the Resource Conservation and Recovery Act[2] (USA) , "*a by-product is a material that is not one of the primary products of a production process and is not solely or separately produced by the production process [...] The term does not include a co-product that is produced for the general public's use that is ordinarily used in the form in which it is produced by the process*". The Waste Framework Directive[1] (Europe) pays special attention to the specification of the term *by-product*. Notably, Article 3 of this directive specifies: "*A substance or object, resulting from a production process, the primary aim of which is not the production of that item, may be regarded as not being waste, [...] but as being a* ***by-product*** *only if the following conditions are met: (a) further use of the substance or object is certain; (b) the substance or object can be used directly without any further processing other than normal industrial practice; (c) the substance or object is produced as an integral part of a production process; and (d) further use is lawful...*".

In addition, the commission of the European communities elaborated an ad-hoc communication aiming to interpret and clarify the distinction between waste and by-products. Based on this document, let us propose the following definitions, illustrated by Figure 2.5, which conciliate all aforementioned legislative references:

Definition 2. *A* ***product*** *is all lawful material deliberately created in a production process. The term includes co-products.*

Definition 3. *A* ***production residue*** *is a material not deliberately produced in a production process, but may or may not be considered as waste.*

Definition 4. *A* ***by-product*** *is a production residue that is not a waste. By definition, by-products are lawful production outputs, whose further use is economically and environmentally sustainable.*

In order to deal efficiently with different outputs produced simultaneously, a number of studies have been conducted in both co-production systems and those with by-products (see Table 3.7). Given the legislative context, the cohabitation of different interpretations of the same words, by-product and co-product, is not surprising. Whatever the used term, let us focus on environmental-friendly and economic recovery of production residues.

3.5.1 Mathematical formulation

Besides the production planning models conceived to address specific industrial applications, several research efforts have been dedicated to proposing generic lot-sizing formulations for production systems with by-products. Except the model given by Sridhar et al. [2014], these generic models trigger production runs for satisfying demands of so-called by-products [Bitran and Leong, 1992, Ağralı, 2012, Lu and Qi, 2011]. Such a strong assumption is inconsistent with the widely accepted meaning of by-products, but can be encountered in production systems with co-products.

For clarifying the differences between co-products and by-products, let us discuss the basic version of the lot-sizing problem for production systems with co-products proposed and formulated by Ağralı [2012]. In the framework of a such system, a main product, indexed by 0, is generated together with K co-products at a known proportion α^k, $\forall k \in [\![1,K]\!]$. By definition, α^0 is equal to 1. Each product k has its own demand d_t^k to be satisfied in every period t. The production launched at a given period t entails a joint fixed setup cost f_t and a unitary production cost p_t^k per product k. Moreover, a holding cost h_t^k is incurred on each product k held in inventory at the end of a time period t. In this setting, the raised problem consists of determining when and how much to produce, over a planning horizon of T periods, while meeting all demands and minimizing the sum of the related costs.

For modeling purposes, denote by: **(i)** X_t: production quantity in a given period $t \in [\![1,T]\!]$, **(ii)** Y_t: binary indicator of production setup in period $t \in [\![1,T]\!]$, and **(iii)** I_t^k: inventory level of product k at the end of period t. The notations

Table 3.8: By-products versus co-products: *Notations*

Indexes and parameters:	
T	Number of time periods
$K+1$	Number of co-products
α^k	Production coefficient of co-product k
p_t^k	Unit production cost of co-product k in period t
f_t	Fixed setup cost in period t
h_t^k	Unit holding cost of co-product k in period t
d_t^k	Demand of co-product k in period t
Decision variables:	
X_t	Production quantity in period t
Y_t	Binary setup indicator for production in period t
I_t^k	Inventory level of co-product k in period t

are summarized in Table 3.8.

$$\text{minimize} \sum_{t=1}^{T} \left[f_t Y_t + \sum_{k=0}^{K} (p_t^k \alpha^k X_t + h_t^k I_t^k) \right] \tag{3.17}$$

subject to:

$$I_{t-1}^k + \alpha^k X_t = I_t^k + d_t^k \qquad \forall t \in [\![1,T]\!], \forall k \in [\![0,K]\!] \tag{3.18}$$

$$I_0^k = 0 \qquad \forall k \in [\![0,K]\!] \tag{3.19}$$

$$X_t \leq \max_{k \in [\![0,K]\!]} \left\{ \frac{d_t^k}{\alpha^k} \right\} Y_t \qquad \forall t \in [\![1,T]\!] \tag{3.20}$$

$$X_t \geq 0, I_t^k \geq 0 \qquad \forall t \in [\![1,T]\!], \forall k \in [\![0,K]\!] \tag{3.21}$$

$$Y_t \in \{0,1\} \qquad \forall t \in [\![1,T]\!] \tag{3.22}$$

The objective function (3.17) minimizes the sum of setup, production and inventory costs. Inventory balance constraints are ensured by equalities (3.18). The set of constraints (3.19) initializes the inventory levels of each product. Each production launch triggers a setup cost via constraints (3.20). Nonnegativity and binary requirements are expressed in constraints (3.21)-(3.22).

The mixed-integer program (3.17)-(3.22) lends itself well to co-production systems, when all production outputs are deliberately produced and their own demands are sufficient to trigger the production process. Given the set of constraints (3.20), this program does not encompass the downstream canalization of production residues. Note that the by-product management does not fall within classical co-production settings.

3.5.2 Complexity and solution approaches

Ağralı [2012] showed that the linear model (3.17)-(3.22) can be reduced to the single-item lot-sizing problem, for which a dynamic programming algorithm running in $O(T\log(T))$ exists. However, extensions of this basic problem can make it intractable and encourage the researchers to develop competitive heuristic approaches, as done for example in the problem case with lost sales [Lu and Qi, 2011].

By virtue of their ease of use and affordability, optimization software products are usually used to solve problems encountered in the industry world [Spengler et al., 1997, Taşkın and Ünal, 2009, Rowshannahad et al., 2018]. Often, existing solvers are not sufficient to face the complexity and the intractability of specific industrial features. A number of heuristic methods can be found in the literature for achieving industrial needs, whether to handle uncertainty [Bitran and Gilbert, 1994] or to deal with the curse of dimensionality of real-world instances [Santos and Almada-Lobo, 2012]. Inspired by issues stemming from industry, several research studies addressed production planning problems under real-life non-linear or non-deterministic features, by passing through a linear approximation step [Bitran and Leong, 1992, 1995, Sridhar et al., 2014].

3.5.3 Industrial implications and discussion

In spite of the confusion in the literature, let us distinguish the phenomenon of conversion of production residues into by-products from that of co-production. One similitude is sure, these two phenomena are typical for process industries: semiconductor fabrication [Bitran and Gilbert, 1994, Rowshannahad et al., 2018], metal processing [Spengler et al., 1997], pulp and paper industry [Santos and Almada-Lobo, 2012], glass manufacturing [Taşkın and Ünal, 2009]. Even if both of them, by-products and co-products, refer to the joint production of multiple outputs in one run, their management principles and goals in production planning are different.

Consistent with the scope of this review, focus on downstream management of lawful outputs, non-deliberately generated during a production process. The conversion of production residues into by-products forms part of the CE drivers. Commonly referred as *by-product synergy* (or *industrial symbiosis* in the industrial ecology literature), this practice lies on the use of a by-product stream produced by one process as input into another process as depicted in Figure 2.5 [Lee and Tongarlak, 2017, Lee, 2012]. In spite of its opportunistic character, by-product synergy is applicable in many industry sectors, ranging from agriculture to manufacturing. For examples of real-life applications, the reader is referred to the database of industrial symbiosis case studies constructed by Evans et al. [2017] within the framework of the European project MAESTRI[12] (under grant agreement No 680570), focused on the energy and resource management systems for improved efficiency in the process industries.

A number of challenges and gaps remain to be tackled:

- *Supporting the decision-making in real-life environments:* As we can infer from Table 3.7, there is still room for theoretical research on production planning with by-products: **(i)** to model generic problems and propose competitive solution methods; **(ii)** to take into account real-life features, such as irregular cost profiles [Rowshannahad et al., 2018], non-linear or non-deterministic production mixture of outputs [Sridhar et al., 2014, Bitran and Gilbert, 1994], uncertain by-product demands [Lee and Tongarlak, 2017]; **(iii)** to explore the implications on production planning of all aspects related to the by-product synergy, namely economic, managerial and environmental [Lee and Tongarlak, 2017].

- *Identifying and understanding synergy mechanisms*: Given the opportunistic use of production residues, the joint production settings strongly depend on the factors that determine the by-product generation. For example, in manufacturing and service environments, production residues are generated as a result of physical characteristics of the production processes, hence: **(i)** the production capacities are usually known; **(ii)** and the generated quantities of main products and by-products can be estimated, since they are correlated. In retail context, the by-product quantity depends on unsatisfied demands of the main products, which are generally unknown, but can be probabilistically expressed [Lee and Tongarlak, 2017]. To sum up, the formalization of industrial evidences and field knowledge plays a key role in supporting, optimizing and facilitating the by-product synergy.

- *Studying and analyzing different policies of joint production:* To give an order of idea, let us consider the eloquent case of the Danish Kalundborg symbiosis[19], which includes more than twelve different entities, linked by tens of heterogeneous flows: energy (e.g. steam, warm condensate), water (e.g. deionized water, used cooling water), and material (e.g. gypsum, biomass). Some of these flows have to be processed immediately being transported by pipeline (e.g. steam, waste water), others can be stored (e.g. gypsum, fly ash) within the limits of the inventory capacity. The coordination between different production processes raises many economic and operations management questions: How to ensure the synchronization between their production activities?

How to share the benefits and costs of a such complementary relationship? How to make the by-product synergy robust for all involved production processes and mitigate their interdependence? How to setup self-sufficient industrial parks in a sustainable manner?, etc.

3.6 Greenhouse gas emissions and energy consumption

3.6.1 Greenhouse gas emissions

Global warming is currently a hot research topic and raises major political, economic, as well as, social concerns. Greenhouse gas (GHG) emissions in the atmosphere have been identified as one of the main contributors to global warming. In order to tackle the issues caused by these gases, a number of climate-oriented action plans have been implemented by governments around the world, e.g.: **(i)** the *Kyoto Protocol*[20] is an international treaty adopted in 1997, which extends the *United Nations Framework Convention on Climate Change*[21] (UNFCCC), that commits states parties to reduce greenhouse gas emissions. As of May 2013, 191 countries and the European Economic Community have ratified the agreement (Canada withdrew in 2012); **(ii)** the *Paris Climate Agreement*, another initiative under the UNFCCC, was ratified in 2016 by 174 out of 197 countries. It aims to maintain the global temperature below 2° Celsius above pre-industrial levels and to limit its increase to 1.5° Celsius[22].

Until not long ago, researchers start to take a serious interest in reducing greenhouse gases, by integrating the carbon emissions generated during the production, transportation and remanufacturing operations in their models. The standard way of measuring carbon footprints is to consider *carbon dioxide equivalent* (CO_2e). The idea behind this measure is to express the impact of each greenhouse gas in terms of the quantity of CO_2 that would create the same global warming potential. That way, the carbon footprint represents the impact of all greenhouse gases using the same metric. In the following, we use the terms *carbon emissions* and *GHG emissions* to consider emissions of all greenhouse gases.

For achieving the aforementioned reduction targets, governments established and deployed various regulation policies [Hong et al., 2016]. As far as the reduction of GHG emissions is concerned, the most common policies are the following:

- *Emissions threshold:* A threshold-based regulatory policy imposes a maximum quantity of carbon emissions, that cannot be violated [Fahimnia et al., 2013, Benjaafar et al., 2013]. The key disadvantage of this policy is the lack of incentives to reduce emissions beyond the required cap of free emissions. Companies can decide to reach the imposed threshold even if they could meet their needs by emitting less greenhouse gas.

- *Carbon pricing:* This regulatory tool taxes and penalizes GHG emissions according to their quantity [Benjaafar et al., 2013, Zakeri et al., 2015]. Aiming at reducing pollution and encouraging more environmentally conscious production processes, this policy presents a number of drawbacks: induce expensive administration costs, stimulate shift production to countries without a such a tax, promote covert operations, etc.

- *Carbon/Emissions trading* is a market-based instrument to reduce GHG. This form of regulation represents a trade-off between the previous two discussed policies and refers to: **(i)** the limitation of the quantity of GHG emissions over a specific time horizon, and **(ii)** the firms granting with so-called *permits* to emit a given quantity of GHG. In thus emerged carbon markets, GHG emissions are traded under *cap-and-trade* schemes or with permits that pay for or offset GHG reductions [Purohit et al., 2016, Benjaafar et al., 2013, Zakeri et al., 2015, Kantas et al., 2015]. Among the numerous regulatory benefits exhibited by these trading systems, they are subject to heavy criticism: difficulty in standardizing the maximum threshold, volatility in emissions allowance prices, speculation in carbon markets, etc.

- *Carbon offsets* policy is a more intricate extension of the emission trading system, enabling companies to invest in so-called *offsets*. A carbon offset is a credit for GHG reductions obtained by one party, that can be purchased and used to compensate the emissions of another party [Benjaafar et al., 2013].

Note that climate-warming emissions arisen from business and government operations are quantified and managed via global standardized frameworks, such as Greenhouse Gas Protocol[23], EcoTransIT World[24], etc.

Given the topicality and the importance of the GHG reduction, there is an extensive literature related to carbon emissions in many research fields. At operational management level, carbon emission concerns are increasingly considered within the framework of various applications, including: facilities location choices in supply chain network design problems (e.g. Mohammed et al. [2017], Das and Posinasetti [2015]), production scheduling and road freight/maritime transportation problems (e.g. Fang et al. [2011], Bektaş and Laporte [2011], Demir et al. [2014], Bouman et al. [2017]), inventory management problems (e.g. Toptal et al. [2014], Hovelaque and Bironneau [2015]), production planning (see Table 3.9).

In particular, reductions in GHG emissions stemming from industrial processes and systems are primordial for reaching worldwide agreed targets related to the climate change mitigation. In accordance with the scope of this survey, let us put the spotlight, in what follows, on production planning problems including GHG emission issues.

Mathematical formulation

As Table 3.9 witnesses, a number of studies have started to take into account GHG emissions issues in production planning problems and to assess their incidence on operational decisions. Depending on the nature of regulation policies previously evoked, carbon emissions considerations can appear in the objective function or constraints. For instance, additional costs due to emissions are usually incorporated in the objective function when carbon pricing and cap-and-trade regulation schemes are under study. Meanwhile, carbon emission limitations due to the cap-and-trade or threshold regulation are modeled by constraints [Benjaafar et al., 2013, Hong et al., 2016, Purohit et al., 2016, Zakeri et al., 2015].

According to the manner in which carbon emissions are allocated over the planning horizon, four classes of carbon emissions constraints can be distinguished in production planning problems: **(i)** *global constraint*: the carbon emissions capacity is available on the whole horizon [Benjaafar et al., 2013, Absi et al., 2013, Retel Helmrich et al., 2015]; **(ii)** *periodic constraint*: the quantity of carbon emissions, which is not used in a given time period, is lost [Absi et al., 2013, Hong et al., 2016, Fahimnia et al., 2013, Kantas et al., 2015, Absi et al., 2016]; **(iii)** *cumulative constraint*: the quantity of carbon emissions, which is not used in a given time period, can be used in the next periods while respecting an upper cumulative capacity [Absi et al., 2013]; **(iv)** *rolling constraint*: the carbon emissions can only be compensated on a rolling period [Absi et al., 2013].

Green production aims to be profitable via environmentally-friendly industrial processes. In this sense, many efforts have been deployed in the quest of both environmentally and economically viable production and transportation (supply) modes. Kantas et al. [2015] investigated multiple sources to produce one final product, in order to identify the less pollutant processes used to transform different raw materials. Towards the same goal, Fahimnia et al. [2013] and Zakeri et al. [2015] showed how to analyze multiple production and/or transportation modes depending on the used technology and the equipment age. As far as the demand is concerned, it can be specific to each production mode [Absi et al., 2013], or common for all possible combinations of production sources and transportation modes [Hong et al., 2016, Absi et al., 2016]. Note that all of these problems with multiple production and/or transportation modes operate with only a single product in most cases. Nevertheless, companies rarely produce only one product type. Among all the reviewed quantitative models considering GHG emissions, only two deal with multiple products, namely Fahimnia et al. [2013], Zakeri et al. [2015].

Finally, as limiting the production capacity makes any problem much more difficult to solve, most of the available models in the literature are uncapacitated (see Table 3.9). However, problems modeling industrial supply chain are still capacitated in order to get closer to the reality [Fahimnia et al., 2013, Kantas et al., 2015, Zakeri et al., 2015].

Table 3.9: Greenhouse gas emissions

Reference	Emission regulation				Emission constraint					MPM	Cap	Type		Resolution			Instance		Application area
	ETh	ET	ETr	EO	PC	CC	GC	RC	TC			D	ND	Ex	App	Sol	R/B	I	
Absi et al. [2013]					✓	✓	✓	✓		✓		✓							G
Benjaafar et al. [2013]	✓	✓	✓	✓			✓		✓			✓				✓	✓		G
Fahimnia et al. [2013]	✓				✓					✓	✓	✓				✓		✓	textile
Kantas et al. [2015]			✓		✓				✓	✓	✓	✓				✓		✓	biofuel
Retel Helmrich et al. [2015]							✓					✓			✓		✓		G
Zakeri et al. [2015]		✓	✓							✓	✓	✓				✓		✓	metal furniture
Absi et al. [2016]	✓				✓					✓		✓		✓					G
Hong et al. [2016]	✓		✓		✓				✓	✓		✓		✓			✓		G
Purohit et al. [2016]			✓						✓				✓			✓	✓		G
Wu et al. [2018]	✓				✓					✓	✓	✓			✓	✓	✓		G
Zouadi et al. [2018]	✓						✓					✓			✓		✓		G
Lamba and Singh [2019]			✓						✓			✓				✓	✓		G
Phouratsamay and Cheng [2019]	✓		✓		✓				✓	✓		✓		✓					G
Shamayleh et al. [2019]		✓										✓			✓		✓		G

ETh: Emission threshold, ET: Emission tax, ETr: Emission trading, EO: Emission offset, PC: Periodic constraint, CC: Cumulative constraint, GC: Global constraint, RC: Rolling constraint, TC: Trading constraint, MPM: Multiple production modes, Cap: Capacitated, D: Deterministic, ND: Non-Deterministic, Ex: Exact, App: Approximate, Sol: Solver, R/B: Random or Benchmark, I: Industrial, G: Generic

Consider for instance an uncapacitated single-item lot-sizing problem with carbon emissions, as done in the paper of Retel Helmrich et al. [2015]. It aims to determine, over a planning horizon of T periods, when and how much to produce a product to satisfy a deterministic demand d_t for every period $t \in [\![1,T]\!]$. At each time period a fixed setup cost f_t and fixed setup emissions $\hat{f}_t$ occur when production occurs in this period. Furthermore, p_t and h_t are unitary production and holding costs, and $\hat{p}_t$ and $\hat{h}_t$ are unitary production and holding emissions, respectively. Carbon emissions are limited by a maximum emission capacity $\hat{C}$ over the whole time horizon.

Table 3.10: Greenhouse gas emissions: *Notations*

Indexes and parameters:	
T	Number of time periods
p_t	Unit production cost in period t
f_t	Fixed setup cost in period t
h_t	Unit holding cost in period t
$\hat{p}_t$	Unit production emission in period t
$\hat{f}_t$	Fixed setup emission in period t
$\hat{h}_t$	Unit holding emission in period t
d_t	Demand for products in period t
$\hat{C}$	Global emission capacity
Decision variables:	
X_t	Production quantity in period t
Y_t	Binary setup indicator of production in period t
I_t	Inventory level in period t

Making use of the notations described above and summarized in Table 3.10, the uncapacitated single-item lot-sizing problem with a global carbon emission constraint can be formally defined as a mixed-integer problem with the following decision variables: X_t represents the production quantity of the product in a given period t, Y_t is a binary indicator of setup for production in period t, and I_t denotes the inventory level of the product at the end of period t, $t \in [\![1,T]\!]$.

$$\text{minimize} \sum_{t=1}^{T} \left(p_t X_t + f_t Y_t + h_t I_t \right) \tag{3.23}$$

subject to:

$$X_t + I_{t-1} = I_t + d_t \qquad \forall t \in [\![1,T]\!] \tag{3.24}$$

$$I_0 = 0 \tag{3.25}$$

$$X_t \leq \sum_{i=t}^{T} d_i Y_t \qquad \forall t \in [\![1,T]\!] \tag{3.26}$$

$$\sum_{t=1}^{T} \left(\hat{p}_t X_t + \hat{f}_t Y_t + \hat{h}_t I_t \right) \leq \hat{C} \tag{3.27}$$

$$X_t, I_t \geq 0 \qquad \forall t \in [\![1,T]\!] \tag{3.28}$$

$$Y_t \in \{0,1\} \qquad \forall t \in [\![1,T]\!] \tag{3.29}$$

The objective function (3.23) minimizes costs. Constraints (3.24) are the inventory balance constraints. Constraint (3.25) fixes the initial inventory level to zero. Constraints (3.26) ensure that, in each period, there is production only if there is a setup. Constraint (3.27) limits global emissions over the whole time horizon. The unitary carbon emission is commonly considered proportional to the quantity of produced and transported units [Absi et al., 2013]. Finally, nonnegativity and binary requirement constraints are given in expressions (3.28)-(3.29).

Complexity and solution approaches

The manner in which carbon emissions constraints are modeled has a determinant impact on the problem complexity:

- *Global constraint:* Due to its similarity with capacity constraints, the lot-sizing problem with global carbon emissions constraints (3.23)-(3.29) is proved NP-hard even for linear cost functions [Retel Helmrich et al., 2015]. The problem with multiple production modes stays NP-hard [Absi et al., 2013]. Note however that, the single-item single-mode lot-sizing problem with global emission constraints can be solved by a pseudo-polynomial algorithm under some assumptions on co-behaving costs [Absi et al., 2013].

- *Periodic constraints:* The uncapacitated multi-modes lot-sizing problem, subject to periodic emission constraints, is proved polynomial and can be solved by a dynamic programming algorithm [Absi et al., 2013, Hong et al., 2016, Phouratsamay and Cheng, 2019].

- *Cumulative and rolling constraints:* The uncapacitated lot-sizing problems with cumulative or rolling emission constraints have only be used with multiple production modes. They are proved to be NP-hard [Absi et al., 2013]. Note that rolling constraints with only one period become similar to periodic constraints, and the problems can be polynomially affordable by a dynamic programming algorithm.

- *Cap-and-trade constraints:* More flexible than emission cap policy, the emission cap-and-trade scheme is a market-based instrument, that allows production entities to relax their emission constraints via the trading of emissions permits. Lot-sizing problems with cap-and-trade constraints can be solved in a polynomial time by a dynamic programming algorithm [Hong et al., 2016].

Industrial implications and discussion

Either constrained by regulation policies or concerned about their green image, *"a substantial number of companies publicly state carbon emission reduction targets..."* [Velázquez-Martínez et al., 2014]. In the production planning literature, one can find lot-sizing problems dealing with carbon emissions in several industry sectors: textile [Fahimnia et al., 2013], metal furniture [Zakeri et al., 2015], bio-fuel [Kantas et al., 2015].

Besides the academic and industrial interest of the reviewed models, the way in which carbon emissions are handled in the literature remains simplistic. Several aspects of the state-of-the-art modeling approach are open to further investigation:

- *Modeling accuracy:* Generally, carbon emissions are modeled using linear or affine functions, while the emissions reduction can exhibit irregular nonlinear trends [Zakeri et al., 2015, Purohit et al., 2016]. It may be constructive to study more deeply the effects of emission parameters against regulatory mechanisms.

- *Towards multi-objective optimization:* The majority of existing studies deal with single-objective optimization problems. Basically, cost minimization and carbon emissions minimization are two conflicting objectives. One has to find the trade-off between the total cost and the total carbon emissions. Considering carbon emissions as a second objective and addressing bi-objective versions of lot-sizing problems with carbon emissions stand out as a worthwhile research avenue to be explored.

- *Emission reduction in both forward and reverse supply chain directions:* In the related literature, carbon emissions are mainly considered in the forward supply chain, but very rarely in the reverse one [Fahimnia et al., 2013]. Note that greenhouse gas emissions also occur during collection, remanufacturing and recycling.

Table 3.11: Energy consumption

Reference	Energy in the objective function depends on ...		Energy capacity	Type		Resolution			Instance		Application area
	Nb of machines	Produced quantity		D	ND	Ex	App	Sol	R/B	I	
Tang et al. [2012]		✓			✓		✓		✓		G
Giglio et al. [2017]	✓	✓		✓			✓		✓		G
Masmoudi et al. [2017]	✓	✓		✓			✓		✓		G
Golpîra et al. [2018]	✓				✓			✓	✓		G
Rapine et al. [2018b]	✓		✓	✓		✓			-	-	G
Rapine et al. [2018a]	✓		✓	✓		✓			-	-	G
Wichmann et al. [2019]	✓			✓				✓		✓	warm and hot forming

D: Deterministic, ND: Non-Deterministic, Ex: Exact, App: Approximate, Sol: Solver, R/B: Random or Benchmark, I: Industrial, G: Generic, -: None

For the sake of completeness, it would be useful to evaluate the emission parameters of each component of a supply chain in both forward and reverse directions.

3.6.2 Energy consumption

An environmentally-friendly industrial activity must reflect an energy-efficient production management, in terms of both costs and GHG emissions involved by the energy consumption. Against the vast amount of literature aiming at making production technologies less energy consuming, it is only recently that energy-aware production planning has received the scientific attention. Among papers operating with managerial actions, the literature review of Biel and Glock [2016] revealed the predominant interest expressed for job allocation and sequencing problems and the deficit of attention manifested for mid-term lot-sizing problems. To provide a more accurate picture of existing studies on energy-aware production planing, Table 3.11 lists and describes all identified references.

Table 3.12: Energy consumption: *Notations*

Indexes and parameters:	
T	Number of time periods
p_t	Unit production cost in period t
$f(M_t^+)$	Fixed start-up cost of M_t^+ machines in period t
h_t	Unit holding cost in period t
d_t	Demand for products in period t
e_t	Quantity of energy necessary to produce one unit of product in period t
w_t	Quantity of energy necessary to start up a machine in period t
E_t	Available quantity of energy in period t
C	Capacity of a machine
Decision variables:	
X_t	Production quantity in period t
I_t	Inventory level in period t
M_t	Number of running machines in period t
M_t^+	Number of machines started up at the beginning of period t

Mathematical formulation

At the limit between tactical and operational levels, the energy efficiency considerations are separately or jointly integrated in lot-sizing problems, as follows:

- *Energy consumption modeling:* For the sake of simplicity, the energy consumption is usually measured via the production activity, i.e. either the size and usage duration of the resources park [Rapine et al., 2018b,a, Masmoudi et al., 2017] or the production quantity [Tang et al., 2012].
- *Energy related costs:* Energy pricing is a complex related matter. Apart from the reasons lied on seasonal cycles and highly market volatility, energy providers intensify the diversification of their pricing schemes with the development of renewable sources of energy. Accordingly, the academics tend to align their models with this economic conjuncture, by considering time-varying energy costs [Masmoudi et al., 2017, Rapine et al., 2018b].
- *Limitations in the available quantity of energy:* The limitations in the available quantity of energy per period or over the entire time horizon can be justified by different application reasons. In particular, that is the case of

renewable energy sources, which require to carefully manage the electricity demand and balance electric grids. Hence, the electric energy can no longer be considered unlimited [Rapine et al., 2018b,a]. To go further in this direction, Masmoudi et al. [2017] envisaged the case of more realistic energy supply contracts, defined in terms of the maximal power and electricity prices over time.

For modeling purposes, consider the energy single-item lot-sizing problem examined by Rapine et al. [2018b]. This problem consists in determining the quantity X_t to be produced over a planning horizon of T periods, in order to satisfy each periodic deterministic demand d_t, $\forall t \in [\![1,T]\!]$. The industrial process is performed on parallel and identical machines. The production capacity depends on the number of running machines M_t, which is bounded by a constant C per machine. The number of machines started-up at the beginning of period t is denoted by M_t^+. I_t represents the inventory level at the end of period t. The available quantity of energy is limited to E_t. Let e_t be the quantity of energy necessary to produce one unit of the considered product. The start-up of one machine in period t consumes a quantity of energy w_t. The production process involves the following costs: **(i)** h_t, the unit holding cost, **(ii)** p_t, the unit production cost, **(iii)** $f(M_t^+)$, the cost to start-up M_t^+ machines. The notations introduced above are summarized in Table 3.12

The energy-aware lot-sizing problem can be modeled as follows:

$$\text{minimize} \sum_{t=1}^{T} \left[f(M_t^+) + p_t X_t + h_t I_t \right] \tag{3.30}$$

$$\begin{aligned}
\text{s.t. } & I_{t-1} + X_t = I_t + d_t && \forall t \in [\![1,T]\!] && (3.31)\\
& X_t \leq C M_t && \forall t \in [\![1,T]\!] && (3.32)\\
& I_0 = 0 && && (3.33)\\
& e_t X_t + w_t M_t^+ \leq E_t && \forall t \in [\![1,T]\!] && (3.34)\\
& M_t^+ \geq M_t - M_{t-1} && \forall t \in [\![1,T]\!] && (3.35)\\
& I_t, X_t \geq 0 && \forall t \in [\![1,T]\!] && (3.36)\\
& M_t^+, M_t \in \mathbb{N} && \forall t \in [\![1,T]\!] && (3.37)
\end{aligned}$$

The traditional lot-sizing constraints are modeled by the first three constraints in the following order: flow conservation constraints (3.31), production capacity restriction (3.32), initial inventory setting (3.33). The energy consumption is limited via inequalities (3.34). Constraints (3.35) link the number of machines started-up at the beginning of t with the number of running machines in t and $t-1$. The definition domains of problem variables are given by constraints (3.36)-(3.37). The cost-based objective function is provided by the expression (3.30).

Complexity and solution approaches

Rapine et al. [2018a] showed that the basic energy-aware lot-sizing problem (3.30)-(3.37) is NP-hard. However, under some strong assumptions (sometimes unrealistic), the program (3.30)-(3.37) can be solved by polynomial-time solution methods [Rapine et al., 2018b,a]. Other studies found in the literature use either standard solvers to solve the resulting mixed-integer programs [Wichmann et al., 2019, Golpîra et al., 2018], or developed heuristics (Relax-and-Fix, LP-based, Lagrangian Relaxation) to come up with feasible solutions [Masmoudi et al., 2017, Giglio et al., 2017, Tang et al., 2012].

Industrial implications and discussion

As Table 3.11 shows, very few research efforts have been dedicated to integrating energy efficiency issues in production planning problems. This topic deserves to be extended in several directions:

- *Theoretical development:* Rapine et al. [2018a] left open the question about the NP-hardness of the mixed-integer program (3.30)-(3.37). Is it NP-hard in the strong sense or in the weak sense? Generally speaking, the energy-aware lot-sizing problem remains open for extensively investigations from a theoretical point of view.

- *Energy consumption versus GHG emissions:* Future studies must take into account the contradictory character of the energy efficiency goals, notably the energy consumption (or its related cost) against greenhouse gas emissions. For example, coal-fired power plants are known to be the largest contributors to the atmospheric carbon dioxide concentrations, which usually generate cheap energy during off-peak consumption periods [Biel and Glock, 2016].

- *Renewable energy resources and energy storage:* One of the main issues of renewable energy resources (such as wind or solar energy sources) relates to their intermittent production and difficulty to store energy. For example, electricity can be stored in batteries or transformed into a storable energy. Excess of electricity production can be used to transform water (H_20) into hydrogen (H_2), i.e. an energy easier to store. This hydrogen can be used to produce electricity when needed or to power hydrogen-based systems (vehicles, facilities, etc.). This way of considering energy poses a number of questions related to lot-sizing for energy management: When to buy energy? When to store it? How much to store, and when and how much to retrieve? In production planning models involving energy-related constraints, loss of energy should be taken into account.

- *Energy production and smart grids:* Since electricity is not easy to store, one may want to align electricity production runs with electricity demands, or to create intermediate energy storage facilities for preserving unused energy. The alignment of demands with productions can be done by setting up some incentives (discounts, premiums, etc.) to move energy demands to periods where renewable energy is available. This mechanism of energy production and consumption leads to new lot-sizing models with special energy cost functions and intermediate energy storage facilities.

  Note that the energy transition appears to be a global environmental priority and a future reality. For instance, GRTgaz, a French natural gas transmission system operator, is currently interested in developing the power-to-gas industry in France within the project Jupiter 1000[25], funded jointly by the European Union, the French Government and the Provence-Alpes-Côte d'Azur Region of France. Power-to-gas aims at transforming the surplus of renewable electricity into a storable energy. For example, the surplus of electricity can be transformed into hydrogen (H_2) that can be used to feed hydrogen installations or vehicles. The produced hydrogen can be also coupled with carbon dioxide (CO_2) to produce methane (CH_4) that can be injected within industrial gas networks. Obtained gases can be also used within power plants to generate electricity when needed. These processes face a major tactical decision problem. It consists in deciding when and how much electricity to transform/store and when and how much to retrieve in order to satisfy different sources of demands. Demands in energy can be expressed in different forms: electricity, gas or liquid. The objective is to optimize the system by minimizing the total cost while meeting environmental objectives.

3.7 Discussion and future research directions

The extensive breadth of studies, carried out to support the transition of industrial processes from a linear towards a circular economy, stands out across the different sections of this chapter. Despite the significant research efforts dedicated to making *circular* production streams, a cross-analysis of the current review shows that a number of gaps remain to be filled and reflections to be conducted.

Disseminate data. Even if the management science is less data-intensive than data sciences, data availability is a critical specification for a transparent and progressive scientific research. Having access to industry-relevant data or to conceptual generic data models reflecting the industrial complexity is of major importance for the scientific community in developing novel credible models and industrially-viable solution methods. To the best of our knowledge, benchmark instances are only available for remanufacturing problems. More efforts are welcome to support the well-posedness of new emerging problems and the analytic reproducibility of existing solution methods.

Characterize and handle different data formats. Decision-making under uncertainty is one of the main issues of the most recovery operations. In upstream operations, the product returns and undesirable production outputs are both qualitatively and quantitatively subject to a high variability. This variability is often conditioned by factors difficult to be explained, controlled or anticipated. Further down the recovery chain, outgoing streams inherit the market volatility and sensitivity to economic and financial fluctuations, so specific for classical linear production modes of operating.

Some of aforementioned factors are measurable and can be quantified from available historical data, industrial evidence, or traceability information provided by new communicating technologies. The extraction of knowledge from available data may give valuable suggestions and contribute to soundly support the decision-making, whether the exhibited knowledge results in deterministic-based, probabilistic or fuzzy formats. In this respect, the scarcity of research studies dealing with non-deterministic or heterogeneous data formats is a major lack. Given the ubiquity of communicating technologies and the recent advances in high-performance computing technologies, it is imperative to fill this gap. On this topic, the European Commission has, through the Horizon 2020 program, funded the FUDIPO[26] project (under grant agreement No 723523), focusing on diagnostics and data reconciliation for improving data-intensive decision making in production planning and process optimization.

Sustainability in production planning. As underlined throughout different sections, numerous extensions are worth to be pursued. In the context of international trade and climate policy, far more attention should be paid to the consideration of environmental implications into production planning including recovery operations.

If the economic and environmental dimensions of the sustainability are more or less studied in the production planning literature, the only found social-aware studies operate in the framework of traditional (continuous-time) economic order quantity (EOQ) models (see e.g. Battini et al. [2014], Andriolo et al. [2016]). The EOQ models are out of the scope of this review.

Besides the economic and environmental impacts, lot-sizing decisions also affect the workers health and security in terms of such human implications as the number of working hours, human fatigue and recovery, learning/forgetting effects, metabolic energy consumption and rest allowance [Andriolo et al., 2016]. The integration of human factors and ergonomics into production planning is a particularly topical subject stressed by the increasing concern for a humanly-friendly production planning. The human well-being and ergonomics in production systems remain to be explored.

To suitably respond to the growing request of the sustainability in the actual society, lot-sizing decisions are expected to adhere simultaneously to all three economic, environmental and social goals. Significant work is still needed to balance economic, environmental and social performances in production planning.

Integrate interrelated problems. Despite the proven benefits of handling various interrelated problems in an integrated way, there is a scarcity of studies that takes simultaneously decisions belonging to different class of problems. A particular emphasis has been placed on emerging transversal problems having a strong production planning connotation. Given the plenty of interdependencies created by the circularity character of reverse logistics, a great deal of research should be done in this sense:

- *From returns collection to serviceable products distribution:* A typical remanufacturing facility includes three different subsequent industrial steps: disassembly, manufacturing/remanufacturing processing, and reassembly. This poses original lot-sizing problems under uncertainty on the quantities of returned products, qualities of products and sub-products, announced quality of returned products.

An original lot-sizing problem would be to integrate both disassembly and remanufacturing in the demand fulfillment of intermediate products. This is typically the case in the automotive industry, in which used vehicle components are remanufactured or refurbished in order to be sold.

Another original problem consists in integrating disassembly with remanufacturing and reassembly. This is mainly the case for remanufactured products. Sometimes these products should be disassembled, sub-products are remanufactured or replaced, and finally these sub-products are reassembled to be sold as new products. This generates lot-sizing problems in which all these processes need to be considered.

- *Carbon emission constraints in circular economy:* As shown in Section 3.6, the majority of studies dealing with carbon emissions addressed classical lot-sizing problems. Note that all circular economy concepts (disassembly, remanufacturing, by-products and co-production, etc.) should be environmentally viable. Accordingly, carbon emissions should become classical constraints and/or objectives in all these problems. This will naturally introduce an overlay of complexity, but will help converging towards environmentally-friendly processes.

- *Towards eco-industrial parks: Self-sufficient industrial parks* (also called *eco-industrial parks*) emerge as an effective approach towards a sustainable growth. Eco-industrial parks offer the same business advantages as classical industrial parks, while using resources more efficiently. As pointed out in Section 3.5.3 and Section 3.6.2, eco-industrial systems offer opportunities for all three dimensions of the sustainable development: **(i)** *economic:* to avoid disposal costs and increase resource efficiency by means of the industrial synergy between different industrial activities, **(ii)** *environmental:* to reduce the raw material consumption and the environmental impact via the exchange of by-products and other collateral resources (energy, water, services, etc.), **(iii)** *social:* to support the regional economic development. Combining the constraints related to all of these flow exchanges and the multiple objectives posed by the sustainable development leads to new original and complex production planning problems. Multiple aspects should be integrated in these problems: resource flow synchronization, decision-making at different time granularity imposed by the production processes, simultaneous coordination of several industrial activities, etc.

Deal with real-life applications and support industries in their transition towards a circular economy. As can be drawn from this review, a number of academic models together with their solution methods are relatively well-posed and investigated. On the flip side, a few industrial case studies have been conducted in the literature, despite the highly industrial character of all recovery operations. Of particular interest are innovative applications which can be attractive for both: **(i)** *academics*, to better apprehend industrial realities and needs, as well as to address complex and ill-structured real-life problems, **(ii)** *practitioners*, to be assisted in solving their industrial problems. Moreover, we believe that the collaboration of these complementary communities is equally important to go even further into the reflection on how to facilitate and flourish the transition towards a circular economy.

3.8 Concluding remarks

This chapter presents a literature review focused on discrete-time optimization models for tactical production planning under the prism of circular economy. The main recovery operations and notions with respect to this concept are clearly defined and discussed. In the light of global environmental pressures, an emphasis is also put on greenhouse gas emissions and energy consumption. Within this framework, we classified the identified papers on the production planning problems dealing with circular economy concepts into four parts: **(i)** disassembly for recycling, **(ii)** product and raw material recycling, **(iii)** by-products and co-production, and **(iv)** greenhouse gas emissions and energy consumption. For each part, definitions for terms related to the topic under study and a clear picture of the existing literature are provided.

All in all, let us summarize the major achievements made in production planning for the circular economy:

- Since the 1990s, new classes of lot-sizing problems have been proposed and studied in the literature to suitably integrate recycling (see Sections 3.3-3.4) and other recovering operations (see Section 3.5). Pushed by legislation and regulations, the two most important drivers of the CE development, the scientific community begins from the 2010's to show serious interest in proposing more environmentally-friendly and energy-efficient production planning solutions (see Section 3.6).

- Generally speaking, the consideration of sustainability issues in operations management problems is attracting an increasing attention in the research field after the mid-1995s (see Section 2.2). No purely sustainable lot-sizing problems can be found in the literature. Acquired by inheritance from linear production schemes, the economic dimension continues to enjoy all the attention in the production planning literature. The newly introduced CE-related lot-sizing problems are strongly economic-oriented (see Sections 3.3-3.4). In addition to economic concerns, environmental considerations have been progressively introduced in the lot-sizing problems only in the last decade (see Section 3.6).

  To the best of our knowledge, the social criterion has been neglected in production planning problems. As concluded by other closely-related studies, this is also the case for other operations management problems [Moreno-Camacho et al., 2019]. Given the actual shift towards a sustainable society, human-aware lot-sizing problems are pending.

- Together with the complexity analysis, both exact and approximate solution methods have been developed to cope with the combinatorial nature of the majority of reviewed problems. Theoretical results and efficient approaches proposed to solve generic problems lay the foundations for the development of tractable and industrial viable decision-supporting tools. Even if the most of the studied problems are deterministic in spite of some real-life settings, the conducted industrial case-studies show the applicability of the current solution methods.

- The findings discussed in Section 3.7 reveal a number of gaps and new insights as to how recovery options can be suitably integrated within traditional production environments for converging towards an environmentally-friendly economy.

This chapter defines the industrial symbiosis as a process by which a by-product of a production unit, i.e. an unavoidable substance generated in the same time as a main product, is used as raw materials by another company. The by-product exchange takes place between two or several related or autonomous companies, which requires to align lot sizing decisions of each involved actor (at least one supplier and one receiver of by-product). The literature is poor concerning this kind of relationship between two actors, despite the potential discussed in Section 3.5.

To cope with this joint production planning problem, we focus on the framing of symbiotic partnerships within a by-product synergy network, involving one supplier and one receiver of by-products. To do this, we first investigate the problem encountered by the supplier, i.e. a lot-sizing problem dealing with the by-products management, in Chapter 4. Then, we extend this work by adding a receiver which also has to take lot-sizing decisions in Chapter 5. Finally, as the collaboration policies can differ from one industrial symbiosis to another depending on the trust between the actors and the need to keep sensitive information private or not, we investigate several collaboration policies for different levels of information sharing, in Chapter 6.

Chapter 4

A single-item lot-sizing problem with a by-product and inventory capacities

The high pace of waste accumulation in landfills and the depletion of scarce natural resources lead us to seek pathways for converting unavoidable production outputs into useful and high added-value products. In this context, we formalize and propose a model for the single-item lot-sizing problem, which integrates the management of unavoidable production residues which are not waste and considered as by-products. During the production process of a main product, a by-product is generated, stored in a limited capacity and transported with a fixed transportation cost. This problem is investigated for two cases of the by-product inventory capacity: time-dependent and stationary. We prove the problem with time-dependent capacities is NP-Hard. To solve it optimally, we develop a pseudo-polynomial time dynamic programming algorithm. For the case with a stationary inventory capacity, a polynomial time dynamic programming algorithm is proposed.

Contents

4.1 Introduction

Given the definitions deeply discussed in Section 3.5, the conversion of production residues into by-products realized by using waste from one industrial process in another one is commonly referred to as *by-product synergy* (see Figure 2.5). According to Section 2.4, this paradigm of joint production offers opportunities for all three dimensions of the sustainable development, economic, environmental and social, by: **(i)** avoiding disposal costs and increasing resource efficiency, **(ii)** reducing raw material consumption, **(iii)** supporting the regional economic development. Over the past two decades, an increasing number of industrial symbiosis networks (also called eco-industrial parks) have been implemented all around the world [Evans et al., 2017]. Let us mention some eloquent specific cases encountered in Europe: **(i)** The Platform for Industry and Innovation at Caban Tonkin[27] (PIICTO) (France) supports the synergies between industrial activities located in the heart of the port of Marseille Fos (exchanges of material and energy flows, pooling of services and equipment), **(ii)** The Kalundborg eco-industrial park[19] (Denmark) regroups separate industries, which use each other's by-products, energy, water and various services, **(iii)** The Deltalinqs[28] (The Netherlands) promotes the industrial ecology to companies located in the Europoort/Botlek harbour area near Rotterdam.

One of the notable physico-chemical properties of by-products affecting their management are the storability, as highlighted in Section 2.4. The by-products can be:

- **Unstorable** (e.g. energy and some gases, such as steam water or waste heat): In order not to be lost, these materials are sent from a production unit to another one by pipelines or via grids, **(i)** either to be directly used (see e.g. the steam synergies in the Kwinana industrial area[29] in Australia, PIICTO[27] or the natural gas synergy in the Kalundborg symbiosis[19]), or **(ii)** to be transformed into storable products (see e.g. the carbon dioxide transformed into hydrogen in Kwinana and the gypsum from sulphur dioxide [Harris et al., 2006]).

- **Storable** (e.g. some gases, liquids and solids): Except the waste water, which is often sent by pipeline due to its large continuous production, they are transported between production units using conveyors or tankers after having been stored. For instance, this is the case for the waste plastics, wood, iron and sludge in Styria in Austria. The interested reader is referred to the overview of Zheng et al. [2013] related to the management of by-product flows in Styria.

  Recall that the storable quantity of by-products can be limited for questions of space or available material, like tanks, silos, lagoons, and of treatment capacity (see e.g. the sludge storability in Kalundborg explained by Larsen et al. [1991]).

Driven by the nature of the exchanged production streams, setting up industrial symbiosis networks implies multiple requirements including: technological feasibility, organizational and operational coordination, and business framing. In particular, the emergence of these networks raises new production planning problems [Herczeg et al., 2018]. In accordance to the lot-sizing literature related to joint production (see Section 3.5), the work presented in the current chapter contributes in this direction by integrating the management of by-products in the classical lot-sizing problem. More precisely, inspired by the sludge synergy in the Kalundborg symbiosis[19], we introduce and deal with the single-item lot-sizing problem with by-products storable in limited quantities, by: **(i)** performing a complexity analysis for time-varying and stationary inventory capacities, and **(ii)** proposing structural properties of optimal solutions together with exact solution methods based on dynamic programming (DP).

The remainder of this chapter is organized as follows. The problem under study is introduced and formalized in Section 4.2. A network flow representation of the problem is proposed in Section 4.3, to better illustrate the findings provided in the following sections. The lot-sizing problem with by-products and inventory capacities is examined for two special cases, namely: **(i)** with time-dependent inventory bounds in Section 4.4, and **(ii)** with constant inventory bounds in Section 4.5. Complexity analysis, structural properties and solution methods are provided for both of these cases. This chapter ends in Section 4.6 with conclusions and discussions on future extensions of this work.

Table 4.1: ULS-B problem: *Notations*

Parameters:	
T	Number of time periods indexed by $t \in \mathscr{T}$
α	Production coefficient of the by-product (w.l.o.g., $\alpha := 1$)
d_t	Demand of the main product in period t
B_t	Inventory capacity of the by-product in period t
p_t	Production cost of the main product in period t
f_t	Fixed setup cost of the main product in period t
h_t	Holding cost of the main product at the end of period t
$\hat{h}_t$	Holding cost of the by-product at the end of period t
g_t	Fixed transportation cost for the by-product at the end of period t
M_t	Big number with $M_t = \min\left\{\sum_{i=t}^{T} d_i; \frac{B_t}{\alpha}\right\}$
Decision variables:	
X_t	Production quantity of the main product in period t
Y_t	Binary setup indicator for production of the main product in period t
I_t	Inventory level of the main product at the end of period t
J_t	Inventory level of the by-product at the end of period t
W_t	Transportation quantity of by-product at the end of period t
Z_t	Binary setup indicator for transportation of the by-product at the end of period t

4.2 Mathematical formulation

Consider a single-item lot-sizing problem dealing with by-products storable in a limited capacity, called ULS-B for short in the rest of this chapter. Let us define the ULS-B problem as illustrated in Figure 4.1, i.e.: Over a planning horizon of T periods, determine when and how much to produce X_t units of a main product at a low cost, while satisfying a deterministic demand d_t and forwarding the quantity of by-products generated with a proportion of $\alpha \in \mathbb{R}^+$ to a further destination, $\forall t \in \mathscr{T}$.

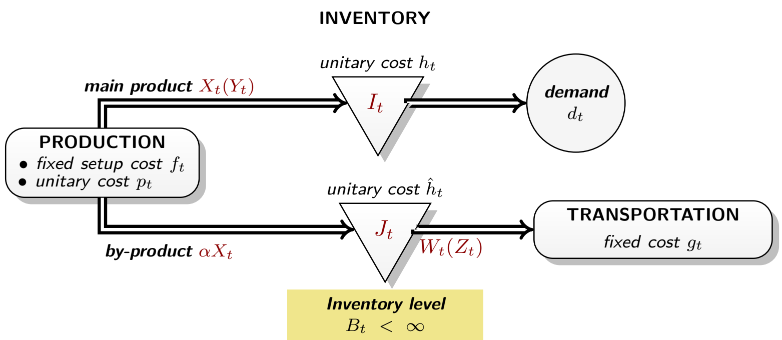


Figure 4.1: ULS-B problem: *Flow of products*, $\forall t \in \mathscr{T}$

The production system involves a fixed setup cost f_t and a unitary production cost p_t per period of time. The surplus quantity of the produced main product can be kept in inventory at a unitary cost h_t from period t to period $t+1$. To stock the unavoidable by-products, a cost $\hat{h}_t$ is charged per unit and period of time. For the end of each period t, the inventory level of the by-product is limited to B_t. The by-product transportation is performed at a fixed cost g_t.

Before proceeding to the problem modeling, let us consider the following assumptions:

- A transportation operation implies the complete emptying of the by-product inventory. This assumption is always valid because of non-negative costs.
- Both inventory levels of the main product and the by-product are assumed null at the end of the planning horizon.

Three main decisions are posed by the ULS-B problem: **(i)** when and **(ii)** how much to produce, as well as **(iii)** when to transport. Accordingly, all other related decisions are implied, namely: inventory levels of the main product and the by-product, as well as the quantities to transport.

Hence, let X_t and W_t be two decision variables that represent the quantity of the main product to produce at period t, and respectively, the quantity of by-products to transport at the end of period t. Denote by Y_t and Z_t two binary decision variables that indicate if the production of the main product takes place in period t, and respectively, if the transportation of the by-product is performed at the end of period t. The inventory levels of the main product and the by-product at the end of period t are represented by I_t and J_t, $t \in \mathscr{T}$.

By making use of the notation summarized in Table 4.1, the ULS-B problem can be formulated as a mixed-integer linear program as follows:

$$\text{minimize} \sum_{t=1}^{T} \left(p_t X_t + f_t Y_t + h_t I_t + \hat{h}_t J_t + g_t Z_t \right) \tag{4.1}$$

subject to:

$$I_{t-1} + X_t - I_t = d_t, \qquad \forall t \in \mathscr{T} \tag{4.2}$$
$$J_{t-1} + \alpha X_t - J_t = W_t, \qquad \forall t \in \mathscr{T} \tag{4.3}$$
$$W_t \leq B_t Z_t, \qquad \forall t \in \mathscr{T} \tag{4.4}$$
$$J_t \leq B_t (1 - Z_t), \qquad \forall t \in \mathscr{T} \tag{4.5}$$
$$X_t \leq M_t Y_t, \qquad \forall t \in \mathscr{T} \tag{4.6}$$
$$I_0 = J_0 = I_T = J_T = 0, \tag{4.7}$$
$$Y_t, Z_t \in \{0, 1\}, \qquad \forall t \in \mathscr{T} \tag{4.8}$$
$$X_t, I_t, J_t, W_t \geq 0, \qquad \forall t \in \mathscr{T} \tag{4.9}$$

The objective function (4.1) minimizes the sum of the following costs: production (fixed and variable), inventory holding and transportation. The set of equalities (4.2) models the flow conservation constraints of the main product. By the same token, constraints (4.3) express the flow conservation of the by-product. Linking constraints (4.4) involve a fixed transportation cost, if the accumulated by-product is transported in that period. Constraints (4.5) empty the by-product inventory level when a transport operation is triggered. In case of production, inequalities (4.6) ensure that a fixed setup cost is paid. Without loss of generality, constraints (4.7) set initial and ending inventories to zero. Non-negativity and binary requirements are given via constraints (4.8)-(4.9).

Remark 1. *Given constraints (4.3)-(4.5), X_t cannot be greater than $\frac{B_t}{\alpha}$. Without loss of generality, let us suppose in what follows that $\alpha = 1$, by setting $B_t := \frac{B_t}{\alpha}, \forall t \in \mathscr{T}$.*

4.3 ULS-B problem: Network flow representation

A solution of a ULS-B problem can be represented by a flow in a special network, as shown in Figure 4.2. In such a network, black nodes depict time periods. The lower part of the network, depicted with solid lines, represents the flow balance equations of the main product. Solid horizontal arcs represent the inventory of the main product. Demands are expressed by the solid inclined outgoing arcs. The upper part of the network, in dashed lines, refers to the flow balance equations related to by-products. Dashed horizontal arcs represent the inventory flows of by-products. Dashed outgoing arcs represent the transportation periods with the associated by-product inventory levels. The absence of horizontal arcs means null inventory levels. The particularity of this network lies in the fact that the flow generated in a production period t is the same in both directions (upper and lower arcs) and is equal to the quantity produced in period $t, t \in \mathscr{T}$.

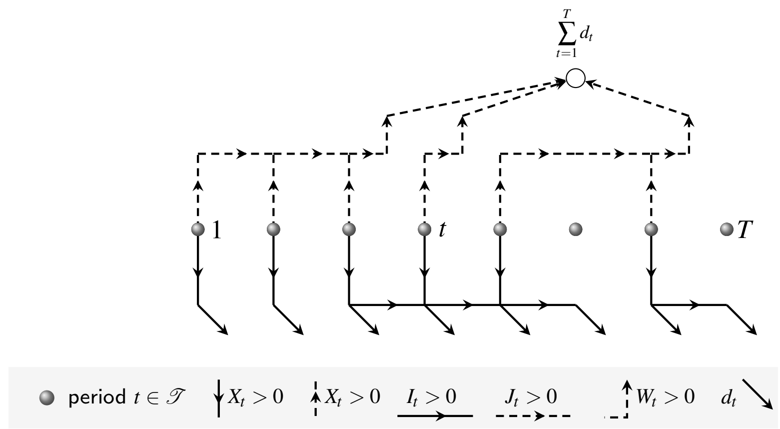


Figure 4.2: ULS-B problem: *Network flow representation*

It is worthwhile to mention that in the ULS-B problem, an extreme solution $S = (X, I, J, W)$ can contain at most two cycle configurations, which differ one from another by the nature of by-product flows, other than those related to production. Accordingly, let us distinguish the following two cases illustrated in Figure 4.3:

Case.1 Solution S contains a cycle including inventory flows of by-products defined between periods k and ℓ with $k < \ell, k, \ell \in \mathscr{T}$.

Case.2 Solution S contains a cycle including transportation flows of by-products defined between periods i and j with $i < j, i, j \in \mathscr{T}$.

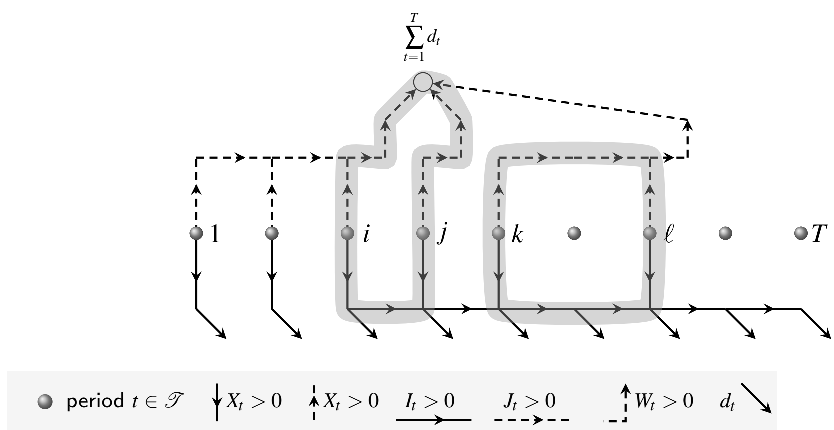


Figure 4.3: Cycle configurations in the ULS-B network flow

Property 2. *There exists an optimal solution of the ULS-B problem, such that arcs corresponding to variables with flows strictly between their lower and upper bounds form a cycle free network.*

Proof. To prove this property, we transform the flow network previously defined (see Figure 4.2) into a classical flow network, which is defined by a directed graph with capacity constraints on arcs, having source and destination nodes. To do so, let us reverse the arcs of the by-product flows (dashed lines). Contrary to the flow network depicted in Figure 4.2, in which production flows are doubled, the newly obtained network is a classical one. The source node is the one on the top, it injects $\sum_{t=1}^{T} d_t$ into the network flow. The flow generated by this quantity traverses the arcs of this new network in order to satisfy demands of all periods.

In classical network flow models with concave costs, extreme flows are cycle free (see e.g. Ahuja et al. [1988], Zangwill [1968]). Since the ULS-B problem can be modeled as a classical network flow problem with concave costs, extreme solutions are cycle free. Recall that extreme flows are those that cannot be represented as a convex combination of two distinct flows. We also know that there exists at least an optimal solution of the ULS-B among these extreme flows. This concludes the proof.

□

4.4 ULS-B problem: The general case

In what follows, let us investigate the general case of the ULS-B problem formalized in the previous section.

4.4.1 Complexity of the ULS-B problem

Theorem 1. *The ULS-B problem with time-dependent inventory capacities of the by-product is $\mathcal{NP}$-Hard.*

Proof. The proof of $\mathcal{NP}$-Hardness of ULS-B is performed by reduction from the capacitated lot-sizing (CLS) problem, the general case of which is known to be $\mathcal{NP}$-Hard [Florian et al., 1980]. The decision version of the CLS problem is defined by:

- a planning horizon of $\{1,2,\dots,\tilde{T}\}$,
- limited production capacities $\tilde{C}_t$, $\forall t \in \{1,2,\dots,\tilde{T}\}$,
- demands $\tilde{d}_t$, $\forall t \in \{1,2,\dots,\tilde{T}\}$,
- a triple cost: fixed setup $\tilde{f}_t$, unit production cost $\tilde{p}_t$, and unit inventory holding cost $\tilde{h}_t$, $\forall t \in \{1,2,\dots,\tilde{T}\}$.

Let $\tilde{X} = (\tilde{X}_1, \tilde{X}_2, \dots, \tilde{X}_T)$ be the vector of produced quantities, and $\tilde{I} = (\tilde{I}_1, \tilde{I}_2, \dots, \tilde{I}_T)$ be the vector of inventory levels during the planning horizon. Denote by $\tilde{Y} = (\tilde{Y}_1, \tilde{Y}_2, \dots, \tilde{Y}_T)$ the production indicator vector. The question posed by the capacitated lot-sizing problem is: Does there exist a production plan $(\tilde{X}, \tilde{I}, \tilde{Y})$ of total cost at most equal to a given value V, which satisfies demands $\tilde{d} = (\tilde{d}_1, \tilde{d}_2, \dots, \tilde{d}_T)$?

An instance I^{CLS} of the capacitated lot-sizing problem can be transformed into an instance I of ULS-B by making the following substitutions $\forall t \in \{1,2,\dots,\tilde{T}\}$:

(S.1) Number of periods: $T = \tilde{T}$,

(S.2) Demands: $d_t = \tilde{d}_t$,

(S.3) Capacities: $B_t = \tilde{C}_t$,

(S.4) Costs related to the main product: $f_t = \tilde{f}_t$, $p_t = \tilde{p}_t$ and $h_t = \tilde{h}_t$,

(S.5) Costs related to the by-product: $g_t = 0$ and $\hat{h}_t = 0$.

Let us show that instance I^{CLS} has an affirmative answer, if and only if, there exists a feasible solution (X,Y,I,J,W,Z) for instance I such that $\sum_{t=1}^{T}\Big(p_tX_t + f_tY_t + h_tI_t + \hat{h}_tJ_t + g_tZ_t\Big) \leq V$. To do this, we prove the conditional relationship between CLS and ULS-B problems related to the solution existence.

$(\Longrightarrow)$ Suppose that instance I^{CLS} has an affirmative answer. Let $(\tilde{X},\tilde{I},\tilde{Y})$ be a production plan, such that $\sum_{t=1}^{\tilde{T}}\Big(\tilde{p}_t\tilde{X}_t + \tilde{f}_t\tilde{Y}_t + \tilde{h}_t\tilde{I}_t\Big) \leq V$. A feasible solution (X,Y,I,J,W,Z) for instance I, such that the total cost is at most equal to V, can be built as follows:

- Produce $X = \tilde{X}$ quantities of the main product, this generates by-product quantities lower than $B = (B_1, B_2, \dots, B_T)$ by virtue of substitution (S.3),
- Hold $I = \tilde{I}$ levels of the main product,
- Transport at the end of each period the entire generated quantity of the by-product during this period.

Given substitutions (S.1)-(S.5), it follows that $\sum_{t=1}^{T}\Big(p_tX_t + f_tY_t + h_tI_t + \hat{h}_tJ_t + g_tZ_t\Big) \leq V$.

$(\Longleftarrow)$ Conversely, assume that instance I has a positive answer, i.e. there exists a production plan (X,Y,I,J,W,Z), which satisfies all demands with a cost at most equal to V. Making use of substitutions (S.1)-(S.5), it can immediately be checked that $\sum_{t=1}^{\tilde{T}}\Big(\tilde{p}_t\tilde{X}_t + \tilde{f}_t\tilde{Y}_t + \tilde{h}_t\tilde{I}_t\Big) \leq V$, where $\tilde{X} = X$, $\tilde{I} = I$ and $\tilde{Y} = Y$. □

4.4.2 Definitions and structural properties of optimal solutions

In this section, let us give several definitions and derive a useful structural property of optimal solutions.

Definition 5 (Production period)**.** *A period $t \in \mathscr{T}$ is called **a production period**, if the production of the main product is performed at this period, i.e. $X_t > 0$.*

Definition 6 (Inventory period)**.** *A period $t \in \mathscr{T}$ is called **an inventory period**, if at least one of the following conditions holds:*

- *The inventory level of the main product at the end of period t is equal to zero, i.e. $I_t = 0$.*
- *The inventory level of the by-product at the end of period t is equal to zero, i.e. $J_t = 0$.*
- *The inventory level of the by-product at the end of period t is equal to its maximum inventory capacity, i.e. $J_t = B_t$.*

Definition 7 (Transportation period)**.** *A period $t \in \mathscr{T}$ is called **a transportation period**, if the by-product transportation takes place at the end of this period, i.e. $W_t > 0$.*

Based on Definitions 5-7 and consistent with the network structure of ULS-B, Property 3 generalizes the classical block decomposition approach introduced by Florian and Klein [1971].

Property 3. *In an optimal solution of the ULS-B problem, there is at most one production period between two consecutive inventory periods.*

Proof. Consider an optimal solution S of ULS-B containing two consecutive inventory periods $j-1$ and ℓ, $j-1,\ell \in \{2,3,...,T\}$. By virtue of Definitions 5-6, period j is either **(i)** a production period with $X_j > 0$ and $j \neq \ell$, or **(ii)** an inventory period with $X_j = 0$ and $j = \ell$, since it inherits one of the inventory period conditions from its predecessor.

Let $j \neq \ell$ be a production period. Suppose, by contradiction, that there exists another production period t between j and ℓ. As $j-1$ and ℓ are two consecutive inventory periods, the inventory levels of both the main product and the by-product are not null, and do not reach the inventory capacity until t. The flows corresponding to the production in periods j and t form thus a cycle. From Property 2, it follows that the considered solution S is not optimal. Hence, the assumption that there exists more than one production period between two consecutive inventory periods is false. $\square$

Table 4.2: ULS-B problem: An instance I of 8 periods and an optimal solution S^*

Parameters values:								
f	600	2,000	600	600	600	400	800	600
p	5	5	5	7	8	4	8	5
h	4	4	4	4	4	4	4	4
$\hat{h}$	1	1	1	1	1	1	1	1
g	2,000	1,000	3,000	4,000	3,000	3,000	1,000	1,000
d	200	300	400	200	100	100	500	200
B	800	800	500	800	1,400	1,400	1,500	100
Values of decision variables in optimal solution S^*:								
X	500	0	400	300	0	100	700	0
Y	1	0	1	1	0	1	1	0
I	300	0	0	100	0	0	200	0
J	500	0	400	700	700	800	0	0
W	0	500	0	0	0	0	1,500	0
Z	0	1	0	0	0	0	1	0
t	1	2	3	4	5	6	7	8

Example 4.1

Consider an instance I of ULS-B with a planning horizon of 8 periods. The values of the problem parameters and an optimal solution S^* are provided in Table 4.2. The network flow representation of S^* is illustrated in Figure 4.4a. The value of optimal solution S^* is 23,100.

Optimal solution S^* contains:

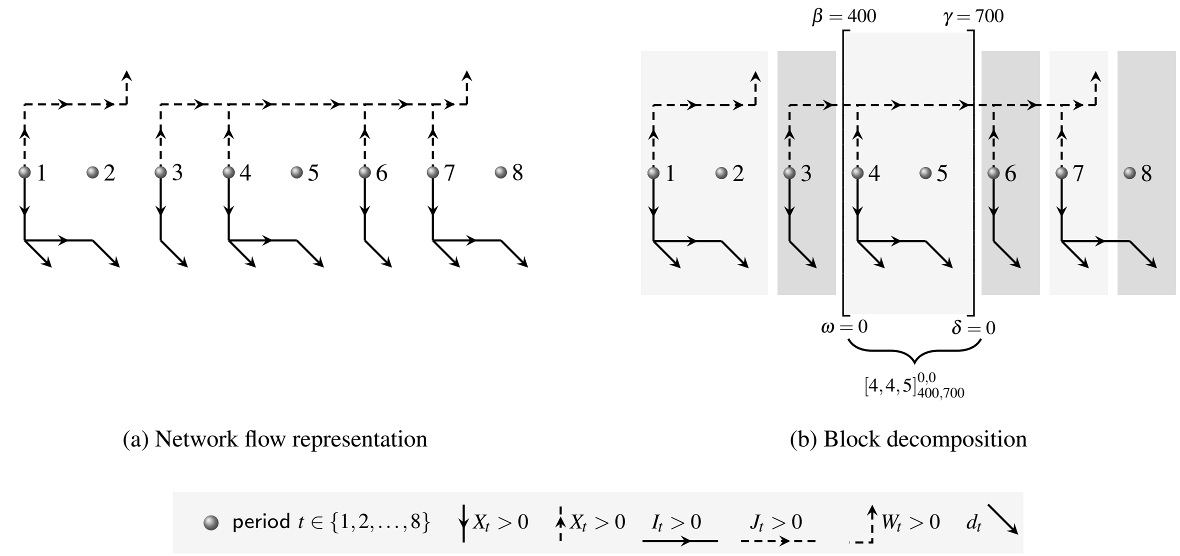
- 6 inventory periods: 2, 3, 5, 6, 7 and 8.
- 5 production periods: 1, 3, 4, 6, 7.

4.4.3 Dynamic programming algorithm

This section presents a pseudo-polynomial dynamic programming algorithm for solving the general case of ULS-B. Property 3 is used to reduce the number of generated states. For the sake of convenience, let us introduce the notion of block to characterize the extreme solutions of the problem under study.

Definition 8 (Block). *The set of time periods $\{j, j+1, \ldots, \ell\}$ between two consecutive inventory periods $j-1$ and ℓ is called a **block**, where $j \leq \ell$, $\forall j, \ell \in \mathscr{T}$.*

In other words, a block formed by two inventory periods j and ℓ implies that: **(i)** $I_{j-1} = 0$ or $J_{j-1} \in \{0, B_{j-1}\}$, **(ii)** $I_\ell = 0$ or $J_\ell \in \{0, B_\ell\}$, and **(iii)** $I_t > 0$ and $0 < J_t < B_t$, $\forall t \in \{j, \ldots, \ell-1\}$. As shown in Property 3, there is at most



(a) Network flow representation

(b) Block decomposition

Figure 4.4: Network flow representation of optimal solution S^* of instance I

one production period k between two consecutive inventory periods $j-1$ and ℓ, $j-1 < k \leq \ell$. Hence, let us represent an extreme solution as a sequence of blocks:

$$[j,k,\ell]^{\omega\gamma}_{\beta\delta},$$

where values ω, γ, β and δ indicate the states of entering and ending inventory levels of both the main product and by-products.

More specifically, a block $[j,k,\ell]^{\omega\gamma}_{\beta\delta}$ is defined by two consecutive inventory periods $j-1$ and ℓ, and at most one production period k is characterized by the following states:

- $\omega \in \{0,1,\cdots,\tilde{M}_j\}$: entering inventory level of the main product at period j, with $\tilde{M}_t = \sum_{i=t}^{T} d_i$, $\forall t \in \{1,\cdots,T\}$ and $\tilde{M}_{T+1} = 0$;
- $\gamma \in \{0,1,\cdots,\tilde{M}_{\ell+1}\}$: ending inventory level of the main product at period ℓ;
- $\beta \in \{0,1,\cdots,N_j\}$: entering inventory level of the by-product of period j, with $N_j = B_{j-1}$, $\forall j \in \{1,\cdots,T\}$ and $N_{T+1} = 0$.
- $\delta \in \{0,1,\cdots,R_\ell\}$: ending inventory level of the by-product of period ℓ before transportation, with $R_\ell = B_\ell$, $\forall \ell \in \{1,\cdots,T-1\}$ and $R_T = 0$.

By convention, if there is no production period between j and ℓ, k is set to 0. The objective value of the optimal policy for a block $[j,k,\ell]^{\omega\gamma}_{\beta\delta}$ is denoted $\varphi^{\omega\gamma}_{\beta\delta}(j,k,\ell)$.

Define the following three functions in order to improve the readability of the subsequent equations:

- The function $\mathscr{P}_t(Q)$ provides the cost of producing the quantity Q at period t:

$$\mathscr{P}_t(Q) = \begin{cases} f_t + p_t Q, & \text{if } Q > 0 \text{ and } t > 0 \\ 0, & \text{if } Q = 0 \text{ or } t = 0 \\ +\infty, & \text{if } Q < 0. \end{cases}$$

- The function $\mathscr{H}_{t,\ell}(Q)$ calculates the inventory cost of storing quantity Q of the main product between period t and period ℓ, while considering the demand satisfaction between these two periods:

$$\mathscr{H}_{t,\ell}(Q) = \begin{cases} \sum_{i=t}^{\ell-1} h_i(Q - d_{ti}), & \text{if } Q \geq d_{t,\ell-1} \\ +\infty, & \text{if } Q < d_{t,\ell-1} \end{cases}$$

where $d_{t\ell} = \begin{cases} \sum_{i=t}^{\ell} d_i, & \text{if } t \leq \ell \\ d_{t\ell} = 0, & \text{if } t > \ell. \end{cases}$

- The function $\hat{\mathscr{H}}_{t,\ell}(Q)$ represents the inventory cost of storing the quantity Q of the by-products between period t and period ℓ.

$$\hat{\mathscr{H}}_{t,\ell}(Q) = \begin{cases} \sum_{i=t}^{\ell-1} \hat{h}_i Q, & \text{if } Q \geq 0 \\ +\infty, & \text{if } Q < 0. \end{cases}$$

Given a block $[j,k,\ell]_{\beta\delta}^{\omega\gamma}$, the value of δ can be expressed in terms of ω, γ, β and $d_{j\ell}$: $\delta = \beta + d_{j\ell} + \gamma - \omega$. Note that if transportation occurs at period ℓ then $\delta = 0$.

By making use of the notations introduced above, the cost of the block $\varphi_{\beta\delta}^{\omega\gamma}(j,k,\ell)$ is given by:

$$\varphi_{\beta\delta}^{\omega\gamma}(j,k,\ell) = \begin{cases} \mathscr{P}_k(d_{j\ell} + \gamma - \omega) + \mathscr{H}_{j,k}(\omega) & \text{if } k \neq 0, d_{j,k-1} \leq \omega, \delta \leq \min_{k \leq i \leq \ell} B_i \\ +\mathscr{H}_{k,\ell+1}(d_{k\ell} + \gamma) + \hat{\mathscr{H}}_{j,k}(\beta) + \hat{\mathscr{H}}_{k,\ell+1}(\delta), & \quad \text{and } \beta \leq \min_{j \leq i \leq k-1} B_i \\ \mathscr{H}_{j,\ell+1}(\omega) + \hat{\mathscr{H}}_{j,\ell+1}(\beta) & \text{if } k = 0, d_{j,\ell} \leq \omega, \beta \leq \min_{j \leq i \leq \ell} B_i \\ & \quad \text{and } \delta = \beta \\ +\infty, & \text{otherwise.} \end{cases}$$

According to Property 3, transportation can only occur at the end of period ℓ. In the contrary case, if transportation occurs at a period $m < \ell$, an inventory period is thus created at period m, fact that breaks down the block structure of $[j,k,\ell]_{\beta\delta}^{\omega\gamma}$. Then,

$$\varphi_{\beta 0}^{\omega\gamma}(j,k,\ell) = \min_{0 \leq \delta \leq B_\ell} \left(\varphi_{\beta\delta}^{\omega\gamma}(j,k,\ell) + g_\ell - \hat{\mathscr{H}}_{\ell,\ell+1}(\delta) \right)$$

Example 4.1 (Continuation): Focus on the *block decomposition*

Consistent with the block decomposition previously introduced, optimal solution S^* can be decomposed into six blocks, as can be seen in Figure 4.4b. Each of them contains at most one production period.

Let $v_t^{\omega\beta}$ be the optimal value of the problem over the periods in $[t,T]$, given that $I_{t-1} = \omega$ and $J_{t-1} = \beta$.

Remark 2. *Given constraint (4.7) of the ULS-B problem, let us initialize the initial and final states of main and by-product inventory level as follows:*

- *Initial states:* $I_0 = J_0 = 0$*, by setting* $B_0 = 0$*;*
- *Final states:* $I_T = J_T = 0$*, by setting* $R_T = 0$ *and* $\tilde{M}_{T+1} = N_{T+1} = 0$.

Proposition 1. *For initial null inventory levels of the main product and the by-product, the optimal cost of the ULS-B problem is equal to v_1^{00} given by the last step of the following algorithm, which proceeds backward in time from period T to 1, $\forall j \in \{T, T-1, \cdots, 1\}$:*

$$v_{T+1}^{\omega\beta} = 0, \forall \omega \in \{0, 1, \ldots, \tilde{M}_{T+1}\}, \forall \beta \in \{0, 1, \ldots, N_{T+1}\}$$

$$\begin{aligned}
v_j^{\omega 0} &= \min_{j \le \ell \le T} \min_{j \le k \le \ell} \left\{ \min_{0 \le \gamma \le \tilde{M}_{\ell+1}} \left\{ \varphi_{00}^{\omega\gamma}(j,k,\ell) + v_{\ell+1}^{\gamma 0}, \quad \varphi_{0R_\ell}^{\omega\gamma}(j,k,\ell) + v_{\ell+1}^{\gamma R_\ell} \right\}, \min_{0 \le \delta \le R_k} \left\{ \varphi_{0\delta}^{\omega 0}(j,k,\ell) + v_{\ell+1}^{0\delta} \right\} \right\} \\
v_j^{\omega N_j} &= \min_{j \le \ell \le T} \min_{j \le k \le \ell} \left\{ \min_{0 \le \gamma \le \tilde{M}_{\ell+1}} \left\{ \varphi_{N_j 0}^{\omega\gamma}(j,k,\ell) + v_{\ell+1}^{\gamma 0}, \quad \varphi_{N_j R_\ell}^{\omega\gamma}(j,k,\ell) + v_{\ell+1}^{\gamma R_\ell} \right\}, \min_{0 \le \delta \le R_k} \left\{ \varphi_{N_j \delta}^{\omega 0}(j,k,\ell) + v_{\ell+1}^{0\delta} \right\} \right\} \\
v_j^{0\beta} &= \min_{j \le \ell \le T} \min_{j \le k \le \ell} \left\{ \min_{0 \le \gamma \le \tilde{M}_{\ell+1}} \left\{ \varphi_{\beta 0}^{0\gamma}(j,k,\ell) + v_{\ell+1}^{\gamma 0}, \quad \varphi_{\beta R_\ell}^{0\gamma}(j,k,\ell) + v_{\ell+1}^{\gamma R_\ell} \right\}, \min_{0 \le \delta \le R_k} \left\{ \varphi_{\beta\delta}^{00}(j,k,\ell) + v_{\ell+1}^{0\delta} \right\} \right\}
\end{aligned} \tag{4.10}$$

where $0 \le \omega \le \tilde{M}_j, 0 \le \beta \le N_j$ *with* $B_0 = 0, d_{T+1} = 0$ *and* $B_{T+1} = 0$.

Proof. By virtue of Property 4, an optimal solution of the ULS-B problem can be partitioned in a series of consecutive blocks. Hence, from the perspective of a given period j, the cost of ULS-B over the periods in $[j, T]$ can be recursively expressed via the sum of: **(i)** the cost of the block, starting at period j and finishing at a period ℓ, and **(ii)** the cost of ULS-B over the periods $[\ell+1, T]$, where $j \le \ell \le T$.

A block is defined by two consecutive inventory periods, each having to respect one of the three conditions specified in Definition 6. Hence, at each period $j \in \{T, T-1, \ldots, 1\}$:

- The recursion formula (4.10) considers all possible couples of inventory level states:
  - $(\omega, 0)$: when transportation is performed at the end of period $j-1$, i.e. $I_{j-1} = \omega$ and $J_{j-1} = 0$;
  - $(0, \beta)$ and (ω, B_{j-1}): when transportation is not performed at the end of period $j-1$, i.e. $I_{j-1} = 0$ and $J_{j-1} = \beta$, or $I_{j-1} = \omega$ and $J_{j-1} = B_{j-1}$, $0 \le \omega \le \tilde{M}_j, 0 \le \beta \le N_j$.

  and, respectively, the corresponding optimal costs are evaluated in recursion formula (4.10): $v_j^{\omega 0}$, $v_j^{\omega B_{j-1}}$ and $v_j^{0\beta}$.
- For each couple of inventory level states $(\bullet, \bullet)$, all of three possible subsequent block structures are examined by the recursion formula (4.10): $[j,k,\ell]_{\bullet 0}^{\bullet\gamma}$, $[j,k,\ell]_{\bullet B_\ell}^{\bullet\gamma}$, and $[j,k,\ell]_{\bullet\delta}^{\bullet 0}$.

Given the exhaustive evaluation of all blocks performed via the recursion formula (4.10), it follows by induction that v_1^{00} provides the optimal cost of the ULS-B problem, where

$$v_1^{00} = \min_{1 \le \ell \le T} \min_{1 \le k \le \ell} \left\{ \min_{0 \le \gamma \le \tilde{M}_{\ell+1}} \left\{ \varphi_{00}^{0\gamma}(1,k,\ell) + v_{\ell+1}^{\gamma 0}, \varphi_{0B_\ell}^{0\gamma}(1,k,\ell) + v_{\ell+1}^{\gamma B_\ell} \right\}, \min_{0 \le \delta \le R_k} \left\{ \varphi_{0\delta}^{00}(1,k,\ell) + v_{\ell+1}^{0\delta} \right\} \right\}$$

. □

Example 4.1 (End): Focus on the *optimal solution computation*

The DP algorithm given in Proposition 1 starts from the last period 8, and proceeds backward in time from period 8 to 1. The optimal value from period 8 with an initial inventory of the main product equal to 200 is:

$$v_8^{200,0} = \varphi_{00}^{200,0}(8,0,8) + v_9^{00} = 0$$

$$v_7^{0,800} = \varphi_{800,0}^{0,200}(7,7,7) + v_8^{200,0} = \varphi_{800,0}^{0,200}(7,7,7)$$

$$\ldots$$

The optimal value of the ULS-B problem for instance I is equal to v_1^{00}, which corresponds to the value of the best succession of blocks:

$$\begin{aligned} v_1^{00} &= \varphi_{0,0}^{0,0}(1,1,2) + v_3^{00} \\ &= \varphi_{0,0}^{0,0}(1,1,2) + \varphi_{0,400}^{0,0}(3,3,3) + \varphi_{400,700}^{0,0}(4,4,5) + \varphi_{700,800}^{0,0}(6,6,6) + \varphi_{800,0}^{0,200}(7,7,7) \end{aligned}$$

Proposition 2. *The optimal value v_1^{00} can be computed in $O\big(T^3(\bar{B}+T\bar{d})^2\big)$, given the pre-calculation of all possible blocks, which can be done in $O\Big(T^3(T^2\bar{d}^2+T\bar{d}\bar{B}+\bar{B}^2)\Big)$, where $\bar{d}=\dfrac{\sum_{i=1}^T d_i}{T}$ and $\bar{B}=\dfrac{\sum_{i=1}^T B_i}{T}$.*

Proof. The cost of each block $\varphi_{\beta\delta}^{\omega\gamma}(j,k,\ell)$ presupposes the evaluation of three functions $\mathscr{P}(\cdot)$, $\mathscr{H}(\cdot)$ and $\hat{\mathscr{H}}(\cdot)$, which can be evaluated in linear time $O(T)$ for any fixed parameters $\omega\in[0,\tilde{M}_j], \gamma\in[0,\tilde{M}_{\ell+1}], \beta\in[0,N_j], \delta\in[0,R_l]$ and each fixed triplet (j,k,ℓ):

- Being an affine function, $\mathscr{P}(\cdot)$ is calculated in constant time.
- Functions $\mathscr{H}(\cdot)$ and $\hat{\mathscr{H}}(\cdot)$ can be evaluated in constant time with pre-calculated terms in $O(T^2)$.

Maximum values of ω and γ are lower than $T\bar{d}$, while values of β and δ are lower than $B_t, \forall t\in\mathscr{T}$ and hence on average lower than $\bar{B}$. Since a block is defined by two inventory periods, each having to respect one of the three conditions given in Definition 6, nine blocks structures can be evaluated before the execution of the dynamic programming algorithm for each fixed triplet (j,k,ℓ) in:

- $O(T^2\bar{d}^2)$: $\varphi_{00}^{\omega\gamma}(j,k,\ell)$, $\varphi_{0B_\ell}^{\omega\gamma}(j,k,\ell)$, $\varphi_{B_{j-1}0}^{\omega\gamma}(j,k,\ell)$ and $\varphi_{B_{j-1}B_\ell}^{\omega\gamma}(j,k,\ell)$;
- $O(T\bar{d}\bar{B})$: $\varphi_{0\delta}^{\omega 0}(j,k,\ell)$, $\varphi_{B_{j-1}\delta}^{\omega 0}(j,k,\ell)$, $\varphi_{\beta 0}^{0\gamma}(j,k,\ell)$ and $\varphi_{\beta B_\ell}^{0\gamma}(j,k,\ell)$;
- $O(\bar{B}^2)$: $\varphi_{\beta\delta}^{00}(j,k,\ell)$.

To sum up, the evaluation of all blocks can be preprocessed in $O\Big(T^3(T^2\bar{d}^2+T\bar{d}\bar{B}+\bar{B}^2)\Big)$, while the overall complexity of computing $v_j^{\omega 0}$, $v_j^{\omega B_{j-1}}$ and $v_j^{0\beta}$, for all $j\in\{T,T-1,\dots,1\}, \omega\in[0,\tilde{M}_{j-1}], \beta\in[0,N_j]$ quantities to $O\big(T^3(\bar{B}+T\bar{d})^2\big)$. □

Several numerical experiments have been conducted on a computer with Intel Xeon CPU E5-2620 2.10 GHz and implemented in C++ using Visual Studio 2013 to give an overview of the DP algorithm given in Proposition 1. Instances with $\bar{d}=30, \bar{B}=50$ and $T\le 6$ can be solved within less than 10 seconds. For instances with more than 7 periods, computational tests have been stopped due to memory overflow.

Remark 3. *The complexity of the dynamic programming algorithm given in Proposition 1 is pseudo-polynomial. Together with Theorem 1, it proofs that the ULS-B problem is weakly $\mathscr{N}\mathscr{P}$-Hard.*

4.5 ULS-B problem with a stationary capacity

In this section, let us examine a special case of the ULS-B problem with a stationary capacity of the by-product inventory level over the planning horizon, called ULS-B^{const}. This constant capacity is denoted by B in the rest of this section.

4.5.1 Structural properties of optimal solutions

For convenience, consider the following definitions.

Definition 9 (Full inventory period)**.** *A period $t\in\mathscr{T}$ is called **full inventory period**, if the inventory levels of the main product and the by-product are equal to zero at the end of period t, i.e. $I_t=0$ and $J_t=0$.*

Definition 10 (Fractional transportation period)**.** *A period $t \in \mathscr{T}$ is called **a fractional transportation period**, if $0 < W_t < B$.*

Definition 11 (Full transportation period)**.** *A period $t \in \mathscr{T}$ is called **a full transportation period**, if $W_t = B$.*

Property 4. *In an optimal solution of the ULS-B^{const} problem, there exists at most one fractional transportation period between two consecutive full inventory periods.*

Proof. The proof is done by contradiction. Let $j-1$ and ℓ be two consecutive full inventory periods in an optimal solution $S = (X, I, J, W)$. Suppose by contradiction that between periods $j-1$ and ℓ there exists two fractional transportation periods t and v such that $j-1 < t < v \leq \ell$. This supposition implies that $I_{j-1} = J_{j-1} = 0$, $I_\ell = J_\ell = 0$, $0 < W_t < B$ and $0 < W_v < B$.

Let u be the last production period before transportation period v. Consistent with the connectivity of the network flow corresponding to optimal solution S, there exists a path between:

- periods u and v, which contains flows strictly between their lower and upper bounds, since u and v are not full inventory periods, i.e. $0 < J_i < B, \forall i \in \{u, u+1, \ldots, v-1\}$;
- periods t and u.

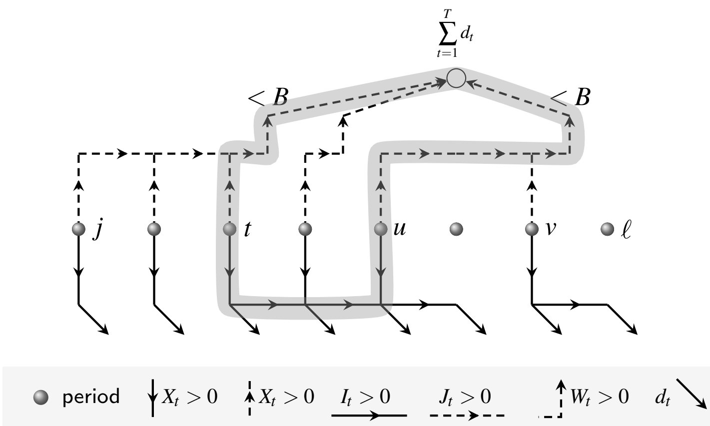


Figure 4.5: ULS-B^{const} with transportation periods between two full inventory periods $j-1, \ell \in \mathscr{T}$: *Network flow representation*

The path traversing flows associated with periods t, u and v forms a cycle with the flows emanated from transportation periods t and v, as illustrated in Figure 4.5. In particular, this path contains flows strictly between their lower and upper bounds, which contradicts Property 2 verified by the optimal solution S. Therefore, the assumption that there exists more than one fractional transportation period between two full inventory periods is false.

□

Example 4.2

Consider an instance I^{const} of ULS-B^{const} problem with a constant capacity $B = 680$. The values of all problem parameters are given in Table 4.2. The optimal value of the objective function is 23,680, and an optimal solution S^*_{const} is provided in Table 4.3. The network flow representation of S^*_{const} is given in Figure 4.6, and includes:

- 3 full inventory periods: 5, 7, 8;
- 1 full transportation periods: 3;

- 3 fractional transportation periods: 2, 7, 8.

Table 4.3: ULS-B^{const} problem: *An optimal solution* S^*_{const}

Decision variables	Values							
X	520	0	680	0	0	600	0	200
Y	1	0	1	0	0	1	0	1
I	320	20	300	100	0	500	0	0
J	520	0	0	0	0	600	0	0
W	0	520	680	0	0	0	600	200
Z	0	1	1	0	0	0	1	1
t	1	2	3	4	5	6	7	8

4.5.2 Dynamic programming algorithm

In the light of Property 4, the optimal solution of ULS-B^{const} can be calculated by decomposing the problem into blocks of periods delimited by full inventory periods.

Definition 12 (Block)**.** *The set of time periods $\{j, j+1, \dots, \ell\}$ between two consecutive full inventory periods $j-1$ and ℓ is called a* ***block****, where $j-1 < \ell$, $\forall j, \ell \in \{2, 3, \dots, T\}$.*

Under Property 4, let us define a block via three periods $[j, \ell]_f$:

- two consecutive full inventory periods $j-1$ and ℓ, i.e. an initial period j with zero entering stocks and a final period ℓ with zero ending stocks;
- a fractional transportation period f, with $j \le f \le \ell$.

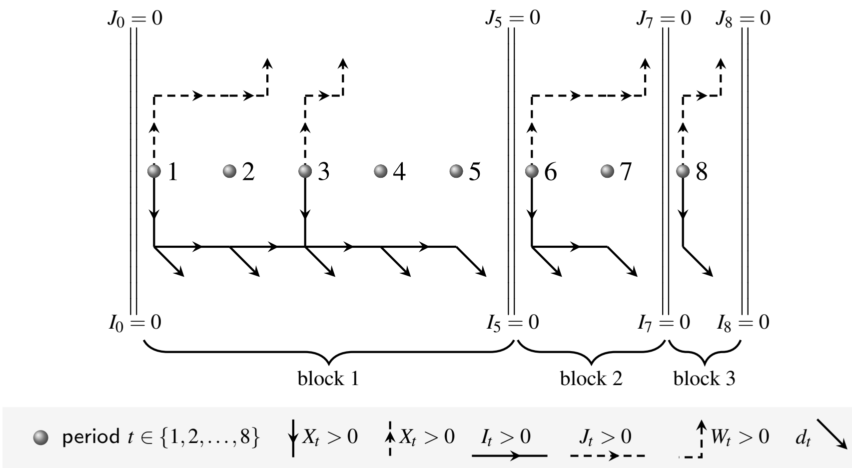


Figure 4.6: Network flow representation of optimal solution S^*_{const} of instance I^{const}

Between two full inventory periods, there is at most one fractional transportation period, all other transportation periods are full transportation periods. Hence, the total production is equal to the total demand between j and ℓ, i.e. $d_{j\ell} = \sum_{t=j}^{\ell} d_t$. Given that $\alpha = 1$, the total generated by-product is also equal to $d_{j\ell}$.

The number of full transportation periods inside a block $[j,\ell]_f$ is equal to $\left\lfloor \frac{d_{j\ell}}{B} \right\rfloor$. Let K denote the total number of transportation periods, full and fractional, inside a block $[j,\ell]_f$, i.e.:

$$K = \left\lceil \frac{d_{j\ell}}{B} \right\rceil.$$

Let Δ be the quantity transported during the fractional transportation period:

$$\Delta = d_{j\ell} - \left\lfloor \frac{d_{j\ell}}{B} \right\rfloor B.$$

There is no fractional transportation period between j and ℓ, if $\Delta = 0$. By convention, f is set to zero and when $\Delta = 0$.

Example 4.2 (Continuation): Focus on the *block decomposition*

Optimal solution S^*_{const} can be decomposed into three blocks, as depicted in Figure 4.6. Let us characterize each block, given that $B = 680$:

- **Block 1:** $d_{1,5} = 1,200$. This block has two transportation periods $K = 2$ including a fractional one. The quantity transported during the fractional transportation period is equal to 520, i.e. $\Delta = 1200 - 680 = 520$.
- **Block 2:** $d_{6,7} = 600$, then $K = 1$ and $\Delta = 600$. There is a fractional transportation period.
- **Block 3:** $d_{8,8} = 200$, then $K = 1$ and $\Delta = 200$. There is a fractional transportation period.

Definition 13 (Sub-Block)**.** *Let $[j,\ell]_f$ be a block. The set of periods $\{s, s+1, \ldots, t\}$, defined by:*

- *$s-1$ and t: two consecutive transportation periods between j and ℓ, $j \le s \le t \le \ell$;*
- *$n \in \{0,1,\ldots,K\}$: number of transportation periods performed before period s since period j.*

*is called **sub-block** and is denoted by $[s,t]_f^{jn}$.*

By virtue of Definition 13, the following information is known for each sub-block of type $[s,t]_f^{jn}$:

- The entering inventory level at period s and the ending inventory level at period t of the by-products are null, since transportation occurs by definition at periods $s-1$ and t:

$$J_{s-1} = J_t = 0 \tag{4.11}$$

- The entering inventory level of the main product in period t equals to:

$$I_{s-1} = \begin{cases} nB & -d_{j,s-1}, \text{ if } f \ge s \text{ or } f = 0 \\ (n-1)B + \Delta & -d_{j,s-1}, \text{ if } f < s \end{cases} \tag{4.12}$$

- The ending inventory level of the main product at period t can be calculated as follows:

$$I_t = \begin{cases} (n+1)B & -d_{jt}, \text{ if } f > t \text{ or } f = 0 \\ nB + \Delta & -d_{jt}, \text{ if } f \le t \end{cases} \tag{4.13}$$

The cost of sub-block $[s,t]_f^{jn}$ can be evaluated by solving a classical single-item lot-sizing problem using an $O(T \log T)$ dynamic programming algorithm (see e.g. Federgruen and Tzur [1991], Wagelmans et al. [1992], Aggarwal and Park [1993]) as follows:

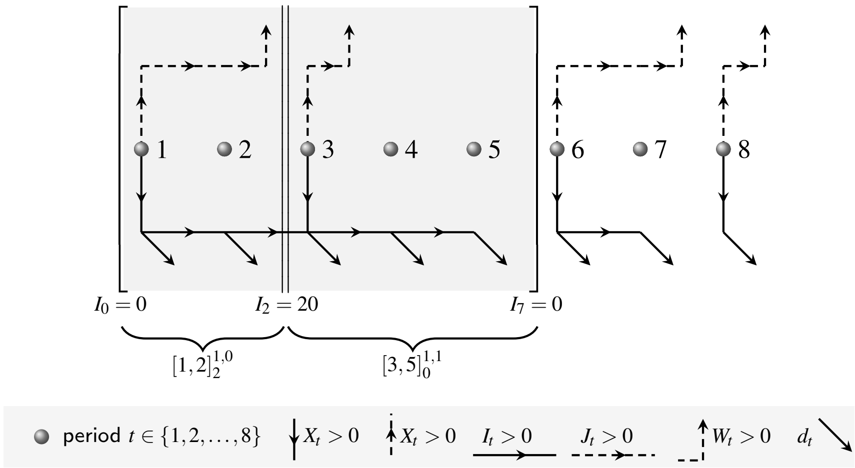


Figure 4.7: Inside the block $[1,5]_2$ with 2 transportation periods in optimal solution S^*_{const} of instance I^{const}

- Calculate the entering and ending inventory levels of the main product via expressions (4.12)-(4.13).
- Pre-treat the inventories of the main product, in order to obtain zero entering and zero ending inventories of the main product.
- Update the production cost at each period i by integrating the inventory costs of the by-product, as follows: $p_i := p_i + \sum_{j=i}^{t-1} \hat{h}_j,\ s \leq i \leq t$.
- Add the cost of the pre-treated initial stock of the main product to this cost.

Example 4.2 (Continuation): Focus on the *sub-block decomposition and evaluation*

The first block $[1,5]_2$, zoomed in Figure 4.7, contains two transportation periods, and can be subsequently decomposed into two sub-blocks. This block contains a fractional transportation period $f = 2$. Given that $B = 680$ and $\Delta = 520$, let us describe the two sub-blocks of block $[1,5]_2$.

- **Sub-block** $[1,2]_2^{1,0}$**:** It contains the fractional transportation period $f = 2$. There are 20 main products in the inventory at the end of period 2. After the pre-treatment step, the demand of the main product becomes 320 in period 2.
- **Sub-block** $[3,5]_0^{1,1}$**:** There is a quantity of main product equal to 20 in the inventory at the end of period 2. After the pre-treatment step, the demand of the main product becomes 380 in period 3. The cost related to the storage of a quantity of main products equal to 20 between periods 2 and 3 is added to the cost of the sub-block $[3,5]_0^{1,1}$.

Denote by $\varphi_f^{jn}(s,t)$ the optimal value of sub-block $[s,t]_f^{jn}$. Figure 4.8 gives an example of a decomposition into sub-blocks. Let f be the fractional transportation period, the optimal cost $v_f^K(j,t)$ from period j to a given transportation period t including n transportation periods is provided by:

$$v_f^n(j,t) = \min_{j \leq s \leq t} \left\{ v_f^{n-1}(j,s-1) + \varphi_f^{j,n-1}(s,t) \right\} + g_t, \tag{4.14}$$

with $v_f^{n-1}(j,j-1) = 0$.

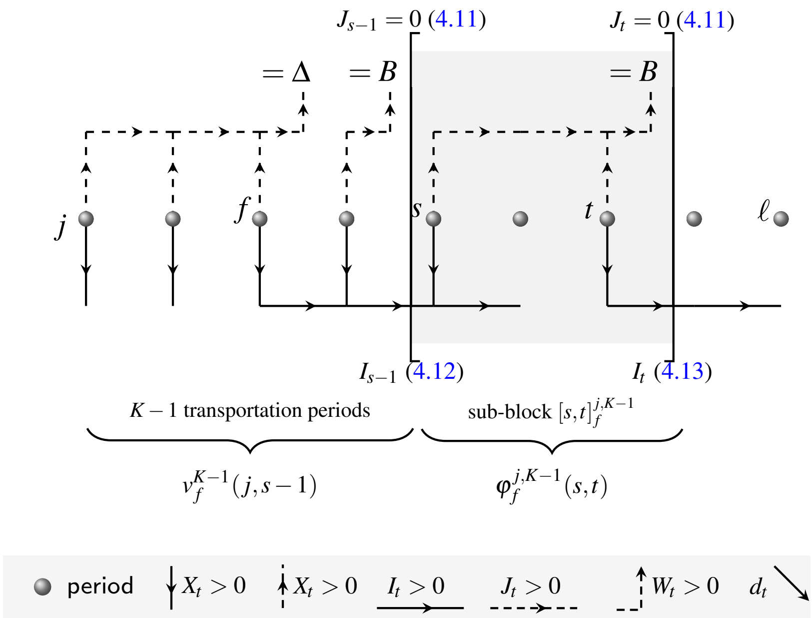


Figure 4.8: Inside a block $[j,\ell]_f$ with K transportation periods: *Network flow representation*

The optimal solution of a block of type $[j,\ell]_f$ is given by $v_f(j,\ell)$:

$$v_f(j,\ell) = \min_{j+K-1 \leq s \leq \ell} \left\{ v_f^K(j,s) + \sum_{t=s}^{\ell-1} h_t d_{t+1,\ell} \right\}$$

The cost of an optimal solution between two full inventory periods $j-1$ and ℓ is the minimum value among all possible costs $v_f(j,\ell)$ between two consecutive full inventory periods $j-1$ and ℓ, for all $f \in \{j, j+1, \ldots, \ell\}$:

$$v(j,\ell) = \begin{cases} \min\limits_{j \leq f \leq \ell} v_f(j,\ell), & \text{if } \Delta > 0 \\ v_0(j,\ell), & \text{if } \Delta = 0 \end{cases}$$

Proposition 3. *For initial null inventory levels of the main product and by-products, the optimal cost of the ULS-B problem is equal to $C(T)$ given by the last step of the following algorithm, which proceeds forward in time from period 1 to T, $\forall \ell \in \mathcal{T}$:*

$$C(0) = 0 \tag{4.15}$$

$$C(\ell) = \min_{1 \leq j \leq \ell} \left\{ C(j-1) + v(j,\ell) \right\} \tag{4.16}$$

Proof. Under Property 4, an optimal solution of ULS-B^{const} can be partitioned in a series of blocks delimited by consecutive full inventory periods. Given a period ℓ, the cost of ULS-B^{const} over the periods from 1 to ℓ can be recursively expressed via the minimum for all $j \in \{1,2,\ldots,\ell\}$ of the sum of: **(i)** the cost up to the previous full inventory period $C(j-1)$, and **(ii)** the cost of the block between two full inventory periods $j-1$ and ℓ.

Recursion formula (4.16) performs the exhaustive evaluation of all possible solution decompositions into blocks of full inventory periods. By induction, it follows that $C(T)$ gives the optimal cost of the ULS-B^{const} problem. □

Example 4.2 (End): Focus on the *optimal solution computation*

The initial equation for the DP algorithm given in Proposition 3 is:

$$C(0) = 0$$

since the initial state cost is 0. The recursive calculation of formula (4.16) is based on the evaluation of blocks of full inventory periods.

Making the same calculation for all blocks highlighted in Figure 4.6 as done for block $[1,5]_2$, we obtain: $v(6,7) = 6,400$ and $v(8,8) = 2,600$. Then, the value of optimal solution S^*_{const} is computed as follows:

$$\begin{aligned} C(8) &= C(7) + v(8,8) \\ &= C(5) + v(6,7) + v(8,8) \\ &= C(0) + v(1,5) + v(6,7) + v(8,8) \\ &= 0 + 14,680 + 6,400 + 2,600 \\ &= 23,680 \end{aligned}$$

Proposition 4. *The optimal value $C(T)$ can be computed in $O(T^6 \log T)$.*

Proof. The complexity of the dynamic programming algorithm given in Proposition 3 is due to the calculation of the cost of sub-blocks $\varphi_f^{j,n-1}(s,t)$. Each sub-block $[s,t]_k^{jn}$ can be evaluated by solving a classical single-item lot-sizing problem using an $O(T \log T)$ dynamic programming algorithm (see e.g. Federgruen and Tzur [1991], Wagelmans et al. [1992], Aggarwal and Park [1993]). Hence, the calculation of cost $v_f^K(j,t)$ is done in $O(T^2 logT)$ for any given periods $f, j, t \in \mathscr{T}$. Given the composition of functions $v_f^K(\bullet,\bullet)$, $v_f(\bullet,\bullet)$, $v(\bullet,\bullet)$ and $C(\bullet)$, it follows that the optimal cost of the ULS-B^{const} $C(T)$ can be computed in $O(T^6 \log T)$.

□

The average CPU times required to find the optimal solution for instances with different numbers of time periods, ranging from 10 to 50, varies between few seconds to hundred of seconds. To sum up, the DP algorithm in Proposition 3 is competitive (with CPU time less than 1 second) for small size instances (up to 20 periods), but quickly becomes out of competition, due to its high CPU time. For problems with 20 to 30 periods this algorithm can be used within a decomposition approach to solve more complex capacitated lot-sizing problems.

4.6 Conclusion and perspectives

Inspired by the circular economy thinking, this chapter introduced and investigated a new version of the lot-sizing problem. The addressed problem is an extension of the classical single-item uncapacitated lot-sizing problem, which integrates explicitly the management of by-products storable in a limited capacity. We showed that ULS-B problem with time-dependent inventory capacities of the generated by-products is weakly $\mathscr{N}\mathscr{P}$-Hard, and provided a pseudo-polynomial dynamic programming algorithm based on the solution decomposition into blocks. For the special case of ULS-B with stationary inventory capacities of the generated by-products, we gave a polynomial dynamic programming algorithm based on the derived structural properties of the optimal solutions.

In spite of its practical interest, the problem under study in this chapter can be extended in several ways:

- **From single-item to multi-item ULS-B problem:** An extension to a multi-item version of the ULS-B problem seems necessary to suitably take into account the industrial realities. For instance, the company which generates the sludge in the Kalunborg symbiosis in Denmark produces multiple kinds of pharmaceuticals with different components, which can vary the composition of the sludge and subsequently its treatment.

- **From a single production unit to an industrial symbiosis network:** The studied generic ULS-B problem is a first step towards the implementation of industrial symbiosis networks. Considering such a relation between two or several industries, gives rise to multiple questions: How to better integrate real-life features (proximity between industries, by-product storability, inventory capacities)? How to coordinate the joint production of multiple industries? How to regulate and formalize the agreements specifying the complementary relationship between industries?

To meet these issues, we extend the ULS-B problem, in the following chapters, from one production unit to two production units, playing the role of supplier and receiver of by-products. More precisely, in Chapter 5, we study a production planning problem raised in an industrial symbiosis context. This two-level problem is addressed in the framework of different collaboration policies. A quantitative analysis of centralized and distributed collaboration policies with a special focus on the information sharing in performed in Chapter 6.

Chapter 5

Lot-sizing for an industrial symbiosis

In this chapter, we study a production planning problem arisen between two production units (PU) within an industrial symbiosis. During the production process of a main product, a production residue is generated by the first PU, which is subsequently either used as raw materials by the second PU, or disposed of. The second PU can also purchase raw materials from an external supplier. The resulting combined production planning problem has been formulated as a two-level single-item lot-sizing problem. We prove that this problem is *NP*-Hard irrespective of the production residue, namely unstorable, or storable with a limited capacity. To efficiently solve this problem, a heuristic based on Lagrangian decomposition is proposed. Extensive numerical experiments highlight the competitiveness of the proposed solution method. The impact of the collaborative framework, in which the production plans of each PU are brought together, has been studied via a comparative analysis of different decentralized and centralized collaboration policies. Valuable insights derived from this analysis are subsequently used to discuss the managerial implications of setting up an industrial symbiosis between a supplier of by-products and its receiver.

Contents

5.1 Introduction

Waste accumulation in landfills, the depletion of finite raw materials and global warming push our generation to shift towards a circular economy and to adopt a number of sustainable practices such as recycling end-of-life products, using renewable energy, reducing greenhouse gas emissions. Since few years, the legislation around the world is evolving to promote the reuse of production residues, particularly those that are no longer allowed in landfills[30].

Industrial symbiosis represents all forms of binding traditionally separate industrial entities in a joint production system, suitable for providing, sharing and reusing resources to create mutual added value. One of the most common beneficial form of symbiotic industrial production is the process by which, by-products of one production unit become raw materials for another, as illustrated in Figure 2.5. Application of the industrial symbiosis allows production residues to be used in a more sustainable way and contributes to the development of a circular economy.

By its nature, industrial symbiosis offers opportunities for the three dimensions of the sustainable development, namely economic, environmental and social, by: **(i)** avoiding disposal costs and increasing resource efficiency, **(ii)** reducing the consumption of raw materials, **(iii)** supporting the regional economic development. As a consequence and due to the success of the existing eco-industrial parks, a number of industrial symbiosis emerges all around the world. In 2017, Evans et al. [2017] referenced 281 implemented by-product synergies and about 150 planned or under feasible study, spread over almost all continents.

All these industrial symbiosis networks have different characteristics (such as storability, transportation mode) and the collaboration policies can differ from one industrial symbiosis to another. There exist cases, where the involved actors develop a mutual trust between themselves and share all their information, enabling thus possible a centralized decision-making related to the by-product exchange. The full information sharing is usually encountered when both actors belong to the same parent company or when a third party manages the industrial symbiosis network. In general, the full information sharing between actors may be difficult to be considered for different reasons such as the requirements to keep sensitive information private or not to reveal the risks related to production disruptions or production recipes of products [Vladimirova et al., 2018, Fraccascia and Yazan, 2018]. Partial and none information sharing are commonly addressed via decentralized collaboration policies. Two main kinds of decentralized collaboration policies with respect to their time frames have been identified (see Section 2.4), namely **(i)** opportunistic for a short-term collaboration, and **(ii)** symbiotic for a long-term collaboration.

To cope with the operation management problems posed by the industrial symbiosis, the current chapter contributes by:

- Introducing and formalizing a lot-sizing problem in the framework of an industrial symbiosis: The addressed problem extends the work done in Chapter 4, by integrating the *receiver* production unit in the management of a by-product generated by the *supplier* production unit. The lot-sizing problem introduced in this chapter enriches the joint production planning literature by its novelty and industrial relevance.

- Studying a two-level lot-sizing problem for different characteristics: The studied problem falls within the class of two-level lot-sizing problems, where the first level corresponds to the problem encountered by the supplier, and the second level corresponds to the problem encountered by the receiver. This is distinguished from classical state-of-the-art problems by its characteristics such as inventory capacities at the supplier level and external intermediate flows between levels.

- Proposing a solution method based on both Lagrangian decomposition and Lagrangian relaxation, whose efficiency and effectiveness are shown via extensive numerical experiments.

- Discussing centralized and decentralized collaboration policies, which can be applied in full and none information sharing settings.

The remainder of this chapter is organized as follows. Section 5.2 reviews the literature covering the class of two-level lot-sizing problems. The generic version of the problem under study is described and a complexity analysis is conducted in Section 5.3. A Lagrangian decomposition approach is proposed in Section 5.4. Different collaboration policies are introduced and addressed in Section 5.5. The competitiveness of the Lagrangian decomposition algorithm is empirically shown by performing computational experiments on two versions of the problem with: storable by-products with a limited capacity and unstorable by-products. Managerial implications of different collaboration policies are discussed in Section 5.6. Concluding remarks and perspectives are provided in Section 5.8.

5.2 Literature review related to two-level lot-sizing problems

In the current chapter, the by-products generated by a production unit are used by another production unit. In order to better apprehend the interactions between these two production units, let us review two-level production planning problems, by placing a special focus on the problems with inventory capacities at the supplier level and external intermediate flows between levels.

The studied problem can be considered as a two-level lot-sizing problem, where the first level corresponds to the problem encountered by the supplier, and the second level corresponds to the problem encountered by the receiver. There are multiple configurations of two-level production planning problems: the production-transportation problem (see e.g. Melo and Wolsey [2010, 2012], Hwang et al. [2016]), the supplier-retailer problem (see e.g. Brahimi et al. [2015], Phouratsamay et al. [2018]) and the One Warehouse Multi-Retailer (OWMR) problem (see e.g. Arkin et al. [1989], Solyalı and Süral [2012]) which are the most studied. As the OWMR problem is different from the problem studied in this chapter, we only take interest of the production-transportation and supplier-retailer problems.

Let us position the problem studied in this chapter within the two-level production planning literature according to its characteristics:

Inventory capacities at the supplier level. The two-level lot-sizing problem with inventory capacities at the retailer level can be polynomially solved using dynamic programming algorithms. Hwang and Jung [2011], Phouratsamay et al. [2018] provided algorithms running in a polynomial time to solve a number of different versions of this problem. Note that the case with inventory capacities at the first level is *NP*-Hard (Jaruphongsa et al. [2004], Brahimi et al. [2015], Phouratsamay et al. [2018]). Jaruphongsa et al. [2004] proposed a two-level problem with demand time-window constraints and stationary inventory bounds at the first level. By allowing demand splitting, Jaruphongsa et al. [2004] solved the problem using a polynomial algorithm based on dynamic programming. Brahimi et al. [2015] solved the supplier-retailer problem with inventory capacities using a Lagrangian relaxation. Phouratsamay et al. [2018] solved the same problem using a pseudo-polynomial algorithm based on dynamic programming. The problem studied in the current chapter is different from Brahimi et al. [2015], Phouratsamay et al. [2018] in the sense that the inventory capacities are not on the product, which has to meet a demand.

External intermediate flows between levels. External intermediate flows between the two levels can occur while considering intermediate demands at the supplier level. Papers dealing with intermediate demands are not numerous. Melo and Wolsey [2010] considered a two-level lot-sizing problem, where an intermediate product is created at the first level, required to meet only the demand in the second level. On the contrary, Zhang et al. [2012a] proposed valid inequalities to solve a multi-echelon lot-sizing problem, where the output of each intermediate echelon has its own external demand to fulfill and can also be used as an input to the next echelon. In the same way, Van Vyve et al. [2014] introduced a problem, where a unique intermediate product is used to multiple outputs. Ahmed et al. [2016] and He et al. [2015] dealt with the multi-stage version of the problem with intermediate demands of the final product as a minimum concave cost flow network problem. Against this background, the novelty of the problem under study lies in the consideration of external intermediate flows and the inventory bounds at the supplier level.

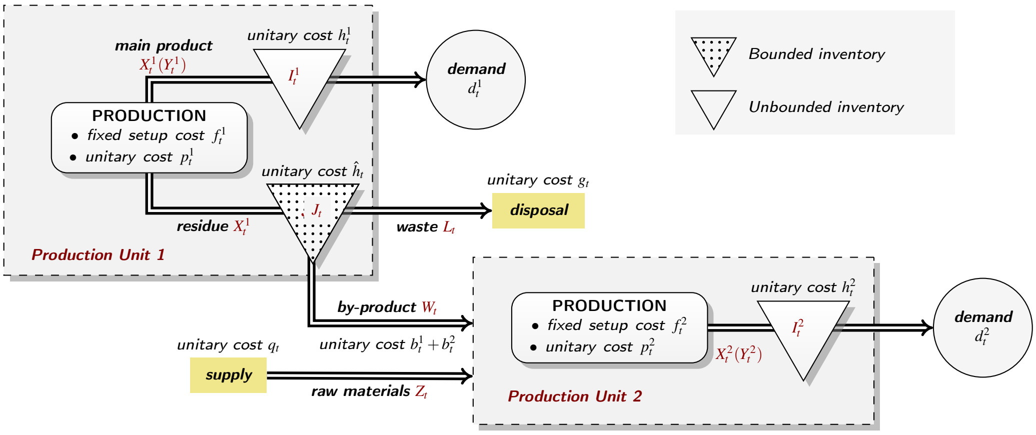


Figure 5.1: Process flow diagram of the ULS-IS problem

5.3 Problem statement

Consider a lot-sizing problem for an industrial symbiosis (ULS-IS), where two production units (PU1 and PU2) have to plan their production over a planning horizon of T periods, as illustrated in Figure 5.1. Each production unit produces a product to meet a deterministic demand. Denote by d_t^1 (resp. d_t^2) the demand of PU1 (resp. PU2) at period $t \in \{1,2,...,T\}$. In addition, during the process of producing a quantity of X_t^1 units of main product in PU1, a quantity of X_t^1 units of by-products is generated. In the same way, to produce X_t^2 units of main product in PU2 at period $t \in \{1,2,...,T\}$, X_t^2 units of raw material are required. The by-product generated by PU1 can be assimilated as the raw material needed to produce the main product of PU2.

To ensure the procurement of raw materials, PU2 can supply its production either with the by-products generated by PU1, or with the raw materials from an external supplier. The quantity of by-products, which is not used by PU2, can be disposed of or stored by PU1, as long as the stored quantity does not exceed a limited capacity B in each period $t \in \{1,2,...,T\}$. The quantity of by-products, transported from PU1 to PU2 at each period $t \in \{1,2,...,T\}$, is denoted by W_t. The quantity of raw materials bought from the external supplier at period $t \in \{1,2,...,T\}$ is denoted by Z_t. Let L_t (resp. J_t) be the quantity of by-products disposed of (resp. stored) in period $t \in \{1,2,...,T\}$.

The management of the exchange of by-products and the supply of raw materials generate the following unitary costs in each period $t \in \{1,2,...,T\}$:

- a unitary disposal cost g_t, paid by PU1,
- a unitary inventory holding cost $\hat{h}_t$ paid by PU1 to store the generated by-products,
- a unitary cost of reusing by-products of PU1 by PU2, decomposed into two unitary costs: b_t^1 (resp. b_t^2) paid by PU1 to prepare by-products for further use (resp. paid by PU2 to transport by-products from PU1 to PU2),
- a unitary purchasing cost q_t of raw materials supplied from an external supplier, paid by PU2.

Moreover, each PU pays the classical lot-sizing costs per period $t \in \{1,2,...,T\}$: a unitary production cost p_t^1 (resp. p_t^2), a fixed setup cost f_t^1 (resp. f_t^2), and a unitary holding cost h_t^1 (resp. h_t^2), paid by PU1 (resp. PU2). The binary setup indicators of production for PU1 and PU2 are denoted by Y_t^1 and Y_t^2, respectively. Let I_t^1 be the inventory level of the main product in PU1 at the end of period t, and I_t^2 be the inventory level of the product in PU2. The problems parameters and variables are summarized in Table 5.1.

In what follows, a number of assumptions are made, without loss of generality:

(A.1) The by-product inventory is null at the end of the planning horizon, i.e. $J_T = 0$.

(A.2) The treatment or transportation cost of by-products imputed to PU1 is lower than their disposal cost performed by PU1, i.e. $b^1 \leq g, \forall t \in \mathscr{T}$.

(A.3) The treatment or transportation cost of by-products imputed to PU2 is lower than its purchasing cost, i.e. $b^2 \leq q, \forall t \in \mathscr{T}$.

(A.4) On average, the by-product inventory holding cost is small enough to make possible the storage of by-products instead of their disposing of, i.e. $\sum_{t=1}^{T} \hat{h}_t \leq \sum_{t=1}^{T} (g_t - b_t^1)$. Otherwise, the problem to solve can be reduced to the problem without intermediate storage of the by-product.

(A.5) The needs for raw materials in PU2 cannot trigger the production in PU1, i.e. $q \leq p^1 + b^1 + b^2, \forall t \in \mathscr{T}$.

(A.6) An available quantity of by-products in PU1 cannot trigger the production in PU2, i.e. $g \leq p^2 + b^1 + b^2, \forall t \in \mathscr{T}$.

(A.7) The by-product has lower value than the main product, i.e. $\sum_{t=1}^{T} \hat{h}_t \leq \sum_{t=1}^{T} h_t^1$.

(A.8) The main product cannot be produced and stored with the single aim of creating gains resulting from the recovering of production residues, i.e. $(g - b^1) \leq h^1$ and $(q - b^2) \leq h^2$.

Note that, if one of Assumptions (A.1)-(A.4) is not verified, the problem becomes trivial. Assumptions (A.5)-(A.8) are made to comply with the definition of by-products. Note that the relaxation of Assumptions (A.5)-(A.8) reduces the problem under study to a lot-sizing problem dealing with co-products (see e.g. Ağralı [2012]).

5.3.1 Straightforward formulation (AGG)

Using the notations given in Table 5.1, the ULS-IS problem can be modeled via the following straightforward formulation:

$$\text{minimize} \sum_{t=1}^{T} (p_t^1 X_t^1 + f_t^1 Y_t^1 + h_t^1 I_t^1 + \hat{h}_t J_t + g_t L_t + b_t^1 W_t) + \sum_{t=1}^{T} (p_t^2 X_t^2 + f_t^2 Y_t^2 + h_t^2 I_t^2 + q_t Z_t + b_t^2 W_t) \tag{5.1}$$

subject to:

$$I_{t-1}^1 + X_t^1 - I_t^1 = d_t^1, \quad \forall t \in \{1,2,\ldots,T\} \tag{5.2}$$
$$I_0^1 = 0, \tag{5.3}$$
$$X_t^1 \leq M_t^1 Y_t^1, \quad \forall t \in \{1,2,\ldots,T\} \tag{5.4}$$
$$I_{t-1}^2 + X_t^2 - I_t^2 = d_t^2, \quad \forall t \in \{1,2,\ldots,T\} \tag{5.5}$$
$$I_0^2 = 0, \tag{5.6}$$
$$X_t^2 \leq M_t^2 Y_t^2, \quad \forall t \in \{1,2,\ldots,T\} \tag{5.7}$$
$$J_{t-1} + X_t^1 = W_t + L_t + J_t, \quad \forall t \in \{1,2,\ldots,T\} \tag{5.8}$$
$$J_0 = J_T = 0, \tag{5.9}$$
$$J_t \leq B, \quad \forall t \in \{1,2,\ldots,T\} \tag{5.10}$$
$$W_t + Z_t = X_t^2, \quad \forall t \in \{1,2,\ldots,T\} \tag{5.11}$$
$$X_t^1, X_t^2, I_t^1, I_t^2, W_t, Z_t, J_t, L_t \geq 0, \quad \forall t \in \{1,2,\ldots,T\} \tag{5.12}$$
$$Y_t^1, Y_t^2 \in \{0,1\}, \quad \forall t \in \{1,2,\ldots,T\} \tag{5.13}$$

The objective function (5.1) minimizes the sum of all costs: production, inventory holding, transportation, disposal and purchasing. Constraints (5.2) and (5.5) model the flow conservation of the main products of PU1 and PU2, respectively. Constraints (5.3) and (5.6) set to zero the initial inventory levels of the main products of both PU1 and PU2. Constraint (5.9) fix the initial and ending inventory levels of the by-product to zero. Constraints (5.4) and (5.7) are production constraints, which ensure that the production launching at a given period entails a setup operation at the same period. The inventory capacity of the by-product is limited by Constraints (5.10). Constraints (5.8) and (5.11) are conservation constraints of flows of by-products and external raw materials: Constraints (5.8) ensure that the production residue generated by PU1 are either disposed of, stored or used, and Constraints (5.11) deal with the purchasing of raw materials required by PU2. Constraints (5.12) and (5.13) are the nonnegativity and binary requirement constraints.

Table 5.1: Summary of the problem parameters

Parameters:	
T	Number of time periods
d_t^1 (d_t^2)	Demand for the main product of PU1 (PU2) in period t
p_t^1 (p_t^2)	Unitary production cost for PU1 (PU2) in period t
f_t^1 (f_t^2)	Fixed setup cost for PU1 (PU2) in period t
h_t^1 (h_t^2)	Unitary holding cost of the main product of PU1 (PU2) in period t
$\hat{h}_t$	Unitary holding cost of the by-product of PU1 in period t
q_t	Unitary cost of purchasing raw materials from an external supplier by PU2 in period t
b_t^1 (b_t^2)	Unitary treatment or transportation cost imputed to PU1 (PU2) for the by-product in period t
g_t	Unitary by-product disposal cost of PU1 in period t
B	By-product inventory capacity in PU1 in each period
M_t^1 (M_t^2)	Big number with $M_t^1 = \sum_{i=t}^{T} d_i^1 \left(M_t^2 = \sum_{i=t}^{T} d_i^2 \right)$
Decision variables:	
X_t^1 (X_t^2)	Production quantity in PU1 (PU2) in period t
Y_t^1 (Y_t^2)	Binary setup indicator for PU1 (PU2) for period t
I_t^1 (I_t^2)	Inventory level of the main product of PU1 (PU2) at the end of period t
J_t	Inventory level of the by-product of PU1 at the end of period t
W_t	Quantity of by-products sent from PU1 to PU2 in period t
Z_t	Quantity of raw materials purchased at an external supplier by PU2 in period t
L_t	Disposal quantity of by-products in period t

5.3.2 Facility location formulation (FAL)

In the lot-sizing literature, the straightforward formulation is very intuitive and easy to understand, but its linear relaxation usually provides a poor dual bound [Brahimi et al., 2017]. As we can fear that the straightforward formulation becomes intractable for large size problems, we propose a disaggregated formulation, called *the facility location model* [Krarup and Bilde, 1977], to link the production variables not only with their production period, but also with their consumption period.

In the ULS-IS problem, there is production of two different main products linked by a flow of storable by-products. Thus, the facility location formulation of the ULS-IS problem requires the introduction of new variables that link production and inventory variables with: **(i)** production periods in PU1, **(ii)** consumption periods of the main product in PU1, **(iii)** periods of using by-products or periods of ordering raw materials (i.e. production periods in PU2) and, **(iv)** consumption periods of the main product in PU2. To do this, we introduce a new set of variables U_{ijkl}, $i \in \{1,...,T\}$, $j \in \{i, i+1,...,T\}$, $k \in \{i, i+1,...,T\}$, $l \in \{k, k+1,...,T\}$, which represent:

1. the quantity of the main product produced in PU1 in period i,
2. to fulfill a fraction of the demand of the main product in period j in PU1,
3. such that the quantity of by-products generated in i is stored until k and sent to PU2 in order to produce the main product of PU2 in period k,
4. to meet the demand of PU2 in period l.

By convention, variables U_{00kl} are used to denote the quantity of raw materials purchased from an external supplier to be used in period k to produce the main product of PU2 for satisfying the demand in period l. We also consider variables U_{ijk0} to represent the quantity of by-products generated by PU1 in period i to satisfy the demand of the main product in period j, which is disposed of after having been stored until period k.

A new parameter a_{ijkl} corresponding to the cost associated with variables U_{ijkl}, $\forall i \in \{0,1,...,T\}$, $\forall j \in \{0,i,i+1,...,T\}$, $\forall k \in \{i,i+1,...,T\}$, $\forall l \in \{0,k,k+1,...,T\}$ is added. Parameter a_{ijkl} is defined as follows:

$$a_{ijkl} = \begin{cases} p_i^1 + b_k^1 + b_k^2 + p_k^2 + \sum_{t=i}^{j-1} h_t^1 + \sum_{t=k}^{l-1} h_t^2 + \sum_{t=i}^{k-1} \hat{h}_t, & \text{if } i,j,l \neq 0 \ (i \leq j, i \leq k \leq l) \\ p_i^1 + g_k + \sum_{t=i}^{j-1} h_t^1 + \sum_{t=i}^{k-1} \hat{h}_t, & \text{if } i,j \neq 0, l = 0 \ (i \leq j, i \leq k) \\ p_k^2 + q_k + \sum_{t=k}^{l-1} h_t^2, & \text{if } i,j = 0, l \neq 0 \ (k \leq l) \\ +\infty, & \text{otherwise.} \end{cases}$$

The facility location formulation of the ULS-IS problem is given below:

$$\text{minimize} \sum_{i=1}^{T}\sum_{j=i}^{T}\sum_{k=i}^{T}\left(\sum_{l=k}^{T} a_{ijkl}U_{ijkl} + a_{ijk0}U_{ijk0}\right) + \sum_{k=1}^{T}\sum_{l=k}^{T} a_{00kl}U_{00kl} + \sum_{t=1}^{T}\left(f_t^1 Y_t^1 + f_t^2 Y_t^2\right) \tag{5.14}$$

subject to:

$$\sum_{i=1}^{j}\sum_{k=i}^{T}\left(\sum_{l=k}^{T} U_{ijkl} + U_{ijk0}\right) = d_j^1, \qquad \forall j \in \{1,2,...,T\} \tag{5.15}$$

$$U_{ijkl} \leq d_j^1 Y_i^1, \qquad \forall i,j,k,l \in \{1,2,...,T\} \ (i \leq j, i \leq k \leq l) \tag{5.16}$$

$$U_{ijk0} \leq d_j^1 Y_i^1, \qquad \forall i,j,k \in \{1,2,...,T\} \ (i \leq j, i \leq k) \tag{5.17}$$

$$\sum_{k=1}^{l}\left(\sum_{i=1}^{k}\sum_{j=i}^{T} U_{ijkl} + U_{00kl}\right) = d_l^2, \qquad \forall l \in \{1,2,...,T\} \tag{5.18}$$

$$U_{ijkl} \leq d_l^2 Y_k^2, \qquad \forall i,j,k,l \in \{1,2,...,T\} \ (i \leq j, i \leq k \leq l) \tag{5.19}$$

$$U_{00kl} \leq d_l^2 Y_k^2, \qquad \forall k,l \in \{1,2,...,T\} \ (k \leq l) \tag{5.20}$$

$$\sum_{i=1}^{t}\sum_{j=i}^{T}\sum_{k=t+1}^{T}\left(\sum_{l=k}^{T} U_{ijkl} + U_{ijk0}\right) \leq B, \qquad \forall t \in \{1,2,...,T\} \tag{5.21}$$

$$U_{ijkl}, U_{00kl}, U_{ijk0} \geq 0, \qquad \forall i,j,k,l \in \{1,2,...,T\} \ (i \leq j, i \leq k \leq l) \tag{5.22}$$

$$Y_i^1, Y_i^2 \in \{0,1\}, \qquad \forall i \in \{1,2,...,T\} \tag{5.23}$$

The objective function (5.14) minimizes the sum of unitary and fixed costs. Constraints (5.15) and (5.18) ensure the demand satisfaction of PU1 and PU2, respectively. Constraints (5.16), (5.17), (5.19) and (5.20) are production constraints, i.e. there is a setup at a given period if there is production at the same period. Constraints (5.21) limit the level of the by-product inventory. The definition domains of decision variables are given in Constraints (5.22)-(5.23).

Note that, experimental tests show that adding the following valid inequalities:

$$\sum_{k=i}^{T}\left(\sum_{l=k}^{T} U_{ijkl} + U_{ijk0}\right) \leq d_j^1 Y_i^1, \quad \forall i,j \in \{1.2,...,T\}, (i \leq j) \tag{5.24}$$

$$\sum_{i=1}^{k}\sum_{j=i}^{T} U_{ijkl} + U_{00kl} \leq d_l^2 Y_k^2, \quad \forall k,l \in \{1.2,...,T\}, (k \leq l) \tag{5.25}$$

allows to substantially reduce the computational time needed to optimally solve the facility location formulation. The computational time is even lower when replacing Constraints (5.16)-(5.17) and (5.19)-(5.20) by Constraints (5.24)-(5.25).

5.3.3 Complexity analysis

In this section, we study the complexity of the ULS-IS problem. To do this, let us: **(i)** consider a particular case of the ULS-IS problem when the by-product is unstorable, **(ii)** show that this particular case is *NP*-Hard by performing a reduction from the classical capacitated lot-sizing problem, and **(iii)** derive the complexity of the general case of the ULS-IS problem. The following proposition holds:

Proposition 5. *The ULS-IS problem with an unstorable by-product ($B = 0$), i.e. with no by-product inventory (ULS-IS-NI) is NP-Hard.*

Proof. The proof of *NP*-Hardness is performed by reduction of ULS-IS-NI from the capacitated lot-sizing (CLS) problem, whose general case is known to be *NP*-Hard [Florian et al., 1980]. The decision version of the CLS problem is defined by:

- a planning horizon of $\tilde{T}$ periods $\{1,2,...,\tilde{T}\}$,
- limited production capacities $\tilde{C}_t, \forall t \in \{1,2,...,\tilde{T}\}$,
- demands $\tilde{d}_t, \forall t \in \{1,2,...,\tilde{T}\}$,
- three cost components: fixed setup costs $\tilde{f}_t$, unit production costs $\tilde{p}_t$ and unit inventory holding costs $\tilde{h}_t, \forall t \in \{1,2,...,\tilde{T}\}$.

Let $\tilde{X} = (\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_T)$ be the vector of produced quantities, and $\tilde{I} = (\tilde{I}_1, \tilde{I}_2, \ldots, \tilde{I}_T)$ be the vector of inventory levels during the planning horizon. Denote by $\tilde{Y} = (\tilde{Y}_1, \tilde{Y}_2, \ldots, \tilde{Y}_T)$ the production indicator vector. The question posed by the CLS problem is: Does there exist a production plan $(\tilde{X}, \tilde{I}, \tilde{Y})$ of total cost at most equal to a given value V, which satisfies demands $\tilde{d} = (\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_T)$?

An instance I^{CLS} of the CLS problem can be transformed into an instance I of ULS-IS-NI by making the following substitutions $\forall t \in \{1,2,\ldots,\tilde{T}\}$:

(S.1) Number of periods: $T = \tilde{T}$;

(S.2) Demands: $d_t^1 = \tilde{C}_t$, $d_t^2 = \tilde{d}_t$;

(S.3) Costs related to the main product of PU1: $f_t^1 = 0$, $p_t^1 = 0$ and $h_t^1 = 1$;

(S.4) Costs related to the product of PU2: $f_t^2 = \tilde{f}_t$, $p_t^2 = \tilde{p}_t$ and $h_t^2 = \tilde{h}_t$;

(S.5) Costs related to by-products of PU1 and raw materials of PU2: $g_t = 0$, $b_t = 0$ and $q_t = V$.

Let us show that instance I^{CLS} has an affirmative answer, if and only if, there exists a feasible solution $(X^1,Y^1,I^1,X^2,Y^2,I^2,W,Z,L)$ for instance I such that:

$$\sum_{t=1}^{T}\left(p_t^1X_t^1+f_t^1Y_t^1+h_t^1I_t^1+g_tL_t+b_t^1W_t\right)+\sum_{t=1}^{T}\left(p_t^2X_t^2+f_t^2Y_t^2+h_t^2I_t^2+q_tZ_t+b_t^2W_t\right)\le V. \tag{5.26}$$

To do this, we prove the conditional relationship between CLS and ULS-IS-NI problems related to the solution existence.

($\Longrightarrow$) Suppose that instance I^{CLS} has an affirmative answer. Let $\tilde{S}=(\tilde{X},\tilde{I},\tilde{V})$ be a production plan, such that $\sum_{t=1}^{\tilde{T}}\left(\tilde{p}_t\tilde{X}_t+\tilde{f}_t\tilde{Y}_t+\tilde{h}_t\tilde{I}_t\right)\le V$.
A feasible solution $(X^1,Y^1,I^1,X^2,Y^2,I^2,W,Z,L)$ for instance I, such that the total cost is at most equal to V, can be built as follows: **(i)** produce $X^1=\tilde{C}$ quantities of the main product in PU1, this generates by-product quantities less than $\tilde{C}$ by virtue of substitution (S.2), **(ii)** $I^1=0$, hold inventory levels of the main product of PU1 to zero according to (S.3), **(iii)** transport to PU2 the quantity of by-product $W_t=\tilde{X}_t$ in each period and dispose of $L_t=\tilde{C}_t-\tilde{X}_t$ by virtue of substitution (S.5), **(iv)** produce $X^2=\tilde{X}$ quantities in PU2, and **(v)** $I^2=\tilde{I}$, hold the levels of the product in PU2 to $\tilde{I}$. Given substitutions (S.1)-(S.5), it follows that equation (5.26) is hold.

($\Longleftarrow$) Conversely, assume that instance I has a positive answer, i.e. there exists a production plan

$$(X^1,Y^1,I^1,X^2,Y^2,I^2,W,Z,L)$$

, which satisfies all demands with a cost at most equal to V. Making use of substitutions (S.1)-(S.5), it can immediately be checked that $\sum_{t=1}^{T}\left(\tilde{p}_t\tilde{X}_t+\tilde{f}_t\tilde{Y}_t+\tilde{h}_t\tilde{I}_t\right)\le V$, where $\tilde{X}=X^2$, $\tilde{I}=I^2$ and $\tilde{Y}=Y^2$.

□

Remark 4. *As the ULS-IS-NI problem is a particular case of the ULS-IS problem, the ULS-IS problem is also NP-Hard.*

5.4 Solution method based on Lagrangian decomposition

Lagrangian decomposition approaches have been successfully used to solve a large variety of optimization problems. The main idea is to decompose a complex problem, often *NP*-Hard, into two or more easy to solve sub-problems. To do this, a set of variables is duplicated and the constraints corresponding to the equivalence of these variables are relaxed and penalized in the objective function by Lagrangian multipliers (see e.g. Fisher [1981]). Lagrangian decomposition provides a lower bound to the initial problem. Upper bounds can be computed using a Lagrangian heuristic, which transforms the obtained infeasible solutions into feasible ones. This procedure is repeated a large number of iterations in order to improve the obtained lower bounds.

The set of variables W_t, which links PU1 and PU2, corresponds to the flows of by-products. Let us duplicate these variables as follows: W_t represents the quantity of by-products to be sent to PU2 in period $t\in\{1,2,\dots,T\}$, and $\bar{W}_t$ represents the quantity of by-products to be recovered from PU1 in period $t\in\{1,2,\dots,T\}$. The straightforward formulation of the problem under study becomes:

$$\text{minimize}\sum_{t=1}^{T}(p_t^1X_t^1+f_t^1Y_t^1+h_t^1I_t^1+\hat{h}_tJ_t+g_tL_t+b_t^1W_t)+\sum_{t=1}^{T}(p_t^2X_t^2+f_t^2Y_t^2+h_t^2I_t^2+q_tZ_t+b_t^2\bar{W}_t) \tag{5.27}$$

subject to:

$$(5.2)-(5.7),(5.9)-(5.10),(5.12)-(5.13) \tag{5.28}$$

$$W_t+L_t+J_t=X_t^1+J_{t-1}, \quad \forall t\in\{1,2,...,T\} \tag{5.29}$$

$$\bar{W}_t+Z_t=X_t^2, \quad \forall t\in\{1,2,...,T\} \tag{5.30}$$

$$\bar{W}_t=W_t, \quad \forall t\in\{1,2,...,T\} \tag{5.31}$$

$$W_t,\bar{W}_t\geq 0, \quad \forall t\in\{1,2,...,T\} \tag{5.32}$$

By relaxing constraints (5.31), the objective function becomes:

$$\begin{aligned} C_{LD}(\lambda) = \min \sum_{t=1}^{T}(p_t^1X_t^1+f_t^1Y_t^1+h_t^1I_t^1+\hat{h}_tJ_t+g_tL_t+b_t^1W_t) \\ +\sum_{t=1}^{T}(p_t^2X_t^2+f_t^2Y_t^2+h_t^2I_t^2+q_tZ_t+b_t^2\bar{W}_t) \\ +\sum_{t=1}^{T}\lambda_t(\bar{W}_t-W_t) \end{aligned}$$

where $\lambda\in\mathbb{R}^T$ is the vector of Lagrangian multipliers.

The problem thus obtained can be separated into two sub-problems SP1 and SP2.

Sub-problem 1 ($SP1(\lambda)$ and $SP1(\lambda,\alpha)$). After applying the Lagrangian decomposition, the sub-problem referring to PU1 can be formulated as follows:

$$\text{minimize} \sum_{t=1}^{T}\left(p_t^1X_t^1+f_t^1Y_t^1+h_t^1I_t^1+\hat{h}_tJ_t+g_tL_t+(b_t^1-\lambda_t)W_t\right) \tag{5.33}$$

subject to:

$$(5.2)-(5.4),(5.8)-(5.10) \tag{5.34}$$

$$X_t^1,I_t^1,J_t,W_t,L_t\geq 0, \quad \forall t\in\{1,2,...,T\} \tag{5.35}$$

$$Y_t^1\in\{0,1\}, \quad \forall t\in\{1,2,...,T\} \tag{5.36}$$

Sub-problem (5.33)-(5.36), called for short $SP1(\lambda)$, corresponds to the single-item lot-sizing problem with a by-product, which can be solved using a time consuming polynomial algorithm when the inventory capacity of the by-product is constant and is proved NP-Hard in the general case (see Chapter 4). Let us apply a Lagrangian relaxation on capacity constraints (5.10). Constraints (5.10) are relaxed and penalized in the objective function by a vector of Lagrangian multipliers denoted $\alpha\in\mathbb{R}_+^T$. The sub-problem, called $SP1(\lambda,\alpha)$ is given as follows:

$$C_{SP1}(\lambda,\alpha)=\min\sum_{t=1}^{T}\left(p_t^1X_t^1+f_t^1Y_t^1+h_t^1I_t^1+(\hat{h}_t+\alpha_t)J_t+g_tL_t+(b_t^1-\lambda_t)W_t-\alpha_tB\right)$$

subject to:

$$(5.2)-(5.4),(5.8)-(5.9)$$

$$X_t^1,I_t^1,J_t,W_t,L_t\geq 0, \quad \forall t\in\{1,2,...,T\}$$

$$Y_t^1\in\{0,1\}, \quad \forall t\in\{1,2,...,T\}$$

Being constant, the term $-\alpha_tB$ can be discarded when solving SP1.

Sub-problem 2 ($SP2(\lambda)$). Sub-problem $SP2(\lambda)$ corresponds to the single-item lot-sizing problem to be solved by PU2 and can be formulated as follows:

$$C_{SP2}(\lambda) = \min \sum_{t=1}^{T} \left(p_t^2 X_t^2 + f_t^2 Y_t^2 + h_t^2 I_t^2 + q_t Z_t + (b_t^2 + \lambda_t)\bar{W}_t \right)$$

subject to:

$$\begin{aligned} &(5.5)-(5.7),(5.11) && \\ &X_t^2, I_t^2, \bar{W}_t, Z_t \geq 0, && \forall t \in \{1,2,\ldots,T\} \\ &Y_t^2 \in \{0,1\}, && \forall t \in \{1,2,\ldots,T\} \end{aligned}$$

Each of the sub-problems given above can be considered as a single-item lot-sizing problem and solved using a dynamic programming algorithm running in $O(T \log T)$ [Federgruen and Tzur, 1991, Wagelmans et al., 1992, Aggarwal and Park, 1993, Van Hoesel et al., 1994]. Note that the following properties characterizing the optimal solutions of sub-problems $SP1(\lambda,\alpha)$ and $SP2(\lambda)$ are true:

- $SP1(\lambda,\alpha)$: In an optimal solution, the generated quantities of by-products can either be disposed of or sent to PU2 after storage.
- $SP2(\lambda)$: An optimal solution, where each production period is supplied with raw materials originating from only one supplier, can be found.

Given the values of the Lagrangian multipliers, costs are updated by setting:

- $p'_t := p_t + \min_{t \leq u \leq T} \{ \sum_{v=t}^{u-1} (\hat{h}_v + \alpha_v) + \min\{b_u^1 - \lambda_u, g_u\}\}$, $\forall t \in \{1,2,\ldots,T\}$, in SP1,
- $p''_t := p_t + \min\{b_t^2 + \lambda_t, q_t\}$, $\forall t \in \{1,2,\ldots,T\}$, in SP2.

Thus, the unitary production cost associated with the uncapacitated lot-sizing problems of $SP1(\lambda,\alpha)$ and $SP2(\lambda)$ are respectively p'_t and p''_t.

Algorithm 5.1 Lagrangian decomposition algorithm

$\lambda_t = 0, \alpha_t = 0, \ \forall t \in \{1,2,...,T\}$
$\pi := \pi_{init}, z_{lb} := 0, z_{ub} := +\infty$
while stopping condition is not met **do**
    `Solve`($SP1(\lambda,\alpha)$) and retrieve X^1, I^1, J, W, L, Y^1 ▷ see Section 5.4
    `Solve`($SP2(\lambda)$) and retrieve $X^2, I^2, \bar{W}, Z, Y^2$ ▷ see Section 5.4
    $z^* \leftarrow C_{SP1}(\lambda,\alpha) + C_{SP2}(\lambda)$
    if $z^* > z_{lb}$ **then**
        $z_{lb} = z^*$
    end if
    $\hat{z}_{ub} \leftarrow$ `LagrangianHeuristic`($z^*, X^1, I^1, J, W, L, Y^1, X^2, I^2, \bar{W}, Z, Y^2$) ▷ see Section 5.4.1
    if $\hat{z}_{ub} < z_{ub}$ **then**
        $z_{ub} = \hat{z}_{ub}$
        $z_{ub} \leftarrow$ `LocalSearch`(z_{ub}, Y^1, Y^2, L, Z) ▷ see Section 5.4.2
    end if
    `UpdateMultipliers()` ▷ see Section 5.4.3
end while

The pseudo-code of the proposed Lagrangian decomposition algorithm is presented in Algorithm 5.1. The goal of Algorithm 5.1 is to successively provide a high number of upper bounds of the ULS-IS problem to move closer to its optimal solution. To do this, at each iteration, sub-problems $SP1(\lambda,\alpha)$ and $SP2(\lambda)$ are independently solved by calling function `Solve(`$SP(\bullet)$`)`, which provides an optimal solution by using the efficient dynamic programming algorithm of Wagelmans et al. [1992] based on dynamic programming, that runs in $O(T\log T)$. Based on these optimal solutions, a upper bound is computed using the Lagrangian heuristic proposed in Section 5.4.1. If there is an improvement, the best found lower and upper bounds are updated and a local search procedure is applied (see Section 5.4.2). Lagrangian multipliers are updated using the sub-gradient method described in Section 5.4.3.

5.4.1 Lagrangian heuristic

Lagrangian decomposition does not generally provide feasible solutions for ULS-SI problem, resulting by merging the obtained solutions of sub-problems $SP1(\lambda,\alpha)$ and $SP2(\lambda)$. The merged solution may violate the relaxed constraints. To recover feasibility, three main phases based on Trigeiro et al. [1989] are executed in the framework of a heuristic formalized in Algorithm 5.2:

- **Phase** 1 *(Smoothing phase)*: The solutions of $SP1(\lambda,\alpha)$ and $SP2(\lambda)$ are crossed to create a feasible solution of the ULS-IS problem. To do this, quantities of by-products are moved in order to comply with inventory capacity constraints.

- **Phase** 2 *(Inventory balance phase)*: To improve the solution obtained after Phase 1, the by-product is moved in the inventory.

- **Phase** 3 *(Improvement phase)*: To improve the solution obtained in Phase 2, production quantities are moved to reduce the quantities of disposed by-products and purchased raw materials.

Phase 1 of Algorithm 5.2 creates feasible solutions of the original problem, by: **(i)** checking and satisfying the by-product inventory capacity constraints, and **(ii)** synchronizing the exchange of by-products between PU1 and PU2. Algorithm 5.2 first disposes of the extra inventory quantities. It ensures subsequently that, if there is a quantity of by-products available in PU1 and production in PU2, then there is an exchange of by-products between PU1 and PU2. Otherwise, no transfer between production units is created. Given Assumptions (A.1)-(A.8), Algorithm 5.2 exploits the property that, no disposal of by-products and purchasing of raw materials can be simultaneously done.

When the by-product is storable with a limited capacity, Phase 2 is applied to balance the by-product inventory in a less myopic way. The inventory balance phase consists of reducing in each period the disposal of or purchasing quantities by: **(i)** computing the cost of moving backward or forward by-product quantities in the inventory, while satisfying capacity constraints (done via the function called `CostInventory()` in line 3, Algorithm 5.2), **(ii)** applying the move corresponding to the minimal cost, if it is strictly negative. After Phase 3, Phase 2 is also called to adjust the inventory quantities of by-products.

Phase 3 aims to move the production backward or forward to reduce the quantities of by-products disposed of and raw materials purchased from an external supplier. For each period, if there is disposal of or purchasing operations, there are 4 actions that can contribute to improve the current solution. Each action is applied twice i.e. with: **(i)** next $t-1$ or $t+1$, and **(ii)** the previous or next production period with respect to period t. Function `CostMove(`Q,t,t'`)` in Algorithm 5.2 computes the cost of moving the quantity Q from period t to period t'. More precisely, the goal is to reduce the disposal of and purchasing quantities in each period t, by:

(C.1) Moving a production quantity of PU1 (resp. PU2) in period t to the next period $t+1$ (or the next production period) in order to reduce the disposal of (resp. the purchasing) of by-products. The cost $cost1$ in Phase 3 of Algorithm 5.2 corresponds to the cost of moving the minimal quantity between **(i)** L_t and I^1_t or **(ii)** Z_t and I^2_t, from the current period t to the next period $t+1$ or the next production period $t' > t$.

(C.2) Moving a production quantity of PU1 (resp. PU2) in period t to the previous period $t-1$ (or the last production period before t) in order to reduce the disposal of (resp. the purchasing) of by-products. The cost of moving

Algorithm 5.2 `LagrangianHeuristic`($z_{lb}, X^1, I^1, J, W, L, Y^1, X^2, I^2, \bar{W}, Z, Y^2$)

Phase 1 – Smoothing phase

for $t = 1$ to T **do**
  if $J_t > B$ **then**
    $L_t \leftarrow J_t - B$, $J_t \leftarrow B$
  end if
  $A_t \leftarrow X_t^1 + J_{t-1}$
  if $A_t > 0$ and $X_t^2 > 0$ and $A_t \leq X_t^2$ **then**
    $W_t \leftarrow A_t$, $\bar{W}_t \leftarrow A_t$, $Z_t \leftarrow X_t^2 - A_t$
  else if $A_t > 0$ and $X_t^2 > 0$ and $A_t > X_t^2$ **then**
    $W_t \leftarrow X_t^2$, $\bar{W}_t \leftarrow X_t^2$, $L_t \leftarrow A_t - X_t^2$
  else if $A_t > 0$ **then**
    $L_t \leftarrow A_t$
  else
    $Z_t \leftarrow X_t^2$
  end if
  Update z_{lb}
end for

Phase 2 – Inventory balance phase

for $t = T$ to 1 **do**
  if $L_t > 0$ or $Z_t > 0$ **then**
    $cost \leftarrow$ `CostInventory`(L_t, t) or `CostInventory`(Z_t, t)
    if $cost < 0$ **then**
      Apply the corresponding move
      $z_{ub} \leftarrow z_{ub} + cost$
    end if
  end if
end for

Phase 3 – Improvement phase

for $t = T$ to 1 **do**
  if $L_t > 0$ or $Z_t > 0$ **then**
    $cost1 \leftarrow$ `CostMove`($\min\{L_t, I_t^1\}, t, t+i$) or `CostMove`($\min\{Z_t, I_t^2\}, t, t+i$)
    $cost2 \leftarrow$ `CostMove`($L_t, t, t-i$) or `CostMove`($Z_t, t, t-i$)
    $cost3 \leftarrow$ `CostMove`($L_t, t+i, t$) or `CostMove`($Z_t, t+i, t$)
    $cost4 \leftarrow$ `CostMove`($\min\{L_t, I_{t-1}^2\}, t-i, t$) or `CostMove`($\min\{Z_t, I_{t-1}^1\}, t-i, t$)
    if $\min(cost1, cost2, cost3, cost4) < 0$ **then**
      Apply the move corresponding to the minimal cost
      $z_{ub} \leftarrow z_{ub} + \min(cost1, cost2, cost3, cost4)$
    end if
  end if
end for
if $B \neq 0$ **then**
  Execute *Phase 2*
end if

quantity L_t or Z_t from the current period t to the previous period $t-1$ or the last production period $t' < t$ is denoted by $cost2$ in Phase 3 of Algorithm 5.2.

(C.3) Increasing the quantity of by-products transported between PU1 and PU2, by moving a production quantity of PU1 (resp. PU2) from the next period $t+1$ (or the next production period after t) to period t in order to reduce the purchasing of raw materials (resp. the disposable of) in period t. The cost of moving quantity L_t or Z_t from period $t+1$ or the next production period t' after t to the current period t is denoted by $cost3$ in Phase 3 of Algorithm 5.2.

(C.4) Increasing the quantity of by-products transported between PU1 and PU2, by moving a production quantity of PU1 (resp. PU2) from the previous period $t-1$ (or the last production period before t) to period t in order to reduce the purchasing of raw materials (resp. the disposable of) in period t. The cost of moving the minimal quantity between L_t or Z_t, and the inventory level of the main products in period $t-1$ from period $t-1$ or the last production period t' before t to the current period t is denoted by $cost4$ in Phase 3 of Algorithm 5.2.

For each possibility (C.1)-(C.4), the satisfaction of capacity constraints is checked. If the inventory capacity prevents from performing a move, the associated cost is set to zero.

In each of the cases (C.1)-(C.4), if the quantity moved equals to the produced quantity, the setup cost is subtracted. Moreover, if a production period is created, a setup cost is added. If the lowest cost thus obtained is negative, the corresponding move is applied and the objective function is updated. Algorithm 5.2 performs backward and is applied twice to adjust the moves corresponding to periods after the current period.

5.4.2 Local search procedure

To improve the obtained upper bounds, a local search procedure is executed. It consists of improving a given feasible solution, by applying a number of neighborhood operators.

The operators are applied on the solution obtained by Algorithm 5.2, and more precisely on the values of binary variables Y_t^1 and Y_t^2 for $t \in \{1,2,...,T\}$ corresponding to the setup indicators:

- The *setup removing* operator aims at grouping the production of two consecutive production periods in the earliest period. This consists of switching the value of a single binary variable from 1 to 0: if $Y_t^i = 1$ and $Y_{t'}^i = 1$, then $Y_t^i = 1$ and $Y_{t'}^i = 0$, for $t,t' \in \{1,2,...,T\}$, $t < t'$, $i \in \{1,2\}$.
- The *setup moving* operator aims at exchanging the setup indicators of two consecutive periods: if $Y_t^i = 1$ and $Y_{t'}^i = 0$, then $Y_t^i = 0$ and $Y_{t'}^i = 1$, for $t,t' \in \{2,3,...,T\}$, $t < t'$, $i \in \{1,2\}$.
- The *setup synchronization* operator matches the production periods of PU1 and PU2: if $Y_t^1 + Y_t^2 = 1$, then $Y_t^1 = 1$ and $Y_t^2 = 1$, for $t \in \{2,3,...,T\}$.
- The *disposal or purchasing removing* operator fixes a setup in PU2, when there is an operation of disposal of in PU1, and a setup in PU1, when there is a purchasing operation in PU2: if $L_t > 0$ then $Y_t^2 = 1$ otherwise if $Z_t > 0$ then $Y_t^1 = 1$, for $t \in \{2,3,...,T\}$.

Based on the above introduced neighborhood operators, the local search procedure is formalized in Algorithm 5.3.

Algorithm 5.3 `LocalSearch`(z_{ub}, Y^1, Y^2, L, Z)

1: $z_{ub} \leftarrow$ `SetupSynchronization`(z_{ub}, Y^1, Y^2)
2: $z_{ub} \leftarrow$ `DisposalRemoving`(z_{ub}, Y^1, Y^2, L)
3: $z_{ub} \leftarrow$ `PurchasingRemoving`(z_{ub}, Y^1, Y^2, Z)
4: $z_{ub} \leftarrow$ `SetupMoving`$(z_{ub}, Y^i,$ `PU_i`$)$, $i \in \{1,2\}$
5: $z_{ub} \leftarrow$ `SetupRemoving`$(z_{ub}, Y^i,$ `PU_i`$)$, $i \in \{1,2\}$

Function `SetupSynchronization(•)` in Algorithm 5.3 applies the *setup synchronization* operator to all periods from period 2 to period T. For each period, if the value of the objective function is improved, the associated solution is kept and serves as a basis for the next iteration. Algorithm 5.3 is composed of the succession of functions corresponding to the neighborhood operators introduced above. The succession, in which the operators are applied, has been empirically determined. To limit the computational time, Algorithm 5.3 is applied only when the upper bound is improved by Algorithm 5.2.

5.4.3 Updating of Lagrangian multipliers

The Lagrangian multipliers used in Algorithm 5.1 are initialized to 0, and then updated using the sub-gradient method. The procedure is named `UpdateMultipliers()` in Algorithm 5.1. Let δ be the sub-gradient composed by the vectors δ^1 and δ^2 such that:

$$\delta_t^1 = \bar{W}_t - W_t, \quad \delta_t^2 = J_t - B, \quad \forall t \in \{1,2,...,T\}.$$

The step size Δ is calculated using the formula:

$$\Delta = \frac{\pi(UB^* - LB)}{\|\delta\|^2}$$

where UB^* is the value of the best known feasible solution and LB is the current lower bound.

The Lagrangian multipliers λ_t are updated using the following formula, $\forall t \in \{1,2,...,T\}$:

$$\lambda_t = \lambda_t + \Delta\delta_t^1, \quad \forall t \in \{1,2,...,T\}.$$

The Lagrangian multipliers α_t are computed as follows:

$$\alpha_t = \begin{cases} \alpha_t + \Delta\delta_t^2, & \text{if } \alpha_t + \Delta\delta_t^2 > 0 \\ 0, & \text{otherwise.} \end{cases} \quad \forall t \in \{1,2,...,T\}$$

Usually, the scalar π is initially equal to 2. This coefficient is divided by 2 whenever the lower bound is not improved in a fixed number of iterations. In our settings, if there is an improvement after each 3 iterations, π is not modified, otherwise π is multiplied by 0.8.

If no improvement is recorded during a fixed number of iterations, a multi-start procedure is called. This procedure consists of taking the values of the last Lagrangian multipliers, multiplying them by random values and continuing the execution of Algorithm 5.1. For the ULS-IS problem, Lagrangian multipliers are multiplied by values between 0.5 and 2. After 600 iterations without improvement, each Lagrangian multiplier is multiplied by a random value comprised between 0.5 and 2.

5.5 Collaboration policies

In this section, let us investigate a number of different collaboration policies in an industrial symbiosis system controlled by two decision makers. As previously discussed, a centralized collaboration policy (i.e. the ULS-IB problem) can be only applied in full information sharing environments. Nowadays, the lack of information sharing remains a major barrier in the expansion of industrial symbiosis networks [Fraccascia and Yazan, 2018]. At the other extreme, a full decentralized decision-making process is not globally consistent, since each decision maker pursues its own local objectives with its local constraints, which does not necessarily maximize the global benefits.

With respect to the centralized collaboration policy modeled via the ULS-IB problem, we study the following baseline collaboration policies: **(i)** a policy without collaboration, **(ii)** a policy expressing an opportunistic collaboration and, **(iii)** two sequential decentralized collaboration policies expressing a symbiotic partnership, when one production unit dominates another one, i.e. makes first its production plan.

The production plans are obtained as follows:

- **No collaboration:** As no interaction is considered between PU1 and PU2, production plans can be found by solving separately sub-problems $SP1(\lambda)$ and $SP2(\lambda)$ with: **(i)** $W_t = 0$, $\forall t \in \{1,2,...,T\}$ in PU1, and **(ii)** $\bar{W}_t = 0$, $\forall t \in \{1,2,...,T\}$ in PU2.

- **Opportunistic collaboration:** This policy presupposes the replacement of raw materials with by-products whenever possible, by matching the production plans of PU1 and PU2 calculated separately. The first iteration of Algorithm 5.1 implements this collaboration policy.

- **Sequential decentralized collaboration policies:** Let us consider the case when one production unit makes its production plan first, then another production unit proceeds to the decision-making accordingly. The management of the by-product flows transported from the supplier PU to the receiver PU is discussed in what follows for both cases, when decisions are made downward, and when decisions are made upward.

Downward sequential decision-making. Let us suppose that PU1 makes its production plan and communicates to PU2 the quantities of by-products available in each period. Subsequently and knowingly, PU2 establishes its informed production plan, by taking advantage of the by-products generated by PU1.

The problem to be solved by PU1 corresponds to the sub-problem $SP1(\lambda)$. It is solved by the dynamic programming algorithm of Wagelmans et al. [1992] that runs in $O(T\log T)$. Let $w_t = W_t$ be the quantity of by-products generated by PU1 and available for PU2, $t \in \{1,2,\ldots,T\}$. The resulting problem for PU2 is a basic supplier selection problem and can be formulated by a mixed integer linear programming model as follows:

$$\text{minimize} \sum_{t=1}^{T} \left(p_t^2 X_t^2 + f_t^2 Y_t^2 + h_t^2 I_t^2 + q_t Z_t + b_t^2 W_t\right)$$

subject to:

$$\begin{aligned}
&(5.5)-(5.7),(5.11) && \\
&W_t \le w_t, && \forall t \in \{1,2,\ldots,T\}\\
&X_t^2, I_t^2, W_t, Z_t \ge 0, && \forall t \in \{1,2,\ldots,T\}\\
&Y_t^2 \in \{0,1\}, && \forall t \in \{1,2,\ldots,T\}
\end{aligned}$$

By virtue of Assumption (A.3), this problem can be solved using a dynamic programming algorithm by setting:

$$p_t(Q) = \begin{cases} (p_t^2 + b_t^2)Q, & \text{if } Q \le w_t\\ p_t^2 Q + b_t^2 w_t + q_t(Q - w_t), & \text{otherwise.}\end{cases}$$

Since the resulting production cost is concave, the algorithm proposed by Wagner and Whitin [1958], running in $O(T^2)$, is used to solve the problem.

After obtaining the production plans for both production units, they are crossed in order to update the disposal quantities L_t, as well as, the quantities of by-products W_t transported between PU1 and PU2.

Upward sequential decision-making. PU2 makes first its production plan and provides their needs in terms of by-products (i.e. recovered raw materials) to PU1. This collaboration policy can thus be reduced to a co-product problem with: **(i)** inventory bounds, **(ii)** a disposal of option, and **(iii)** possible lost sales on the by-product, since PU1 is allowed to not meet the demands for by-products.

The problem addressed by PU2 is a classical lot-sizing problem and can be solved using an algorithm running in $O(T\log T)$ (see e.g. Wagelmans et al. [1992]). Let $w_t = W_t$ be the quantity of raw materials required by PU2 in period t. The problem to be solved by PU1 can be formulated as follows.

$$\text{minimize} \sum_{t=1}^{T} \left(p_t^1 X_t^1 + f_t^1 Y_t^1 + h_t^1 I_t^1 + \hat{h}_t J_t + g_t L_t + b_t^1 W_t \right)$$

subject to:

$$\begin{aligned}
&(5.2)-(5.4),(5.8)-(5.10) & \\
&W_t \leq w_t, & \forall t \in \{1,2,\ldots,T\} \\
&X_t^1, I_t^1, J_t, L_t, W_t \geq 0, & \forall t \in \{1,2,\ldots,T\}
\end{aligned}$$

This model is solved using a commercial solver. Once the production plan is made by PU1, quantities of by-products W_t are recovered by PU2 to adjust the real quantities of raw materials received from PU1 and those purchased from an external supplier.

5.6 Experimental results

5.6.1 Instances generation

Computational results have been conducted on heterogeneous instances randomly generated. All the costs are supposed to be stationary. The tested data sets are built for two time horizon lengths $T \in \{24, 96\}$. The horizon length defines the size of the problem. Parameters p^1, p^2, $h^1 \neq 0$, b^1, b^2, g and q are randomly generated between 0 and 10, while respecting Assumptions (A.1)-(A.8). To define setup costs f^1 and f^2, and holding cost h^2, critical parameters are identified, namely: ratio Δ between PUs, Setup cost-Holding cost Ratio (SHR), demands d^1 and d^2, and inventory capacity of by-products B:

- $\Delta = \dfrac{h^2}{h^1}$ aims at linking the holding costs of PU1 and PU2. This ratio can be *low* ($\Delta = 0.75$), *medium* ($\Delta = 1$) or *high* ($\Delta = 1.25$). Note that Δ is insightful to reveal the impact of one PU to the production plan of another PU.
- SHR is a well-known parameter in the lot-sizing literature (see e.g. Trigeiro et al. [1989]). This ratio has an impact on the average number of time periods between two consecutive setups, known as the Time Between Order (TBO). SHR links the setup and holding costs. As far as we consider a problem involving two different PUs, an SHR is generated for each PU, $SHR1$ for PU1 and $SHR2$ for PU2, which take their values in the set $\{3,4,5\}$.
- Demands d_t^1 and d_t^2, which have an impact on the size of production units, can be: **(i)** *low*: generated following a normal distribution with an average of 50 and a standard deviation of 10, $\forall t \in \{1,2,...,T\}$, **(ii)** *medium*: generated following a normal distribution with an average of 100 and a standard deviation of 20, $\forall t \in \{1,2,...,T\}$, or **(iii)** *high*: generated following a normal distribution with an average of 200 and a standard deviation of 40, $\forall t \in \{1,2,...,T\}$. We denote by $\bar{d}^1$ and $\bar{d}^2$ the average demands of PU1 and PU2, respectively.
- The by-product inventory capacity B can be: **(i)** *tight*: randomly generated around $1.2\bar{d}^1$, **(ii)** *large*: randomly generated around $3\bar{d}^1$, or **(iii)** *null*: when the by-product is unstorable, i.e. $B = 0$.

Given SHR and holding costs of both PUs, setup costs f^1 and f^2 can be computed via:

$$f^1 = \frac{1}{2} h^1 (SHR1)^2 \bar{d}^1$$
$$f^2 = \frac{1}{2} h^2 (SHR2)^2 \bar{d}^2$$

Data sets for each time horizon length $T \in \{24, 96\}$ are generated by combining all possible values of the critical parameters discussed above. By generating 10 instances for each class, the total number of generated instances per $T \in \{24, 96\}$ is $10 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 7{,}290$.

5.6.2 Design of experiments

We carried out the comparison between the following approaches:

- `AGG`: Straightforward formulation (5.1)-(5.13) of the ULS-IS problem solved via CPLEX.
- `FAL`: Facility location formulation (5.14)-(5.23) of the ULS-IS problem solved via CPLEX.
- Several variants of Lagrangian decomposition Algorithm 5.1 are tested:
  - `LD`: Lagrangian heuristic (i.e. Algorithm 5.2) without any local search procedure.
  - `LD-LS`: Lagrangian heuristic (i.e. Algorithm 5.2) embedding the local search procedure given by Algorithm 5.3.
  - `LD-MS`: Lagrangian heuristic (i.e. Algorithm 5.2) within a multi-start procedure, called when there is no improvement after a fixed number of iterations. This procedure consists of taking the values of the last Lagrangian multipliers, multiplying them by random values, and continuing Algorithm 5.1.
  - `LD-MS-LS`: Lagrangian heuristic (i.e. Algorithm 5.2) embedding the local search procedure given by Algorithm 5.3 within a multi-start procedure.
  - `LD-LP`: The Lagrangian heuristic given in Algorithm 5.2 is replaced by the resolution of the linear program obtained by fixing the production periods according to the solutions of sub-problems $SP1(\lambda, \alpha)$ and $SP2(\lambda)$.
  - `LD-LP-MS-LS`: `LD-LP` embedding the local search procedure given by Algorithm 5.3 within a multi-start framework.

Algorithm 5.1 is stopped after 1,000 iterations. For large size problems, this stopping criterion is strengthened by a time limit fixed at 10 seconds (s). The gap between the upper bound (UB) and the lower bound (LB) is given by the formula:

$$Gap = 100 \times \frac{UB - LB}{UB}$$

Numerical tests were carried out on a computer with Intel Xeon e5-2620 2.1GHz CPU with 32GB RAM. Models `AGG` and `FAL` and linear programs in `LD-LP` and `LD-LP-MS-LS` were solved using IBM ILOG CPLEX 12.6. The Lagrangian decomposition algorithm was implemented using the C++ programming language on Microsoft Visual Studio 2013.

5.6.3 Numerical results

The goal of the conducted experiments is manifold: **(i)** to evaluate the impact of critical parameters, **(ii)** to discuss the contribution of the different procedures described above, **(iii)** to show the competitiveness of the proposed solution method with respect to `AGG`, **(iv)** to characterize the generated instances.

On average, `LD` outperforms `LD-LP`, regardless the used improvement procedure. For this reason, regarding the variants based on linear programming solving, only `LD-LP` and `LD-LP-MS-LS` are kept to empirically study the performance of the proposed solution method.

Algorithm 5.1 and its variants. By summarizing Table 1, Table 5.2 provides the distribution of the gaps between UB and LB for the different variants of Algorithm 5.1. The multi-start procedure reduces the average gap of at least 0.01% but its improvement is too low to be noticed on maximum gaps after rounding of the results. On the contrary, the local search procedure reduces the maximum gaps from 3.47 (resp. 4.83) to 3.30 (resp. 3.54) in the unstorable (resp. storable) case. In the same way, the CPU time increases when using the local search procedure (multiplied by 2 or even 3), while remaining acceptable (always below 1s). The combination of the multi-start and local search procedures is particularly efficient when the by-product is storable. The average gap decreases from 0.67% to 0.52%.

The performance of the multi-start and local search procedures can be also observed when comparing the results obtained by `LD-LP` and `LD-LP-MS-LS`. The difference of gaps between `LD-MS-LS` and `LD-LP-MS-LS` are: **(i)** *rather high* for $B \neq 0$, `LD-LP-MS-LS` being twice slower, **(ii)** *rather high* for $B = 0$, `LD-LP-MS-LS` being three times slower.

To sum up, `LD-MS-LS` outperforms other variants of Algorithm 5.1 in terms of both the solution quality and computational time, and is considered by default in what follows.

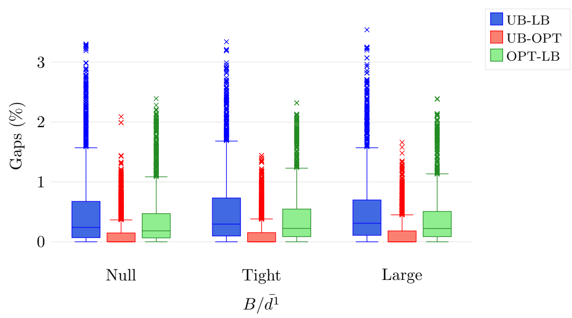
Table 5.2: Algorithm 5.1 and its variants: *Gap distribution (in %) between UB and LB for small size instances*

B	Variant	Mean	Standard deviation	Max	Median	CPU time
Null	`LD`	0.56	0.69	3.47	0.26	0.22
	`LD-MS`	0.53	0.66	3.47	0.25	0.22
	`LD-LS`	0.52	0.65	3.30	0.25	0.72
	`LD-MS-LS`	0.49	0.63	3.30	0.24	1.16
	`LD-LP`	1.00	1.18	6.07	0.50	3.70
	`LD-LP-MS-LS`	0.66	0.86	4.92	0.27	3.61
Non-null	`LD`	0.67	0.75	4.84	0.37	0.40
	`LD-MS`	0.65	0.74	4.83	0.36	0.40
	`LD-LS`	0.53	0.60	3.54	0.31	0.83
	`LD-MS-LS`	0.52	0.59	3.54	0.30	0.85
	`LD-LP`	0.86	1.00	6.86	0.47	2.21
	`LD-LP-MS-LS`	0.62	0.74	4.51	0.34	2.29

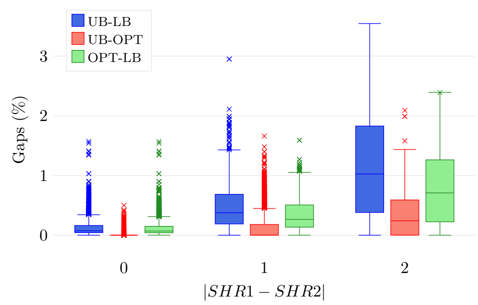
Analysis of the critical parameters. Table 3 in Appendix shows that the tightness of the by-product inventory capacity has a high impact on the CPU time spent by CPLEX to solve `AGG` and `FAL`. The more the capacity is tight, the higher are computational times needed to solve `AGG` and `FAL`.

One can also remark that the closer the values of $SHR1$ and $SHR2$, the faster the problem is to solve. On the contrary, when the average demand of PU1 is close to the average demand of PU2, the optimal solution is found after a higher CPU time. Moreover, when $SHR1$ and $SHR2$ both increase, the CPU times needed to solve `AGG` and `FAL` increase (e.g. for `AGG`, it is around 0.30s for $SHR1 = SHR2 = 3$, and 0.60s for $SHR1 = SHR2 = 5$). The value of Δ intensifies the impact of the parameters on the CPU time, since it operates with the difference between the holding costs, and accordingly between the setup costs. Apart from this fact, the impact of Δ is negligible.

The last row of Table 3 highlights that the time spent by CPLEX to solve `FAL` is: **(i)** on average higher than for `AGG`, **(ii)** inhomogeneous and can be very high for some classes of instances (16.18s in average for $SHR1 = 5$, $SHR2 = 3$ with a tight capacity). We conclude that `FAL` formulation is less efficient than `AGG` for most instances, and consequently it is not used to solve large-size instances.



(a) Gaps as a function of $B/\bar{d}^1$



(b) Gaps as function of $|SHR1 - SHR2|$

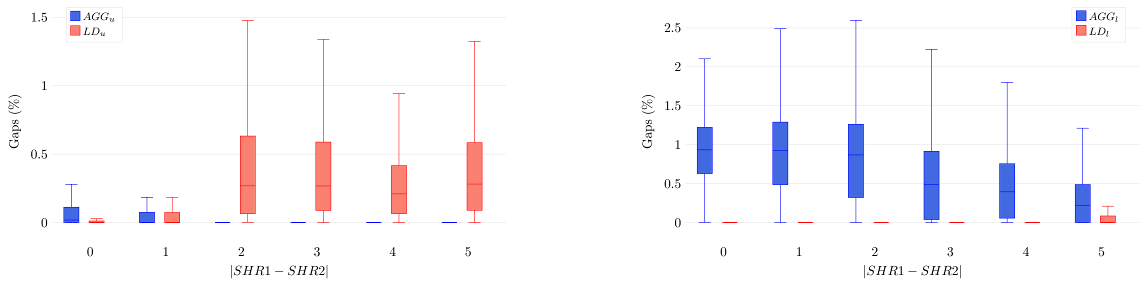
Figure 5.2: Distribution of gaps (%) for $T = 24$: `LD` *versus* `AGG`

***Focus on the effectiveness of* `LD-MS-LS` *versus* `AGG`.** The solution quality provided by `LD-MS-LS` is studied on small instances with respect to the value of B, $SHR1$ and $SHR2$. Figure 5.2 provides the boxplots showing the distributions of gaps between: **(i)** UB and LB for `LD-MS-LS`, denoted by `UB-LB`, **(ii)** UB and optimal solution OPT, denoted by `UB-OPT`, **(iii)** OPT and LB, denoted by `OPT-LB`.

To evaluate the effectiveness of `LD-MS-LS`, no time limit has been imposed to CPLEX. The higher gaps are observed for instances with high computational times for AGG (see Table 2). When $SHR1 = SHR2$, gaps are very closed to 0 in more than three quarters of instances (see Figure 5.2b). The higher the difference between $SHR1$ and $SHR2$ is, the significant the gaps are. For 75% of instances with $|SHR1 - SHR2| = 2$, `UB-OPT` is below 0.5%. Gaps `UB-LB` are quite large, but rarely exceed 2%. For $B \neq 0$, the gaps `UB-OPT` slightly increase with the inventory capacity, whereas the gaps `OPT-LB` tend to decrease (see Figure 5.2a). The impact of $B/\bar{d}^1$ is relatively low: whatever the by-product inventory is, 75% of gaps `UB-OPT` and `OPT-LB` are below 0.5%.

***Focus on the efficiency of* `LD-MS-LS` *versus* `AGG`.** Let UB_{LD} and LB_{LD} (resp. UB_{AGG} and LB_{AGG}) be the lower and upper bounds obtained by Algorithm 5.1 (resp. formulation `AGG`). The maximum CPU time allowed for CPLEX has been limited to 10s.

To compare the quality of the primal and dual bounds, let us define: **(i)** $\texttt{LD}_\texttt{u} = UB_{LD} - UB^*$ and $\texttt{AGG}_\texttt{u} = UB_{AGG} - UB^*$ where $UB^* = \min\{UB_{AGG}, UB_{LD}\}$, **(ii)** $\texttt{LD}_\texttt{l} = LB^* - LB_{LD}$ and $\texttt{AGG}_\texttt{l} = LB^* - LB_{AGG}$, where $LB^* = \max\{LB_{AGG}, LB_{LD}\}$. These gaps are provided in Table 4.



(a) Gaps to UB

(b) Gaps to LB

Figure 5.3: `LD` versus `AGG` as function of $|SHR1 - SHR2|$ for $T = 96$

In general, `AGG` provides better upper bounds than `LD-MS-LS`. However, when $SHR1 = SHR2$, $\texttt{LD}_\texttt{u}$ is very close to $\texttt{AGG}_\texttt{u}$ for a lower running time. In particular, when $B = 0$, `LD-MS-LS` provides upper bounds almost as good as $\texttt{AGG}_\texttt{u}$ in only around 1.5s against around 8.8s taken by `AGG`. Also, the lower bounds obtained by `LD-MS-LS` are generally better than $\texttt{AGG}_\texttt{l}$, when $B = 0$. On the contrary, $\texttt{LD}_\texttt{l}$ is relatively poor compared to $\texttt{AGG}_\texttt{l}$ for $B \neq 0$. The more SHRs increase, the more the gaps are significant.

Challenging instances. In the light of the findings previously discussed, let us challenge `AGG` and `LD-MS-LS`, by increasing the values of SHRs up to 8. The new created instances are discussed in terms of the heterogeneity in SHRs and the maximum value between $SHR1$ and $SHR2$.

- **High heterogeneity in SHR:** As Figure 5.2b shows, gaps to UB of `LD-MS-LS` increase when $SHR1$ and $SHR2$ are very different. For instances with $|SHR1 - SHR2| \geq 2$, `AGG` provides better UBs than `LD-MS-LS` (see Figure 5.3a) . However, LB_{AGG} is very poor compared to LB_{LD}. This makes `LD-MS-LS` globally better than `AGG` in terms of closeness between bounds. For $|SHR1 - SHR2| = 5$, gaps to UB reach more than 2%, that makes the corresponding instances difficult to solve.
- **High dispersion in SHR:** Focus now on instances with $SHR1, SHR2 \in \{6, 7, 8\}$ and $|SHR1 - SHR2| \leq 1$. They correspond to instances with high setup costs, which may make difficult the synchronization between PUs. The

Table 5.3: Gaps to UB (in %): $T = 96$, $SHR1, SHR2 \in \{6,7,8\}$ and $|SHR1 - SHR2| \leq 1$

$SHR1$	$SHR2$	**Null** B		**Tight** B		**Large** B	
		$\texttt{AGG}_\texttt{u}$	$\texttt{LD}_\texttt{u}$	$\texttt{AGG}_\texttt{u}$	$\texttt{LD}_\texttt{u}$	$\texttt{AGG}_\texttt{u}$	$\texttt{LD}_\texttt{u}$
6	6	0.02	0.03	0.04	0.03	0.04	0.06
6	7	0.02	0.07	0.08	0.06	0.07	0.09
7	6	0.02	0.11	0.08	0.08	0.06	0.14
7	7	0.02	0.03	0.11	0.01	0.08	0.03
7	8	0.04	0.03	0.15	0.01	0.12	0.02
8	7	0.02	0.05	0.12	0.02	0.1	0.04
8	8	0.03	0.01	0.14	0	0.13	0.01
Average CPU time		10.0	1.2	10.0	7.2	10.0	7.6

obtained results are provided in Table 5.3. `AGG` outperforms `LD-MS-LS` only when one of the SHRs equals to 6. In all other cases, `LD-MS-LS` provides better feasible solutions than `AGG`, especially when B is tight.

5.7 Managerial implications and research perspectives

In this section, let us discuss the economic and environmental opportunities induced by the exchange of by-products between two production units, by examining five baseline collaboration policies, namely:

- `No_Co`: *No collaboration*, i.e. no symbiotic partnership is considered between production units. The by-products generated by PU1 are disposed of, and raw materials used by PU2 are purchased from an external supplier. Let the costs obtained in the framework of this policy be called *nominal costs*.
- `Full_Co`: *Full collaboration*, i.e. the exchange of by-products are planned in the framework of a centralized collaboration policy. No other policy can provide better gains. Let the costs obtained in the framework of this policy be called *centralized costs*.
- `Opp_Co`: *Opportunistic collaboration*, i.e. the exchange of by-products is being done by taking advantage of a fortunate matching between the production plans of the supplier (PU1) and the receiver (PU2).
- Two sequential decentralized collaboration policies:
  - `PU1_First`: *Downward sequential collaboration*,
  - `PU2_First`: *Upward sequential collaboration*.

The aforementioned policies were addressed in Section 5.5, where the used models and the associated solution methods are detailed. For the sake of simplicity and without loss of generality, the quantitative impact of each collaboration policy is evaluated on small size instances, i.e. for $T = 24$ periods and $SHR1, SHR2 \in \{3,4,5\}$.

The gains of each production unit i are calculated with respect to its nominal cost c^i obtained outside any symbiotic partnership, as follows $(1 - c^i_p/c^i) \times 100$, where c^i_p is the cost of production unit i obtained in the framework of a collaboration policy denoted by p, $p \in \{\texttt{Full\_Co}, \texttt{Opp\_Co}, \texttt{PU1\_First}, \texttt{PU2\_First}\}$, $i \in \{1,2\}$. The environmental gain represents the proportion of by-products which is reused compared to the total quantity of reusable by-product. The gains obtained for each of the aforementioned policy are provided in Table 5.4.

Even an opportunistic exchange of by-products creates value-added benefits for both PUs, reaching from slightly over 2.4% on average to a maximum of more than 10%. Moreover, it leads to the saving of 35% of the reusable quantity of by-products. It is worthwhile to observe that the more the decision-making is informed, the more significant are the benefits of each production unit and related to the environment. Both of the sequential decentralized collaboration policies double the savings procured by exchanging by-products, being far from the centralized costs (`Full_Co`) within a distance of around 1% for the economic gains and 10% for the environmental benefit. In the same line of thought, in the PU2_First policy, PU1 knows the by-product needs of PU2. In this case, this knowledge helps PU1 improving its gains by 1% compared to the gains obtained by policy PU1_First. In our experiments, it appears that the knowledge of PU2 about the availability of by-products (in the PU1_First policy) does improves significantly its gains compared to the gains obtained with the PU2_First policy. This means that sometimes the by-product supplier can obtain higher gains when moving at second instance. It is also important to mention that in the Full_Co policy, even if the average and the maximal gains are higher than the gains obtained by other policies, PU1 can lose up to 1.0% and PU2 can lose up to 0.7% of their total nominal costs. In this case, to make the proposed solution acceptable by both PUs, compensation mechanisms have to be considered. Some perspectives related to these mechanisms are addressed at the end of this section.

Table 5.4: Collaboration policies: *Gains of PU1 and PU2 (in %) against* No_Co

PU	B	Opp_Co			PU1_First			PU2_First			Full_Co		
		min	mean	max	min	mean	max	min	mean	max	min	mean	max
PU1	Null	0.1	2.8	9.5	0.3	6.4	21.7	0.1	5.5	20.6	-0.9	7.0	21.6
	Tight	0.1	3.1	10.1	0.3	7.1	21.0	0.1	6.6	18.7	-0.2	7.9	22.0
	Large	0.1	2.9	10.1	0.2	7.0	21.0	0.1	6.8	18.9	-1.0	7.9	22.0
PU2	Null	0.1	2.4	9.6	0.1	5.0	22.6	0.2	5.5	23.1	-0.7	5.9	23.0
	Tight	0.1	2.4	8.7	0.1	5.2	21.4	0.4	6.1	22.0	0.2	6.3	21.8
	Large	0.0	2.4	8.7	0.0	5.1	21.4	0.4	6.2	22.0	0.2	6.4	21.8
Env.	Null	11.3	35.5	61.0	20.5	82.3	100	11.3	82.7	100	65.1	97.5	100
	Tight	11.3	35.0	61.0	15.1	82.7	100	15.3	87.8	100	70.4	98.6	100
	Large	3.2	33.6	61.0	7.0	81.3	100	15.3	89.8	100	75.8	98.8	100

Impact of technology characteristics on collaboration policies. Figures 5.4-5.5 highlight the economic impact of the by-product storability with respect to $(SHR1 - SHR2)$ and $(\bar{d}^1 - \bar{d}^2)$.

Figure 5.4 (resp. Figure 5.5) reports the average relative gains of each PU obtained by policy Full_Co against No_Co as function of $(SHR1 - SHR2)$ (resp. $(\bar{d}^1 - \bar{d}^2)$) for storable and unstorable by-products. Once again compared to policy No_Co, Figures 5.6a-5.6b (resp. Figures 5.6c-5.6d) report the relative gains of each PU as function of $(SHR1 - SHR2)$ (resp. $(\bar{d}^1 - \bar{d}^2)$) for the three production policies: Full_Co, PU1_First, PU2_First.

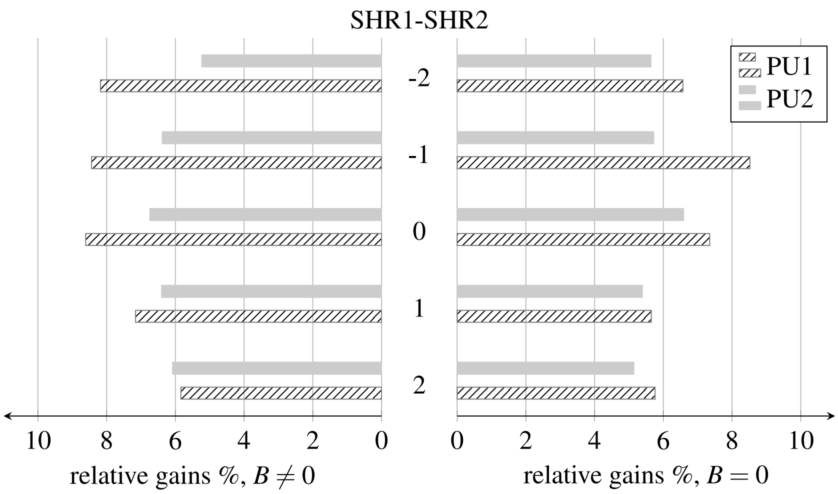


Figure 5.4: Relative gains (in %) as function of $(SHR1 - SHR2)$: *Policy* Full_Co

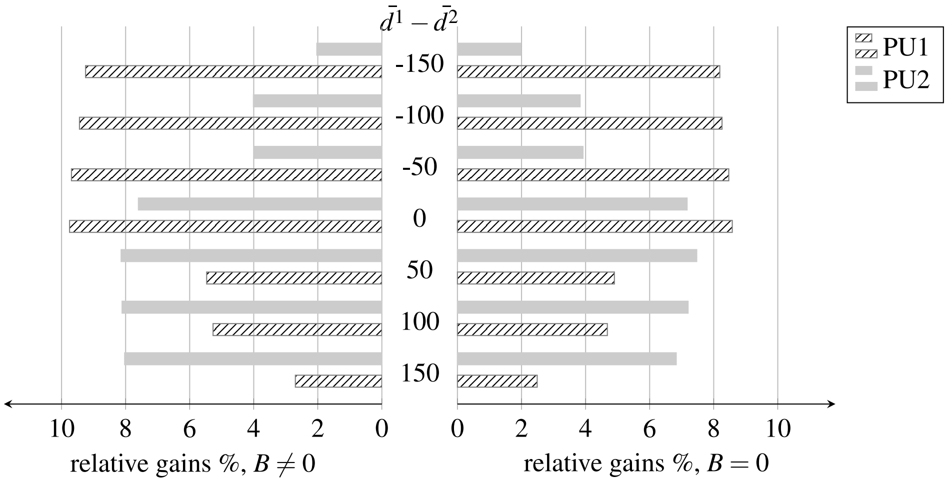


Figure 5.5: Relative gains (in %) as function of $(\bar{d}^1 - \bar{d}^2)$: *Policy* `Full_Co`

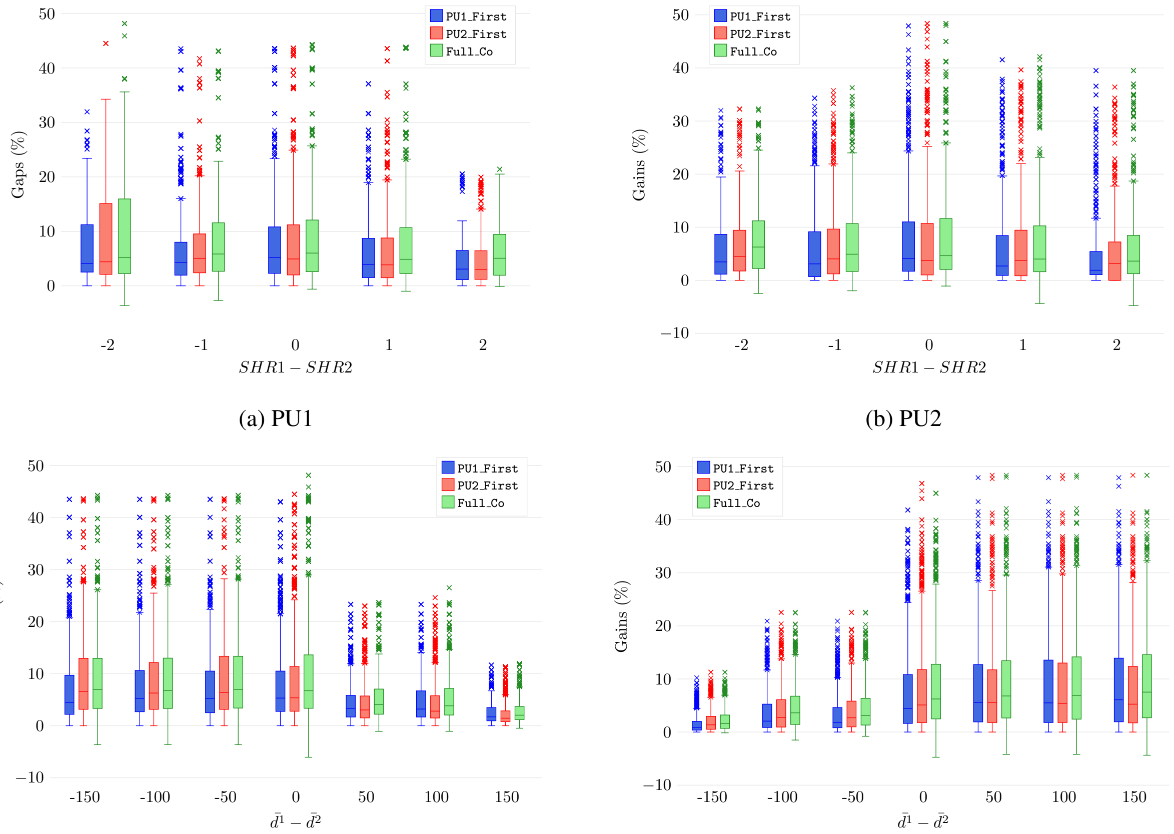
First, let us analyze the impact of $(SHR1 - SHR2)$ on obtained gains regarding the storability or not of by-products. From Figure 5.4, we can notice that when the by-product is unstorable and $(SHR1 - SHR2)$ is high, PU2 has more gains than PU1. This can be explained by the fact that when the SHR is high for PU1 and low for PU2, PU2 has more flexibility to align its production with the one of PU1. When the situation is inverted, i.e. $(SHR1 - SHR2)$ is very small, the average relative gain of PU1 is higher than the one of PU2. In this case, PU1 has more flexibility to align its production with the one of PU2. When the by-product is storable, we notice that the relative gains of PU1 are always higher than those of PU2 regardless the value of $(SHR1 - SHR2)$. This can be explained by the flexibility of PU1, which has more freedom to manage the by-product inventory level. From Figures 5.6a-5.6b, we notice that the higher gains are obtained when the SHR of both PUs are close, i.e. $SHR1 \approx SHR2$. Negative gains are obtained when the difference between the SHR of both PUs is high, i.e. $|SHR1 - SHR2| = 2$.

Focus now on the impact of the balance between average demands of production units on benefits obtained in the framework of different policies. As Figure 5.5 corroborates in the case of policy `Full_Co`, the higher the difference between demands, the more the gains of PUs are unbalanced. From Figures 5.6c-5.6d, it appears to be more beneficial for each PU when the production unit, which has the largest demand, establishes its production plan first. This finding differs from that encountered in classical supply chains, where the primacy in the decision-making ensures the maximum benefits.

- `PU1_First` and $\bar{d}^1 \gg \bar{d}^2$: Having to meet large demands, PU1 generates and makes available large quantities of by-products. This configuration is convenient for PU2 requiring relatively small quantities of by-products.
- `PU2_First` and $\bar{d}^2 \gg \bar{d}^1$: The case when PU2 has to meet greater demands than PU1 is favorable for PU1, since PU2 will tend to deplete the relatively small quantities of by-products generated by PU1.
- $\bar{d}^1 \approx \bar{d}^2$: When demands of production units are balanced, the primacy in decision-making has no drastic effects on costs.

Discussions on industrial symbiosis-based collaboration policies. As previously highlighted, the collaboration schemes applied to coordinate traditional supply chains may not be appropriate to regulate the exchange of by-products between a number of production units.

Apart from the attractiveness in terms of global economic benefits, the centralized collaboration policy suffers from the disadvantage of not being always equitable, as shown in the analysis results provided in Table 5.4. One of the state-of-the-art remedies to deal with the misalignment of benefits is to explicitly add and operate with financial flows within the network. The centralized policy may be improved by sharing benefits between production units in



(a) PU1 (b) PU2

(c) PU1 (d) PU2

Figure 5.6: Gains (in %) as function of $(SHR1 - SHR2)$ and $(\bar{d}^1 - \bar{d}^2)$

the form of side payments given by one PU to compensate financial losses incurred by another PU [Daquin et al., 2019].

As far as the decentralized policies are considered, contract schemes serve to align the interests of each production unit, by rigorously framing the by-product transfer and avoiding relationships based on dominance. The global benefits induced by these contracts are situated between the values of solutions obtained by `PU_First`.

One of the most sensitive and crucial points in making successful an industrial symbiosis partnership is the information sharing. Nowadays, a growing number of IT platforms is implementing not only: **(i)** to facilitate the access to information about the by-product location and availability, but also **(ii)** to support the framing of collaboration schemes. Let us mention a couple of platforms dedicated to fostering the industrial symbiosis [Vladimirova et al., 2019]: **(i)** SYNERGie 4.0 Platform and Database, promoted by International Synergies[31], **(ii)** MAESTRI Toolkit, EPOS Toolbox, Sharebox or SYNERGie 2.0 Platform, etc, developed in the framework of European projects (respectively MAESTRI[12], EPOS[33] and Sharebox[32] projects), and **(iii)** Industrial Symbiosis Data Repository Platform[34], an open source platform. It is worthwhile to underline the importance of the following questions posed by Fraccascia and Yazan [2018] in achieving the zero-waste goal via the industrial symbiosis: *"What is sensitive information for a company? Which type of information is non-sensitive for a company to implement industrial symbiosis based cooperation? Is the sensitive information really sensitive enough to motivate the limitation for its non-disclosure?"* In line with these issues and as future research, it would be very insightful to evaluate the value of information availability within the system of coordinates defined by the baseline collaboration policies investigated in this chapter.

5.8 Conclusion and perspectives

Inspired by the circular economy paradigm, this chapter introduced and investigated a new version of a two-level single-item lot-sizing problem (ULS-IS), posed by an industrial symbiosis network including two production units. We proposed two formulations to model the two-level single-item lot-sizing problem in the framework of a centralized coordination policy. We showed that ULS-IS problem is *NP*-Hard. A solution method based on Lagrangian decomposition has been proposed. Extensive numerical experiments have been conducted on small and large instances to study the competitiveness and the tractability of the proposed solution method and its variants.

From a managerial point of view, two sequential decentralized collaboration policies have been investigated against two extreme configurations, namely: no collaboration, and full collaboration based on a centralized decision-making. Valuable evidence for policy makers has been discussed, and a number of perspectives has been suggested for further research. In particular, the comparison between centralized and decentralized collaboration policies puts the spotlight on some issues. The centralized collaboration policy has the advantage of being economically and environmentally interesting at the price of requiring full knowledge of problem parameters, which is difficult to achieve in real-life settings. Regarding the decentralized collaboration policies introduced in the current chapter, they do not require information sharing, being based on a single contract, and can lack flexibility. To go further in this study, two collaboration policies for partial information sharing are proposed in Chapter 6: **(i)** a game-theoretic collaboration policy for one-sided asymmetric information sharing and, **(ii)** a contractual-based collaboration policy obtained via a negotiation-based scheme managed by a blinded mediator, for symmetric feedback sharing.

Chapter 6

Collaborative lot-sizing for industrial symbiosis

Industrial symbiosis is promoted as a sustainable way to convert production residues into high added-valued products. The by-product synergy is a particular configuration of an industrial symbiosis system, where the by-products generated by a production unit are used as raw materials by another production unit. The by-product exchange takes place between two or several related or independent companies, which requires to align lot-sizing decisions of each involved actor. To cope with this joint production planning problem, we focus on the framing of symbiotic partnerships within a by-product synergy network, involving one supplier and one receiver of by-products. The collaboration policies can differ from one industrial symbiosis to another. In this chapter, we investigate several collaboration policies for different levels of information sharing designed by using various approaches: **(i)** centralized and decentralized collaboration policies based on mixed-integer programming, for full and no information sharing, **(ii)** a game-theoretic collaboration policy for one-sided asymmetric information sharing, **(iii)** a contractual-based collaboration policy obtained via a negotiation-based scheme managed by a blinded mediator, for symmetric feedback sharing.

Contents

6.1 Introduction

Industrial symbiosis is one of the sustainable ways to convert production residues into useful and added-value products. This is a collaborative form between companies based on the *exchange of physical flows*, such as production residues or other secondary resources (e.g. water, and energy), and/or the *sharing of services* like knowledge, logistics, expertise [Lombardi and Laybourn, 2012]. According to the Waste Framework Directive[14], production residues are defined as materials that are not deliberately produced during a production process, and can be divided into two broad classes of products: **(i)** *by-products*, i.e. lawful production residues unavoidably obtained as an integral part of a production process, ready for a certain use without further transformation, and **(ii)** *wastes*, i.e. production residues, which are not by-products.

The particular configuration of an industrial symbiosis system, where the by-products generated by a production unit are used as raw materials by another production unit, is called *by-product synergy*. The by-product exchange can take place either within a single parent entity, or between two or several different autonomous companies. The resulting network includes at least two actors: **(i)** a *supplier*, which generates by-products, and **(ii)** a *receiver*, which uses them. In the absence of a single parent entity, the intervention of a third party can be required to ensure the coordination between the supplier and the receiver of a by-product, by means of collaboration policies.

We distinguish two main types of collaboration policies specific to industrial symbiosis with respect to their time frames: opportunistic (short-term/one-time) and symbiotic (long-term/perennial) (see Chapter 5). This chapter focuses on a symbiotic linkage between one supplier and one receiver of by-products, i.e. on long-term collaborations.

Albrecht [2009] discusses different collaboration mechanisms, that can be encountered in a supply chain for long-term production collaborations. There exist cases, where the involved actors develop a mutual trust between themselves and share all their information, enabling thus possible a centralized decision-making related to the by-product exchange. The full information sharing is usually encountered when both actors belong to the same parent company, or when a third party manages the industrial symbiosis network. In general, the full information sharing between actors may be difficult to be considered for different reasons such as the requirements to keep sensitive information private, or not to reveal the risks related to production disruptions or production recipes of products. In the following, we classify the collaborative mechanisms according to the level of information sharing:

- **Full information sharing**: A centralized collaboration policy is possible in case of a full trust between actors. Full information sharing allows for a centralized alignment of lot-sizing decisions. Generally, it happens when actors belong to the same parent company. It has the advantage of being globally economically optimal.

- **Partial information sharing:** Under these settings, some sensitive information are kept private. The negotiation scheme and the game-theoretic based collaborations policies are often used for partial information sharing. The negotiation process can be conducted with or without a mediator (i.e. a third party).

- **No information sharing:** It is often encountered in hierarchical collaboration policies based on a single contract and presupposes a leader and a follower. It leads to a lack of flexibility for the actor that follows the contract.

In general, the full information sharing between actors may be difficult to be considered for different reasons such as the requirements to keep sensitive information private or not to reveal the risks related to production disruptions or production recipes of products. Within an industrial symbiosis system, no information sharing can be restrictive for all the actors, that may lead to no collaboration if the production plan of the follower does not include the reuse of by-products. To satisfy the interest of each actor, the number of contracts submitted for negotiation can be high. The generation of attractive contracts requires partial information sharing of the local information of partners.

Generally, not all parameters related to lot-sizing decisions are very sensitive [Fraccascia and Yazan, 2018]. By nature, an industrial symbiosis network is based on a win-win collaboration between actors, that have interest to share the non-sensitive information, like demands and capacities of their production processes. Holding and setup costs are usually sensitive and need to be kept private. Depending on the sensitive information of each actor, two types of partial information sharing can occur: asymmetric and symmetric.

In case of an asymmetric information sharing, the actor which has the most information on the other one can propose a set of contracts. For symmetric information sharing, an impartial IT platform may be implemented to manage contract proposals and pilot the negotiation process. To encourage actors to accept a given contract, contracts can include incentives like side payments (i.e. amount of money).

Nowadays, a growing number of IT platforms is being implemented not only: **(i)** to facilitate the access to information about the by-product location and availability, but also **(ii)** to support the framing of collaboration schemes. Let us mention some platforms dedicated to fostering the industrial symbiosis [Vladimirova et al., 2019]: **(i)** SYNERGie 4.0 Platform and Database, promoted by International Synergies[31], **(ii)** MAESTRI Toolkit, EPOS Toolbox, Sharebox or SYNERGie 2.0 Platform, etc., developed in the framework of three European projects (MAESTRI[12], EPOS[32] and Sharebox[33]), and **(iii)** Industrial Symbiosis Data Repository Platform[34], an open source platform. These IT platforms are often secured and the divulged information are not accessible by other actors. However, not all companies still prefer to communicate their sensitive information even to their own detriment.

This chapter extends the contribution of Chapter 5, by studying the collaborative lot-sizing problem in an industrial symbiosis framework. More precisely, the contributions of this chapter are as follows:

- Investigating several collaboration policies in an industrial symbiosis framework for different levels of information sharing designed by appropriate mathematical methods: **(i)** a game-theoretic collaboration policy for one-sided asymmetric information sharing, **(ii)** a contractual-based collaboration policy obtained via a negotiation scheme managed by a blinded mediator, for symmetric information sharing (see Section 6.2).

- Analysing these collaboration policies for partial information sharing compared to several baseline centralized and decentralized collaboration policies for full and no information sharing, using mixed-integer linear programming. These collaboration policies are discussed according to three dimensions: the satisfaction of involved actors, the environmental impact and economic benefits.

The remainder of this chapter is organized as follows. Section 6.2 reviews the literature on: **(i)** game-theoretic collaboration policies for asymmetric information sharing in the framework of lot-sizing decisions and, **(ii)** negotiation schemes in lot-sizing problems. The problem under study is stated and the associated centralized policy is introduced in Section 6.3. A game-theoretic collaboration policy for one-sided asymmetric information sharing is explored in Section 6.4. Section 6.5 describes the proposed negotiation-based scheme managed by a blinded mediator for symmetric information sharing. The soundness of the proposed approaches for both asymmetric and symmetric information sharing are shown in Section 6.6 via extensive numerical experiments. Managerial implications of the collaboration policies introduced in this chapter are discussed in Section 6.7. Finally, concluding remarks and perspectives are provided in Section 6.8.

6.2 Literature review

Very little consideration has been given to industrial symbiosis networks in the production planning literature (see Chapter 3). To the best of our knowledge, no paper deals with partial information sharing and advanced collaboration policies. More broadly, the literature review proposed in this section focuses on the collaboration policies in classical supply chains under different settings of partial information sharing: asymmetric and symmetric. For more details on the production planning problems in general, the reader is referred to the literature reviews of Quadt and Kuhn [2008], Buschkühl et al. [2010], Díaz-Madroñero et al. [2014], Brahimi et al. [2017], Melega et al. [2018].

6.2.1 One-sided asymmetric information sharing: Leader-follower

One-sided asymmetric information sharing means that one actor, called a *leader*, has more information than the other ones, called *followers*. The leader is able to propose a contract and the follower adapts. The collaboration policy based on leadership for no information sharing is well-known is the lot-sizing literature. However, regarding partial information sharing, the literature is scarce. Generally, the unknown sensitive information is related to local costs. Among these local costs, some can be estimated. For others, intervals to which the costs belong can and have to be estimated.

Regarding the collaboration mechanisms, the majority of found papers dealing with one-sided asymmetric information sharing study a lot-sizing problem with two actors: a supplier and a retailer. The proposed lot-sizing problems in the literature usually minimize the supplier's costs, who plays the role of the leader, and propose a side payment option. For instance, Kerkkamp et al. [2019] formalize a lot-sizing problem with continuous estimated intervals of the unknown parameters, while Phouratsamay et al. [2020] and Mobini et al. [2019] study a lot-sizing problem with discrete intervals defining the unknown parameters.

The problems studied in this chapter for one-sided asymmetric information sharing are adaptations from the literature to an industrial symbiosis network with: **(i)** the integration of side payments, **(ii)** discrete time space, **(iii)** discrete and time-dependent costs of the leader, and **(iv)** unknown costs of the follower are considered constant by the leader. Contrary to the classical studies, each actor can be the leader in the framework of an industrial symbiosis. This chapter deals with the minimization of the supplier's costs and the receiver's costs alternately.

6.2.2 Symmetric information sharing: Negotiation process

Recall that the coordination mechanisms between two or more actors are complex and can lead to centralized or decentralized collaboration policies depending on the level of information sharing: **(i)** full information sharing, **(ii)** partial information sharing and, **(iii)** no information sharing. For a complete literature review of these coordination mechanisms with mathematical programming models for decentralized decision-making, the reader is referred to Rius-Sorolla et al. [2020]).

This chapter introduces a negotiation scheme between two actors. This negotiation can take two forms. In the first one, one of the actors manages the collaboration, i.e. it proposes the contracts and chooses when the negotiation is finished (see e.g. Dudek and Stadtler [2007], Li et al. [2011], Ogier et al. [2015]). In this configuration, there is a leader and the other actors do not have the same power. In the second form of negotiation, an impartial mediator, for instance an IT platform, manages all the procedure. It has the advantage of being fair. Let us focus in this chapter on a negotiation procedure managed by an impartial mediator. In the lot-sizing literature dealing with a negotiation procedure managed by a mediator, the blinded mediator knows the demands, but does not know the local costs. Generally, the negotiation procedure is based on heuristics and meta-heuristics. The following of the literature review is performed with respect to the following main steps of a conventional negotiation procedure:

1. **First contract generation and contract contents:** Generally, in the lot-sizing literature dealing with a negotiation procedure managed by a mediator, a contract corresponds to a complete solution. The first contract is generated by the mediator, by randomly choosing the values of decision variables. The solution is then sent to each actor.

2. **Feedback to mediator:** In order to converge towards a viable contract, the mediator needs a feedback on each proposed contract by each actor. This feedback can take the form of: **(i)** an information on the quality of the contract (e.g. a vote) or, **(ii)** a counter-proposal. The configuration, where the actors offer counter-proposals, is scarce in the negotiation-based schemes managed by a mediator [Reiß and Buer, 2014]. In the majority of the papers, only the mediator provides contracts.

3. **Use of the feedback for a new contract generation:** In the literature, feedback is used to update the parameters of the solution approach, generally a meta-heuristic or heuristic. It allows, for instance, to update the pheromone matrix in ant colony based approaches [Homberger and Gehring, 2011] or the temperature in simulated annealing approaches [Eslikizi et al., 2015].

4. **Choice of the best contract (acceptance rule):** Generally speaking, the negotiation procedure with a mediator stops after: a fixed number of iteration, or computational time limit, or when no improvement occurs after a

given number of iterations. The contract chosen at the end is the current economically best one or the best rated one.

In order to encourage an actor to choose a contract, which may be not interesting for him but beneficial from a global point of view, side payments can be computed after the recovery of the feedback of each actor [Buer et al., 2015, Eslikizi et al., 2015, Homberger et al., 2015].

Contributions of the chapter. The work proposed in the current chapter addresses the symbiotic partnerships within a by-product synergy network. Consequently, each actor belongs its own independent supply chain. To allow more flexibility, the contracts proposed by the mediator in this chapter are partial, which is also the case in the negotiation procedure without third party proposed in [Dudek and Stadtler, 2005, 2007, Li et al., 2011, Ogier et al., 2015]. More precisely, the mediator sends to each Production Unit (PU) the by-product exchanges plan. Moreover, our approach differs from the literature as it is based on the estimation of costs, and not on the values of decision variables. Contrary to the literature, which uses the feedback of the actors to update the parameters of the solution approach, the approach proposed in the current chapter is based on a Monte Carlo method, and the feedback are only used to choose the final contract. Finally, to take advantage of the industrial symbiosis network structure, the choice of the final contract is not done based only on an economic criterion, but it is discussed with respect to three criteria: economic, environmental and the satisfaction of PUs. Note that, contrary to existing studies on classical supply chains, if the proposed contracts are not satisfactory for all actors, they can choose to reject the collaboration.

6.3 Sensitive information

In the current chapter, we propose collaboration policies for partial information sharing in the framework of an industrial symbiosis network composed of one supplier and one receiver of by-products, introduced in Chapter 5. For a description of the lot-sizing problem for industrial symbiosis under study, the reader is referred to Section 5.3. Focus now on the identification of sensitive information for the ULS-IS problem.

In accordance with Fraccascia and Yazan [2018], the non-sensitive information of the ULS-IS problem are: the quantity of available by-products at the first PU (PU1), and the required quantity of by-products at the second PU (PU2). These information are sufficient to calculate the environmental impact related to by-products exchange and to identify the potential by-product synergies. Note that these quantities are proportional to the demand of main products, which is supposed known. The internal costs of each PU, needed for production planning, are considered sensitive. We assume that sensitive internal costs vary in intervals inferred from the market knowledge. Due to the low deviation in costs over time and the difficulty to estimate their fluctuation, the unknown parameters are supposed constant by the contract generator.

Table 6.1: Classification of the information related to ULS-IS problem

Known parameters	Estimated parameters	Unnecessary parameters
• Demands: d_t^1, d_t^2, $\forall t \in \mathscr{T}$ • By-product inventory capacity: B	• Setup costs: f^1, f^2 (i.e. SHR^1, SHR^2) • Inventory holding costs: h^1, h^2, $\hat{h}$ • Gains of reusing the by-product: $b^1 - g$, $b^2 - q$	• Production costs: p^1, p^2 • Disposal cost: g • Purchasing cost: q

In the following, we will classify the different parameters into three categories: **(i)** known, **(ii)** to estimate, and **(iii)** unnecessary. As reported in Table 6.1, in the ULS-IS problem, we suppose that the available information are: **(i)** demands d_t^1 and d_t^2 in each period t and, **(ii)** by-product inventory capacity B. Internal costs are considered sensitive and estimated as intervals. All costs are supposed constant over time, so we remove the time index. When trying to estimate the local costs of PU1, the objective function of PU1 can be expressed as follows:

$$\begin{aligned}
C^1 &= \sum_{t=1}^{T}(p^1X_t^1 + f^1Y_t^1 + h^1I_t^1 + \hat{h}J_t + gL_t + b^1W_t) \\
&= p^1\sum_{t=1}^{T}X_t^1 + g\sum_{t=1}^{T}L_t + b^1\sum_{t=1}^{T}W_t + \sum_{t=1}^{T}(f^1Y_t^1 + h^1I_t^1 + \hat{h}J_t) \\
&= p^1d_{1T}^1 + g\sum_{t=1}^{T}(X_t^1 - W_t) + b^1\sum_{t=1}^{T}W_t + \sum_{t=1}^{T}(f^1Y_t^1 + h^1I_t^1 + \hat{h}J_t) \\
&= (p^1+g)d_{1T}^1 + \sum_{t=1}^{T}(f^1Y_t^1 + h^1I_t^1 + \hat{h}J_t + (b^1-g)W_t)
\end{aligned}$$

Similarly, when trying to estimate the local costs of PU2, the objective function of PU2 can be expressed as follows:

$$C^2 = \sum_{t=1}^{T}(p^2X_t^2 + f^2Y_t^2 + h^2I_t^2 + qZ_t + b^2W_t) = (p^2+q)d_{1T}^2 + \sum_{t=1}^{T}\left[f^2Y_t^2 + h^2I_t^2 + (b^2-q)W_t\right]$$

Production costs p^1 and p^2 being constant, they can be neglected. As long as quantity d_{1T}^1 (resp. d_{1T}^2) of main products is produced, $p^1d_{1T}^1$ (resp. $p^2d_{1T}^2$) has to be paid. In the same way, disposal and purchasing costs g and q become unnecessary, since we estimate the gains of reusing the by-product (b^1-g) and (b^2-q) instead of purchasing and disposal costs when estimating sensitive costs.

As mentioned in the previous chapter, the setup cost depends on the inventory holding cost, the average demand and the Setup cost-Holding cost Ratio (SHR). SHR is a well-known parameter in the lot-sizing literature (see e.g. Trigeiro et al. [1989]). This ratio has an impact on the average number of time periods between two consecutive setups, known as the Time Between Order (TBO). SHR links the setup and holding costs. Hence, the setup cost can be computed based on: the average demand $\bar{d}$ known by the negotiator, inventory holding cost β and the SHR, as follows:

$$\mathtt{f}(\beta, SHR, \bar{d}) = \frac{1}{2}\beta\Big(SHR\Big)^2\bar{d} \tag{6.1}$$

According to Assumptions (A.1)-(A.8), the definition domains of the main parameters that can impact our approaches are defined as follows:

1. **Setup cost-holding cost ratios of PU1 (resp. PU2) SHR^1 (resp. SHR^2):** take their values in the set $\{1,2,...,SHR^{max}\}$.

2. **Inventory holding costs of the main products denoted by β^1 and β^2:** Without loss of generality, let us set $\beta^1 = 1$ and express the other parameters in function of β^1. The value of β^2 provides the significance of PU2 compared to PU1. β^2 is defined in the continuous interval $[\beta^{min}, \beta^{max}]$.

3. **Inventory holding cost of the by-product φ:** By Assumption (A.7), the by-product inventory holding cost in PU1 is lower than the main product inventory holding cost. To keep the scale, we fix $\varphi \in [0, \beta^1[$. This interval is continuous. Note that, when the by-product is unstorable, φ is fixed to zero.

4. **Gain γ of reusing the by-product instead of disposing of it and purchasing the raw material (Estimation of $b-g-q$):** Note that γ is smaller or equal to 0, because it corresponds to a gain. The gain γ can be separated into two coefficients γ^1 and γ^2 such that: **(i)** γ^1 corresponds to the estimation of the gain of PU1 (b^1-g) and, **(ii)** γ^2 represents the estimation of the gain of PU2 (b^2-q). In accordance with other coefficients, let us define $\gamma^1 \in [-\beta^1, -\varphi]$ given Assumptions (A.4) and (A.8), and $\gamma^2 \in [-\beta^2, 0]$ given Assumption (A.8).

5. **Setup costs α^1 and α^2:**

$$\begin{aligned}
\alpha^1 &= \mathtt{f}(\beta^1, SHR^1, \bar{d}^1) \\
\alpha^2 &= \mathtt{f}(\beta^2, SHR^2, \bar{d}^2)
\end{aligned}$$

6.4 One-sided asymmetric information sharing: Game-theoretic collaboration policy

This section explores lot-sizing problems for industrial symbiosis in case of one-sided asymmetric information sharing. The production unit having the information about lot-sizing decisions of the other one is called the leader. The production unit, which does not have information about the planning of its interlocutor, is called the follower. The leader and the follower aim to collaborate, i.e. to synchronize their production plans in order to reuse the by-product generated by the supplier. For this purpose, the leader proposes a menu of contracts. A contract is composed of a production plan, and a potential side payment, i.e. an amount of money given by the leader to the follower to encourage him/her to accept the contract. The two following configurations are studied:

- *PU1 has the leadership* and proposes a menu of contracts to PU2. This situation is realistic by the fact that the by-product is created by the first production unit that has to get rid of it, even if it has to pay for that.
- *PU2 has the leadership*. The relevance of this situation lies in the fact that PU1 can have an interest to adapt its production according to production of PU2 in order to get ride of the by-product.

Both configurations are explained more deeply in Sections 6.4.1 and 6.4.2. In the following, in order to make the problem tractable using mixed-integer programming, we suppose that the unknown parameters can be estimated by the leader using a set of scenarios $\Theta = \{1, 2, \ldots, |\Theta|\}$. Each scenario $\theta \in \Theta$ corresponds to a realization of the unknown parameters.

We also suppose that from the leader point of view, a scenario $\theta \in \Theta$ has a probability $\mathbb{P}(\theta)$ of occurring. All decision variables introduced in Chapter 5 are also indexed by the scenario index. All unknown parameters of the follower are also indexed by the scenario index. We also introduce new decision variables $z(\theta)$ that represent the side payment given by the leader to the follower, corresponding to scenario $\theta \in \Theta$.

We also introduce $C^*(\theta)$, which represents the optimal value of the follower without collaboration if scenario $\theta \in \Theta$ occurs. This value is obtained by solving the single-item lot-sizing problem of the follower without collaboration, by fixing the unknown parameters to the associated scenario. $C(\hat{\theta}/\theta)$ is the value of the optimal solution of the follower when the follower applies the production plan corresponding to scenario $\hat{\theta}$, whereas his/her true estimated parameters correspond to scenario $\theta \in \Theta$.

6.4.1 The supplier has the lead

In this section, the supplier of by-products is the leader and the receiver is the follower. As discussed in Table 6.1, the unknown parameters depend on the scenario θ. They are given by: $h^2(\theta)$, $f^2(\theta)$ and $(b^2 - q)(\theta)$, $\forall \theta \in \Theta$.

In the following, we assume, without loss of generality, that only one parameter is unknown and thus corresponds to a scenario realization. The following situations can be investigated:

- The inventory holding cost of the main product of PU2 is unknown, i.e. $h^2(\theta) = h^2_\theta$, $f^2(\theta) = \mathtt{f}(h^2_\theta, SHR^2, \bar{d}^2)$ and $(b^2 - q)(\theta) = b^2 - q$, $\forall \theta \in \Theta$,
- SHR of PU2 is unknown, i.e. $h^2(\theta) = h^2$, $f^2(\theta) = \mathtt{f}(h^2, SHR^2_\theta, \bar{d}^2)$ and $(b^2 - q)(\theta) = b^2 - q$, $\forall \theta \in \Theta$,
- The gain of reusing the by-product of PU1 instead of purchasing raw materials from an external supplier is unknown, i.e. $h^2(\theta) = h^2$, $f^2(\theta) = f^2$ and $(b^2 - q)(\theta) = (b^2 - q)_\theta$, $\forall \theta \in \Theta$.

The goal of the supplier is to propose a set of contracts, each one corresponds to a scenario realization in order to minimize his/her average estimated cost. The supplier has to ensure that each contact provides to the receiver a total production cost at least as good as the cost obtained without collaboration. The supplier should also ensure that the receiver does not lie on his/her true costs. The problem that the supplier has to solve can be formulated by the following mixed-integer linear program:

$$\min \sum_{\theta \in \Theta} \mathbb{P}(\theta) \left[\sum_{t=1}^{T} \left(f_t^1 Y_t^1(\theta) + p_t^1 X_t^1(\theta) + h_t^1 I_t^1(\theta) + \hat{h}_t I_t^1(\theta) + b_t^1 W_t(\theta) + g_t L_t(\theta) \right) + z(\theta) \right] \tag{6.2}$$

$$\text{s.t.} \quad C(\hat{\theta}/\theta) = \sum_{t=1}^{T} \left[f^2(\theta) Y_t^2(\hat{\theta}) + h^2(\theta) I_t^2(\hat{\theta}) + (b^2 - q)(\theta) W_t(\hat{\theta}) \right], \qquad \forall \hat{\theta}, \theta \in \Theta \tag{6.3}$$

$$C(\theta/\theta) - z(\theta) \leq C^*(\theta), \qquad \forall \theta \in \Theta \tag{6.4}$$

$$C(\theta/\theta) - z(\theta) \leq C(\hat{\theta}/\theta) - z(\hat{\theta}), \qquad \forall \hat{\theta}, \theta \in \Theta \tag{6.5}$$

$$I_{t-1}^1(\theta) + X_t^1(\theta) - I_t^1(\theta) = d_t^1, \qquad \forall \theta \in \Theta, \forall t \in \mathscr{T} \tag{6.6}$$

$$I_0^1(\theta) = J_0(\theta) = J_T(\theta) = 0, \qquad \forall \theta \in \Theta \tag{6.7}$$

$$X_t^1(\theta) \leq M_t^1 Y_t^1(\theta), \qquad \forall \theta \in \Theta, \forall t \in \mathscr{T} \tag{6.8}$$

$$I_{t-1}^2(\theta) + X_t^2(\theta) - I_t^2(\theta) = d_t^2, \qquad \forall \theta \in \Theta, \forall t \in \mathscr{T} \tag{6.9}$$

$$I_0^2(\theta) = I_T^2(\theta) = 0, \qquad \forall \theta \in \Theta \tag{6.10}$$

$$Y_t^2(\theta) \leq X_t^2(\theta) \leq M_t^2 Y_t^2(\theta), \qquad \forall \theta \in \Theta, \forall t \in \mathscr{T} \tag{6.11}$$

$$W_t(\theta) + L_t(\theta) + J_{t-1}(\theta) = X_t^1(\theta) + J_t(\theta), \qquad \forall \theta \in \Theta, \forall t \in \mathscr{T} \tag{6.12}$$

$$W_t(\theta) + Z_t(\theta) = X_t^2(\theta), \qquad \forall \theta \in \Theta, \forall t \in \mathscr{T} \tag{6.13}$$

$$J_t(\theta) \leq B, \qquad \forall \theta \in \Theta, \forall t \in \mathscr{T} \tag{6.14}$$

$$X_t^1(\theta), X_t^2(\theta), I_t^1(\theta), I_t^2(\theta), W_t(\theta), Z_t(\theta), L_t(\theta), z(\theta) \geq 0, \qquad \forall \theta \in \Theta, \forall t \in \mathscr{T} \tag{6.15}$$

$$C(\hat{\theta}/\theta) \geq 0, \qquad \forall \hat{\theta}, \theta \in \Theta \tag{6.16}$$

$$Y_t^1(\theta), Y_t^2(\theta) \in \{0,1\}, \qquad \forall \theta \in \Theta, \forall t \in \mathscr{T} \tag{6.17}$$

The objective function (6.2) minimizes the sum of all costs related to PU1: fixed setup costs, unitary production, inventory holding, disposal and transportation costs, and the side payment paid to PU1. The set of constraints (6.3) defines the sum of all the PU2's lot-sizing costs related to scenario $\theta \in \Theta$, when PU1 proposes a contact related to scenario $\hat{\theta} \in \Theta$. Constraints (6.4) ensure that the contracts proposed to PU2 are always advantageous for PU2, i.e. the total incurred cost of PU2 is at least as good as the cost without collaboration. The set of constraints (6.5) pushes PU2 to accept the contract corresponding to its true costs, i.e. PU2 does not have interest in lying. The sets of constraints (6.6)-(6.8) correspond to the lot-sizing constraints related to PU1. The sets of constraints (6.9)-(6.11) are the classical lot-sizing constraints related to PU2. Constraints (6.12)-(6.13) are the flow conservation constraints of the by-product linking the two production units. Constraints (6.14) limit the by-product inventory capacity in each period. Finally, Constraints (6.15), (6.16) and (6.17) define the decision variables.

6.4.2 The receiver has the lead

In this section, the receiver is the leader and the supplier is the follower. As discussed in Table 6.1, the unknown parameters depend on the scenario θ. They are given by: $h^1(\theta)$, $f^1(\theta)$ and $(b^1 - q)(\theta)$, $\forall \theta \in \Theta$.

As in the previous section, we assume, without loss of generality, that only one parameter is unknown and thus corresponds to a scenario realization. The following situations can be investigated:

- The inventory holding cost of the main product of PU1 is unknown, i.e. $h^1(\theta) = h_\theta^1$, $f^1(\theta) = \mathtt{f}(h_\theta^1, SHR^1, \bar{d}^1)$ and $(b^1 - q)(\theta) = b^1 - q$, $\forall \theta \in \Theta$,
- SHR of PU1 is unknown, i.e. $h^1(\theta) = h^1$, $f^1(\theta) = \mathtt{f}(h^1, SHR_\theta^1, \bar{d}^1)$ and $(b^1 - q)(\theta) = b^1 - q$, $\forall \theta \in \Theta$,
- The gain of reusing the by-product of PU1 instead of purchasing raw materials from an external supplier is unknown, i.e. $h^1(\theta) = h^1$, $f^1(\theta) = f^1$ and $(b^1 - q)(\theta) = (b^1 - q)_\theta$, $\forall \theta \in \Theta$.

The problem faced by the receiver is exactly the one that is faced by the supplier when the supplier has the lead. The problem that the receiver has to solve, can be formulated as the following mixed-integer linear program:

$$\min \sum_{\theta \in \Theta} \mathbb{P}(\theta) \left[\sum_{t=1}^{T} \left(f^2 Y_t^2(\theta) + p_t^2 X_t^2(\theta) + h_t^2 I_t^2(\theta) + b_t^2 W_t(\theta) + q_t Z_t(\theta) \right) + z(\theta) \right] \tag{6.18}$$

$$\text{s.t. } (6.4) - (6.9), (6.11) - (6.17) \tag{6.19}$$

$$C(\hat{\theta}/\theta) = \sum_{t=1}^{T} \left[f^1(\theta) Y_t^1(\hat{\theta}) + h^1(\theta) I_t^1(\hat{\theta}) + \hat{h}(\theta) J_t(\hat{\theta}) + (b^1 - g)(\theta) W_t(\hat{\theta}) \right], \quad \forall \hat{\theta}, \theta \in \Theta \tag{6.20}$$

$$I_0^2(\theta) = 0, \quad \forall \theta \in \Theta \tag{6.21}$$

$$I_T^1(\theta) = 0, \quad \forall \theta \in \Theta \tag{6.22}$$

Model (6.18)-(6.22) is symmetrical with regard to Model (6.2)-(6.17). The objective function (6.18) minimizes the sum of all the costs associated with PU2. The set of constraints (6.20) defines the sum of all the PU1's lot-sizing costs related to scenario $\theta \in \Theta$, when PU2 proposes a contract related to scenario $\hat{\theta} \in \Theta$. Constraints (6.21) fixes the initial inventory of the main product of PU2 to zero. The ending inventory of the main product of PU1 is emptied by Constraints (6.22).

6.5 Symmetric information sharing: Negotiation process managed by a mediator

In the case of one-sided asymmetric information sharing, it is quite natural to consider that the leader manages the network. In the case of symmetric information sharing, there is no perfect leader. In this section, a negotiation-based scheme managed by a blinded mediator is the most suitable. In the following, we introduce a negotiation procedure managed by a blinded mediator operating in the context of an industrial symbiosis.

6.5.1 General scheme

As announced in Section 6.3, we suppose that the mediator only knows the demands of PU1 and PU2, d_t^1 and $d_t^2, \forall t \in \mathscr{T}$, and by-product inventory capacity B. The goal of the mediator is to propose contracts economically and environmentally attractive for PU1 and PU2. To propose good contracts, the mediator has to estimate accurately the unknown parameters. consequently, the mediator will suppose that all parameters are constant. The mediator has then to estimate the following parameters:

- Setup Cost-Holding Cost ratios $SHR^1, SHR^2 \in \{1, 2, ..., SHR^{max}\}$;
- Inventory holding costs: $\beta^2 \in [\beta^{min}, \beta^{max}]$ and $\varphi \in [0, \beta^1[$;
- Gains of reusing the by-product instead of disposing of it (PU1) or purchasing raw materials (PU2): $\gamma^1 \in [-\beta^1, -\varphi]$ and $\gamma^2 \in [-\beta^2, 0]$.

The different steps of the negotiation process are provided in Algorithm 6.1, which can be separated into two main phases:

- **Phase 1** *(SHR estimation):* The first phase of Algorithm 6.1 aims at estimating the value of SHR^1 and SHR^2 by a brute force search. Contracts are generated for each value of SHR^1 and SHR^2, while all the other costs are fixed. The values of SHR^i which lead to the higher score are kept for Phase 2. If several values of SHR^i provides the higher score, the interval composed by these values is kept.

- **Phase 2** *(Monte Carlo simulation process):* This phase is based on a Monte Carlo simulation process. It provides a large number of contracts generated by randomly choosing values of $\beta^2, \gamma^1, \gamma^2$ and φ, while SHR^1 and SHR^2 are fixed by Phase 1. These contracts are evaluated by each PU, and those which are not dominated are kept for the final choice.

Algorithm 6.1 Negotiation procedure operated by a mediator based on a Monte Carlo sampling

Create the list of contracts $listContracts = \emptyset$

Phase 1 – SHR estimation phase

for each PU $i \in \{1,2\}$ **do**
  for all $SHR^i \in \{SHR^i_{min}, \dots, SHR^i_{max}\}$ **do**
    mediator: $S^i_r \leftarrow$ `GenerateFirstContract(`SHR^i`)` ▷ see Section 6.5.2
    mediator: send contract S^i_r to PU i
  end for
end for
each PU i: $score^i_r \leftarrow$ `Evaluate`$_i$`(`S^i_r`)` ▷ see Section 6.5.3
mediator: compute $SHR^i := \underset{SHR^i_{min} \le r \le SHR^i_{max}}{\operatorname{argmax}} score^i_r$ for each PU i

Phase 2 – Monte Carlo phase

while stopping condition not met **do**
  mediator: $S_r \leftarrow$ `GenerateContract(`SHR^1, SHR^2`)` ▷ see Section 6.5.2
  mediator: send contract S_r
  each PU i: $score^i_r \leftarrow$ `Evaluate`$_i$`(`S_r`)` ▷ see Section 6.5.3
  $listContracts$.push_back(S_r)
end while
mediator: $S_{best} \leftarrow$ `ChooseBestContract(`$listContracts$`)` ▷ see Section 6.5.4

6.5.2 Contract generation by the mediator

The generation of a contract consists in solving a variant of the following mixed-integer linear Program:

$$\min \sum_{t=1}^{T} (\beta^1 I^1_t + \beta^2 I^2_t + \alpha^1 Y^1_t + \alpha^2 Y^2_t + (\gamma^1 + \gamma^2) W_t + \varphi J_t) \tag{6.23}$$

$$\text{s.t. } (5.2) - (5.13) \tag{6.24}$$

$$I^1_T = 0 \tag{6.25}$$

$$I^2_T = 0 \tag{6.26}$$

Function `GenerateFirstContract(•)`

The goal of this first step is to guess the true values of SHR^1 and SHR^2. Recall that each PU has to rate each proposed contract, and provides a score between a minimal value and maximal value. The idea is that the mediator proposes a first set of contracts without disposal, nor external purchasing for each value of SHR. To this end, the inventory holding cost related to the by-product is set to a constant value different from zero, i.e. $\varphi = 1$, the gains are set to zero, i.e. $\gamma^i = 0, \forall i \in \{1,2\}$. A contract is expressed as an exchange plan of by-products.

The mediator has to solve the following model for PU1:

$$\min \sum_{t=1}^{T} (I_t^1 + \alpha^1 Y_t^1 + J_t + L_t)$$

$$\text{s.t. } (5.2)-(5.4), (5.8)-(5.10), (5.12)-(5.13)$$

For PU2, the model to solve is the following:

$$\min \sum_{t=1}^{T} (I_t^2 + \alpha^2 Y_t^2 + Z_t)$$

$$\text{s.t. } (5.5)-(5.7), (5.11)-(5.13)$$

Note that solving these models leads to contracts, which are not necessarily feasible for each PU. The contract which has the true setup cost will lead to the higher, score since it correspond to the `zeroWaste` policy. In that way, the intervals of definition of SHR^1 and SHR^2 are reduced to accelerate the Monte Carlo simulation procedure.

Function `GenerateContract(•)`

This function generates randomly the unknown parameters and solves a mathematical problem for given values of SHR^1 and SHR^2. Recall that the setup costs α_i are calculated based on $\beta_i, SHR^i, \bar{d}^i$. The mediator solves the model (6.23)-(6.26) with the given values of the parameters, and returns a contract (Function `SolveModel(•)`). A contract is expressed as an exchange plan of by-products. Algorithm 6.2 provides the main steps of the proposed procedure: **(i)** generate randomly the unknown parameters, **(ii)** compute the setup costs α^1 and α^2 and, **(iii)** solve the problem to derive the exchange plan. Note that, SHR^1 and SHR^2 are fixed in the intervals defined in Phase 1 of Algorithm 6.1.

Algorithm 6.2 `GenerateContract(`SHR^1, SHR^2`)`

1: $(\beta^2, \gamma^1, \gamma^2, \varphi) \leftarrow$ Randomly select values of the parameters in their definition intervals
2: **for all** $i \in \{1,2\}$ **do**
3: $\quad \alpha^i \leftarrow \mathtt{f}(\beta^i, SHR^i, \bar{d}^i)$
4: **end for**
5: $S \leftarrow$ `SolveModel(`$\alpha^1, \alpha^2, \beta^2, \gamma, \varphi$`)`
6: **return** S

6.5.3 Contract evaluation

At this step, each production unit has to evaluate the contracts provided by the mediator. Function $\mathtt{Evaluate}_i(S)$ provides a score calculated based on the local costs of production unit $i \in \{1,2\}$, when implementing contract S. A contract S provides the quantity w_t of by-products exchanged in each period $t \in \mathscr{T}$.

Each production unit $i \in \{1,2\}$ estimates its internal optimal cost $C_{eval}^i(S_r)$ for each contract S_r, and transform it into a score $score_r^i$. $C_{eval}^i(S_r)$ is obtained by solving a lot-sizing problem associated with PU i given the exchange plan S_r. The mathematical model that PU1 (resp. PU2) has to solve is given by (6.27)-(6.31) (resp. (6.32)-(6.36)).

Evaluation in PU1

$$C^1_{eval}(S_r) = \min \sum_{t=1}^{T} (p^1_t X^1_t + f^1_t Y^1_t + h^1_t I^1_t + \hat{h}_t J_t + g_t L_t + b^1_t W_t) \tag{6.27}$$

$$\begin{aligned} \text{s.t. } & (5.2)-(5.4), (5.8)-(5.10) & & (6.28)\\ & W_t = w_t, & \forall t \in \mathscr{T} \quad & (6.29)\\ & X^1_t, I^1_t, W_t, J_t, L_t \geq 0, & \forall t \in \mathscr{T} \quad & (6.30)\\ & Y^1_t \in \{0,1\}, & \forall t \in \mathscr{T} \quad & (6.31) \end{aligned}$$

Evaluation in PU2

$$C^2_{eval}(S_r) = \min \sum_{t=1}^{T} (p^2_t X^2_t + f^2_t Y^2_t + h^2_t I^2_t + q_t Z_t + b^2_t W_t) \tag{6.32}$$

$$\begin{aligned} \text{s.t. } & (5.5)-(5.7), (5.11) & & (6.33)\\ & W_t = w_t, & \forall t \in \mathscr{T} \quad & (6.34)\\ & X^2_t, I^2_t, W_t, Z_t \geq 0, & \forall t \in \mathscr{T} \quad & (6.35)\\ & Y^2_t \in \{0,1\}, & \forall t \in \mathscr{T} \quad & (6.36) \end{aligned}$$

Scoring an exchange plan Each PU calculates its own internal optimal cost, but this cost is not provided to the mediator. Instead of providing the true estimated cost, each PU will provide a score. Each score is calculated based on two internal scores.

- $C^i_{nominal}$: No collaboration, i.e. no symbiotic partnership is considered between production units. The by-products generated by PU1 are disposed of, and raw materials used by PU2 are purchased from an external supplier. Let the costs obtained in the framework of this policy, denoted by `No_Co`, be called nominal costs (see Chapter 5).
- $C^i_{zeroWaste}$: This local cost is calculated under perfect settings (not necessarily feasible), when there is no by-product disposal of and no external purchasing of raw materials.

Note that $C^i_{zeroWaste} \leq C^i_{nominal}$, $\forall i \in \{1,2\}$.

Each PU $i \in \{1,2\}$ gives back a score $score^i_r$ to the provided exchange contract S_r. The interval of each score is $]-\infty, score_{max}]$, where $score_{max}$ is the maximal score. The scores are calculated as follows:

$$score^i_r = score_{max} \frac{C^i_{nominal} - C^i_{eval}(S_r)}{C^i_{nominal} - C^i_{zeroWaste}} \tag{6.37}$$

Note that:

- If $C^i_{eval}(S_r) = C^i_{zeroWaste}$ then $score^i_r = score_{max}$.
- If $C^i_{zeroWaste} \leq C^i_{eval}(S_r) \leq C^i_{nominal}$ then $0 \leq score^i_r \leq score_{max}$.
- If $C^i_{eval}(S_r) \geq C^i_{nominal}$ then $score^i_r \leq 0$.

In addition to the scores provided by each PU, the mediator computes an environmental score $score^{exc}_r$ corresponding to the quantity of exchanged by-products. It is computed as follows, for each contract $r \in \{1,...,R\}$:

$$score^{exc}_r = score_{max} \frac{\sum_{t=1}^{T} w_t}{\min\{d^1_{1T}, d^2_{1T}\}} \tag{6.38}$$

If there is no collaboration between PU1 and PU2, i.e. $\sum_{t=1}^{T} W_t = 0$, $score^{exc}_r = 0$. On the contrary, if $\sum_{t=1}^{T} W_t = \min\{d^1_{1T}, d^2_{1T}\}$, $score^{exc}_r = score_{max}$.

6.5.4 Choice of the best contract

When Algorithm 6.1 terminates, the mediator has to choose the final contract to implement among all R proposed contracts. We propose to sort the generated contracts according to three criteria: **(i)** a satisfaction criterion based on the scores returned by PUs (Sat), **(ii)** an environmental impact score based on $score_r^{exc}$ (Env), and **(iii)** an economic score, calculated based on the estimated unknown parameters (Eco).

Satisfaction criterion. This criterion is based on the global satisfaction of the PUs. To satisfy both PUs, the chosen contract must induce the highest cumulative scores. To do this, we compute the satisfaction criterion (Sat_r) for each contract $r \in \{1,...,R\}$ as follows:

$$Sat_r = \frac{score_r^1 + score_r^2}{2} \tag{6.39}$$

Environmental impact. Industrial symbiosis allows to reduce waste and raw materials extraction, having thus a positive environmental impact. In our study, the environmental principle aims at maximizing the quantity of exchanged by-products. To do this, the environmental impact (Env_r) can be defined as follows, for each contract $r \in \{1,...,R\}$:

$$Env_r = score_r^{exc}$$

Economic impact. The economic principle is the objective of the centralized collaboration policy. The goal is to select the contract, which minimizes the global cost without taking into account the cost shared between production units. Recall that the mediator does not know the true values of local costs associated to contracts, but can use the estimated parameters to determine the contract having the minimal global cost. Let C_r be the global cost of contract $r \in \{1,...,R\}$ according to the mediator:

$$C_r = \sum_{t=1}^{T} \left[\beta^1 I_t^1 + \beta^2 I_t^2 + \alpha^1 Y_t^1 + \alpha^2 Y_t^2 + (\gamma_1 + \gamma_2) W_t + \varphi J_t\right]$$

To properly determine the value of a given contract with respect to the aforementioned criteria, the estimated global cost is normalized to scale the range $[0, score_{max}]$. Note that the mediator does not know the `zeroWaste` and `nominal` costs. Consequently, the range of the estimated global cost C_r is computed based on the best and worst costs of the proposed contracts. Let C_{min} and C_{max} be the extreme values of the range in which the estimated global costs vary:

$$C_{min} = \min_{1 \le r \le R} C_r$$
$$C_{max} = \max_{1 \le r \le R} C_r$$

The value of the economic criterion Eco_r $\forall r \in \{1,...,R\}$ is evaluated as follows:

$$Eco_r = score_{max}\left(1 - \frac{C_r - C_{min}}{C_{max} - C_{min}}\right)$$

A weight is given to each criterion by the mediator. Hence, the final contract proposed by the mediator maximizes the weighted sum of the three criteria. It is determined via:

$$R^* = \underset{1 \le r \le R}{\text{argmax}} \left(\mu_{Sat} Sat_r + \mu_{Env} Env_r + \mu_{Eco} Eco_r\right) \tag{6.40}$$

where μ_{Sat}, μ_{Env} and μ_{Eco} are the weights describing the importance given to the three considered criteria (satisfaction, environmental, and economic), such that $\mu_{Sat} + \mu_{Env} + \mu_{Eco} = 1$. Function `ChooseBestContract(•)` in Algorithm 6.1 returns a single resulting contract R^*.

6.6 Numerical experiments

6.6.1 Design of experiments

Computational experiments have been conducted on 7,290 benchmark instances generated in Section 5.6, for a planning horizon length $T = 24$, on a computer with Intel Xeon e5-2620 2.1GHz CPU with 32GB RAM. In these instances, all the costs are stationary and the parameters satisfy Assumptions (A.1)-(A.8). Recall that these instances are classified into different classes depending on the values of critical parameters:

- $\Delta = \frac{h^2}{h^1}$ aims at linking the holding costs of PU1 and PU2. This ratio can be *low* ($\Delta = 0.75$), *medium* ($\Delta = 1$) or *high* ($\Delta = 1.25$). Note that Δ is insightful to reveal the impact of one PU on the production plan of another PU.

- SHR is a well-known parameter in the lot-sizing literature (see e.g. Trigeiro et al. [1989]). This ratio has an impact on the average number of time periods between two consecutive setups, known as the Time Between Order (TBO). SHR links the setup and holding costs. As far as we consider a problem involving two production units, an SHR is generated for each PU, SHR^1 for PU1 and SHR^2 for PU2, which take their values in the set $\{3,4,5\}$.

- Demands d_t^1 and d_t^2, which have an impact on the size of production units, can be: **(i)** *low*: generated following a normal distribution with an average of 50 and a standard deviation of 10, $\forall t \in \mathscr{T}$, **(ii)** *medium*: generated following a normal distribution with an average of 100 and a standard deviation of 20, $\forall t \in \mathscr{T}$, or **(iii)** *high*: generated following a normal distribution with an average of 200 and a standard deviation of 40, $\forall t \in \mathscr{T}$. We denote by $\bar{d}^1$ and $\bar{d}^2$ the average demands of PU1 and PU2, respectively.

- The by-product inventory capacity B can be: **(i)** *tight*: randomly generated around $1.2\bar{d}^1$, **(ii)** *large*: randomly generated around $3\bar{d}^1$, or **(iii)** *null*: when the by-product is unstorable, i.e. $B = 0$.

For each PU $i \in \{1,2\}$, given SHR^i, holding cost h^i and average demand $\bar{d}^i$, setup cost f^i can be computed via formula (6.1):

$$f^i = \mathtt{f}(h^i, SHR^i, \bar{d}^i)$$

All the mathematical programs are solved using IBM ILOG CPLEX 12.6. The characteristics of the negotiation procedure are set as follows: **(i)** the stopping condition corresponds to a fixed number of contracts to generate, **(ii)** $score_{max} = 10$. For the collaboration policies based on the game theory, the interval of estimation of the unknown parameter is computed based on its true value θ^* and with respect to two parameters:

- ***Range:*** $2\Delta_\theta\theta^*$ is the range of the definition domain of unknown parameters, i.e. $\underline{\theta} = \theta^* - \Delta_\theta\theta^*$, $\overline{\theta} = \theta^* + \Delta_\theta\theta^*$. We fixed Δ_θ in the interval $\in \{0.2, 0.5, 0.8\}$.

- ***Size of the interval:*** $|\Theta|$ corresponds to the number of discrete values in interval Θ. Let $|\Theta|$ takes its values in $\{3,7\}$. Note that, when $|\Theta| = 3$, $\Theta = \{\underline{\theta}, \theta^*, \overline{\theta}\}$.

Regarding the collaboration policy based on negotiation, we have fixed the followed values: $SHR^{max} = 7$, $\beta^1 = 1$, $\beta^{min} = 0.75$ and $\beta^{max} = 1.25$.

6.6.2 Result analysis

In this section, we discuss the economic and environmental benefits induced by the exchange of by-products between two production units, by examining the following three collaboration policies for partial information sharing:

- Two game-theoretic collaboration policies for asymmetric information sharing:

  - `PU1_Leader`: The collaboration policy introduced in Section 6.4.1, i.e. the supplier is the leader and the receiver is the follower.
  - `PU2_Leader`: The collaboration policy introduced in Section 6.4.2, i.e. the receiver is the leader and the supplier is the follower.

- `Nego`: The contractual-based collaboration policy obtained via the proposed negotiation-based scheme managed by a blinded mediator, for symmetric information sharing, introduced in Section 6.5.

These collaboration policies are discussed with respect to the baseline collaboration policies for none and full information sharing, introduced in Section 5.7, namely:

- `No_Co`: *No collaboration*, i.e. no symbiotic partnership is considered between production units. The by-products generated by PU1 are disposed of, and raw materials used by PU2 are purchased from an external supplier. Let the costs obtained in the framework of this policy be called *nominal costs*.

- `Full_Co`: *Full collaboration*, i.e. the exchange of by-products are planned in the framework of a centralized collaboration policy. No other policy can provide a better global cost. We call the costs obtained in the framework of this policy: *centralized costs*.

- `Opp_Co`: *Opportunistic collaboration*, i.e. the exchange of by-products is being done by taking advantage of a fortunate matching between the production plans of the supplier (PU1) and the receiver (PU2).

- Two sequential decentralized collaboration policies:

  - `PU1_First`: PU1 makes its production plan first and then PU2 makes its production plan with respect to the available quantities of by-products in PU1,
  - `PU2_First`: PU2 makes its production plan first and then PU1 makes its production plan with respect to the quantities of by-products required by PU2.

For the sake of simplicity and without loss of generality, the impact of each collaboration policy is quantified on small size instances, i.e. for $T = 24$ periods and $SHR^1, SHR^2 \in \{3,4,5\}$.

The gain of each production unit i is calculated with respect to its nominal cost c^i obtained outside any symbiotic partnership, as follows $(1 - c^i_p/c^i) \times 100$, where c^i_p is the cost of production unit i obtained in the framework of a collaboration policy denoted by p, $p \in \{$`Full_Co`, `Opp_Co`, `PU1_First`, `PU2_First`$\}$, $i \in \{1,2\}$. The gains obtained for each of the aforementioned policies are provided in Table 5.4 in Section 5.7. The environmental gain represents the proportion of by-products, which is reused compared to the total quantity of generated by-product. Note that the environmental gain is directly related to the local costs associated with the management of production residues and raw materials, i.e. the economic and environmental benefits are correlated.

Game-theoretic based collaboration policy

In the following, we first discuss the obtained results using the game-theoretic based collaboration policy with respect to the quality of estimated data. Then, we analyze the gains of each PU regarding the definition of the leader (`PU1_First` and `PU2_First`). Recall that in this experiments, we consider only one unknown parameter at a time, namely: SHR^2, SHR^1, h^1, h^2, $b^2 - q$, $b^1 - g$, or $\hat{h}$.

Discussion on the interval of estimation. Table 6.2 summarizes the gaps of `PU1_First` and `PU2_First` to the collaboration policy `No_Co` with respect to the estimated parameter (SHR^2, SHR^1, h^1, h^2, $b^2 - q$, $b^1 - g$, or $\hat{h}$.) and the characteristics of the associated definition interval ($|\Theta|$ and Δ_θ). This gap is calculated as follows: $100 \times (c^1 + c^2 - c^1_p - c^2_p)/(c^1 + c^2)$, where $p \in \{$`PU1_Leader`, `PU2_Leader`$\}$ and c^i is the nominal cost of production unit i.

When analyzing Table 6.2, we can first notice that when the unknown parameter is the gain of reusing the by-product (b^1-g or b^2-q), the gap to `No_Co` is the highest. Regarding the interval of estimation of the unknown parameter, we notice that the number of values in the discrete definition set has a low impact on the global economic gain of the industrial symbiosis network. On the contrary, the large range leads to a significant difference in terms of gain. The more the interval is tight, the more the economic gain is low.

Table 6.2: Game theory based collaboration policies: Gap to `No_Co` (in %) depending on the unknown parameter and the interval of estimation

	ξ	h^2			SHR^2			b^2-q					
`PU1_Leader`	Δ_θ / $\lvert\Theta\rvert$	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8			
	3	4.2	4.6	4.9	4.2	4.6	4.4	4.3	5.1	5.8			
	7	4.2	4.6	4.9	4.1	5.0	5.1	4.3	5.1	5.8			
	ξ	h^1			SHR^1			b^1-g			$\hat{h}$		
`PU2_Leader`	Δ_θ / $\lvert\Theta\rvert$	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8
	3	3.5	3.9	4.2	3.4	3.8	3.5	3.8	4.8	5.7	3.2	3.3	3.4
	7	3.5	4.0	4.2	3.5	4.2	4.2	3.8	4.8	5.7	3.2	3.3	3.4

ξ: Estimated cost, Δ_θ: Standard deviation of the minimal and maximal values defining Θ

Discussion on collaboration policies managed by a leader. The gain of each production unit i is calculated in the same way as for the baseline collaboration policies, i.e. with respect to its nominal cost c^i obtained outside any symbiotic partnership, as follows $(1-c^i_p/c^i)\times 100$, where c^i_p is the cost of production unit i obtained in the framework of a collaboration policy denoted by p, $p \in$ {`PU1_Leader`, `PU2_Leader`}, $i \in \{1,2\}$. The gains obtained for each of the aforementioned policies and for each estimated parameter (SHR^2, SHR^1, h^1, h^2, b^2-q, b^1-g, or $\hat{h}$.) are provided in Tables 6.3 and 6.4. The environmental gain represents the proportion of by-products, which is reused compared to the total quantity of generated by-product. The side payment is computed as a proportion of the cost of the leader given to the follower.

First, we perform a comparison with the collaboration policies based on a leadership, `PU1_First` and `PU2_First` summarized in Table 5.4 in Section 5.7. One can note that environmentally speaking, `PU1_Leader` and `PU2_Leader` are comparable to the full collaboration policy `Full_Co` with a gain of 98.4% on average in the worst case.

Now, if we focus on the economic indicators, we can notice that in collaboration policies based on game theory, the leader has gains higher than the gains obtained with other collaboration policies. For instance, with collaboration policy `PU1_Leader`, the gain of PU1 is at least 0.2% better than `Full_Co`, when the by-product is unstorable, and 0.6% better than `Full_Co`, when the by-product is storable with a limited capacity, in average. With the collaboration policy `PU2_Leader`, PU2 increases its economic gains by 0.4% in average.

As regards the results of the follower, when the estimated parameter is the SHR or the inventory holding cost, we obtain gains below those obtained with collaboration policy `Opp_Co`, despite the side payments (e.g. 2.2% in average for PU2 when it is the follower against 2.4% with the collaboration policy `Opp_Co`).

Note that, the collaboration policy `PU2_Leader` provides better gains for the follower when the by-product is unstorable than when the by-product is storable with a limited capacity.

When the unknown parameters are $q-b^2$ or $g-b^1$, the gain of the follower is better than the one obtained with the opportunistic collaboration policy `Opp_Co` but it does not reach the gain provided by the sequential collaboration policy where the follower makes its production plan second (e.g. with the collaboration policy `PU1_Leader` the gain of PU2 is 3.3% in average against 2.4% for `Opp_Co` and 5.1% for `PU1_First`).

From Tables 6.3 and 6.4, we can conclude that the collaboration policies `PU1_Leader` and `PU2_Leader` do not provide an equitable distribution of the costs, but allow both the leader and follower to obtain financial savings.

Table 6.3: `PU1_Leader` against `No_Co`: *Economic and environmental gains (in %)*

Collaboration policy `PU1_Leader`										
Gains	B	h^2			SHR^2			$g-b^2$		
		min	avg	max	min	avg	max	min	avg	max
PU1	$=0$	0.4	7.3	22.0	0.5	7.4	22.0	0.4	7.2	22.0
	>0	0.6	8.6	23.3	0.6	8.6	23.3	0.6	8.5	23.3
PU2	$=0$	0.1	2.1	11.0	0	2.1	20.0	0.1	3.2	18.5
	>0	0.1	2.2	10.8	0	2.2	18.6	0.1	3.4	17.6
Env.	$=0$	39.3	97.7	100	55.9	98.4	100	37.6	97.4	100
	>0	54.2	98.6	100	66.7	99.0	100	39.3	98.5	100
Pay.	$=0$	0	0	1.8	0	0	2.2	0	0	4.7
	>0	0	0	1.6	0	0	1.9	0	0	4.0

avg: Average, Env.: Environmental, Pay.: Side payment

Table 6.4: `PU2_Leader` against `No_Co`: *Economic and environmental gains (in %)*

Collaboration policy `PU2_Leader`													
Gains	B	h^1			SHR^1			$g-b^1$			$\hat{h}$		
		min	avg	max	min	avg	max	min	avg	max	min	avg	max
PU1	$=0$	0.1	2.4	15.4	0	2.4	18.2	0.1	3.6	17.6			
	>0	0	1.7	9.9	0	1.2	15.6	0.1	4.2	18.6	0	0.5	6.4
PU2	$=0$	0.4	6.3	23.1	0.9	6.4	23.1	0.4	6.3	23.1			
	>0	0.6	6.8	22.3	0.6	6.8	22.3	0.4	6.8	22.3	0.6	6.8	22.3
Env.	$=0$	41.0	97.3	100	52.7	98.1	100	37.1	97.1	100			
	>0	81.9	99.9	100	87.5	99.9	100	65.6	99.3	100	75.7	99.9	100
Pay.	$=0$	0	0	1.7	0	0	0.9	0	0	4.2			
	>0	0	0	1.3	0	0	1.0	0	0	4.6	0	0	1.4

avg: Average, Env.: Environmental, Pay.: Side payment

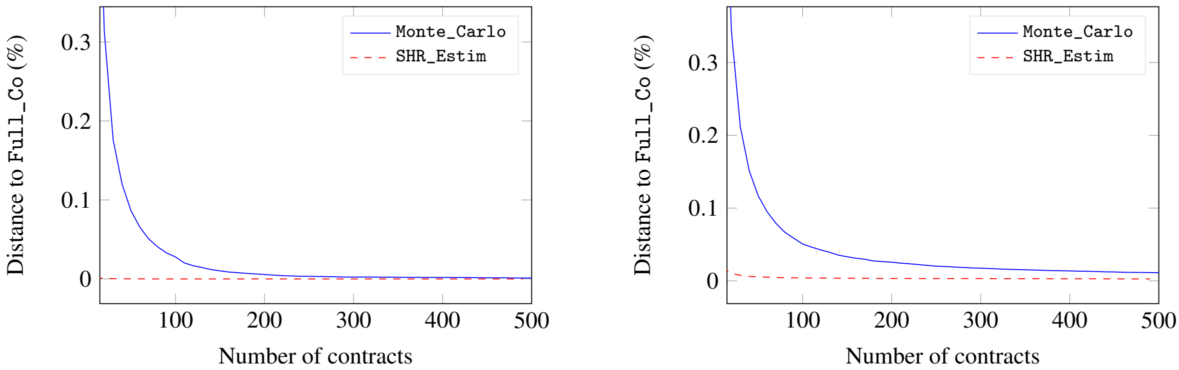
Negotiation procedure managed by a mediator (Algorithm 6.1)

In this, section we perform some experiments in order to analyze the proposed negotiation managed a mediator in the case of symmetric information sharing. The goal of this section is twofold: **(i)** to evaluate the impact of the critical parameters, and **(ii)** to show the industrial soundness of the proposed approach. To do this, we carry out the comparison between the following approaches:

- `SHR_Estim`: Negotiation procedure (Algorithm 6.1).
- `Monte_Carlo`: Negotiation procedure (Algorithm 6.1) without Phase 1, i.e. based only on the Monte Carlo simulation procedure. It is a basic approach, which consists in randomly generating a large number of contracts to converge towards efficient and attractive contracts by virtue of the law of large numbers.
- `Full_Co`: Centralized collaboration policy (i.e. Model (5.1)-(5.13)) of the ULS-IS problem.

`Full_Co` provides the best contract that the mediator can proposed in terms of economic benefits. Consequently, to discuss the competitiveness of `SHR_Estim` against `Monte_Carlo`, we compute for each contract, its gap to `Full_Co` using following formula: $(c_p^1 + c_p^2 - c) \times 100/c$, where c_p^1 (resp. c_p^2) is the cost of PU1 (resp. PU2) for the approach $p \in \{$`SHR_Estim`, `Monte_Carlo`$\}$ and c is the global cost associated with `Full_Co`. For each instance, only the minimal gap to `Full_Co` is kept.

Note that the computational time needed to solve both versions of the negotiation scheme depends linearly on the number of proposed contracts. For instance, the computational time of `SHR_Estim` and `Monte_Carlo` for 50 contracts is about 6 seconds on average.



(a) $B/\bar{d}^1 = 0$ (b) $B/\bar{d}^1 \neq 0$

Figure 6.1: Average gap to `Full_Co` (in %)

***SHR_Estim* versus *Monte_Carlo*.** Figure 6.1 provides the average gap to `Full_Co` for `SHR_Estim` and `Monte_Carlo` for different numbers of proposed contracts for the two following situations: when the by-product is unstorable ($B/\bar{d}^2 = 0$), and when it is storable with a limited capacity ($B/\bar{d}^2 \neq 0$). When the by-product is unstorable, the gap of `SHR_Estim` to `Full_Co` converges quickly to very small values (below 0.001%). When the by-product is storable with a limited capacity, the gap of `SHR_Estim` to `Full_Co` starts from 0.019%, and converges to values below 0.003% after 200 contracts. The gap of `SHR_Estim` at the first iteration is better than the results obtained with `Monte_Carlo` after 500 contracts which reaches 0.011%. To summarize, `SHR_Estim` allows us to reduce the number of proposed contracts, and consequently the computational time. `SHR_Estim` provides better results than `Monte_Carlo`, whatever the storability of the by-product is. Due to the stabilization of the results for `SHR_Estim` after 200 contracts, the number of contracts is fixed to 200 in the rest of experiments.

Focus on the critical parameters. As illustrated in Figure 6.1, the negotiation procedure described in Algorithm 6.1 is very competitive (the average gap to `Full_Co` is very close to 0%). Figure 6.2 exhibits boxplots representing the distribution of the minimal gap of `SHR_Estim` and `Monte_Carlo` to `Full_Co` of each instance, depending on the critical parameters of instances ($B/\bar{d}^1$, $SHR^1 - SHR^2$, $\bar{d}^1 - \bar{d}^2$). Note that the results are not discussed with respect to Δ since as shown in Section 5.6, its impact on the results is very low. Let us mention, first, that all the gaps to `Full_Co` are below 0.9% for `SHR_Estim` and 1.5% for `Monte_Carlo`. Moreover, Figure 6.2 shows that the gap of at least 75% of instances is at 0% for both `SHR_Estim` and `Monte_Carlo`. Figure 6.2 shows that `SHR_Estim` is persistently better than `Monte_Carlo`. In the following we focus on the distribution of gaps depending on the critical parameters:

- **By-product inventory capacity:** The more the by-product inventory capacity is close to the average demand of PU1 ($\bar{d}^1$), the more the gap of `SHR_Estim` to `Full_Co` is high. For the particular case, where the by-product is unstorable, the gap is below 0.05% for all instances. Note that in this configuration, the difference between the gap of `Monte_Carlo` and the gap of `SHR_Estim` is the highest.

- **Setup cost-holding cost ratios SHR^1 and SHR^2:** The more SHR^1 and SHR^2 are different, the more the gaps to `Full_Co` can be high. We can also notice when $SHR^1 > SHR^2$ the gaps are more significant than when $SHR1 < SHR^2$.

- **Average demands $\bar{d}^1$ and $\bar{d}^2$:** When $\bar{d}^1 > \bar{d}^2$, the gap of `SHR_Estim` to `Full_Co` is very close to 0% for all instances. When $\bar{d}^1 < \bar{d}^2$, gaps are below 0.3%. Finally, for instances having $\bar{d}^1 = \bar{d}^2$, `SHR_Estim` provides the worst results even if the gaps of `SHR_Estim` are better than those provided by `Monte_Carlo`.

Focus on the choice of the final contract. Recall that at the end of the procedure a set of contracts is obtained and the true evaluation of each contract at the PU level is not available for the mediator. The mediator can only

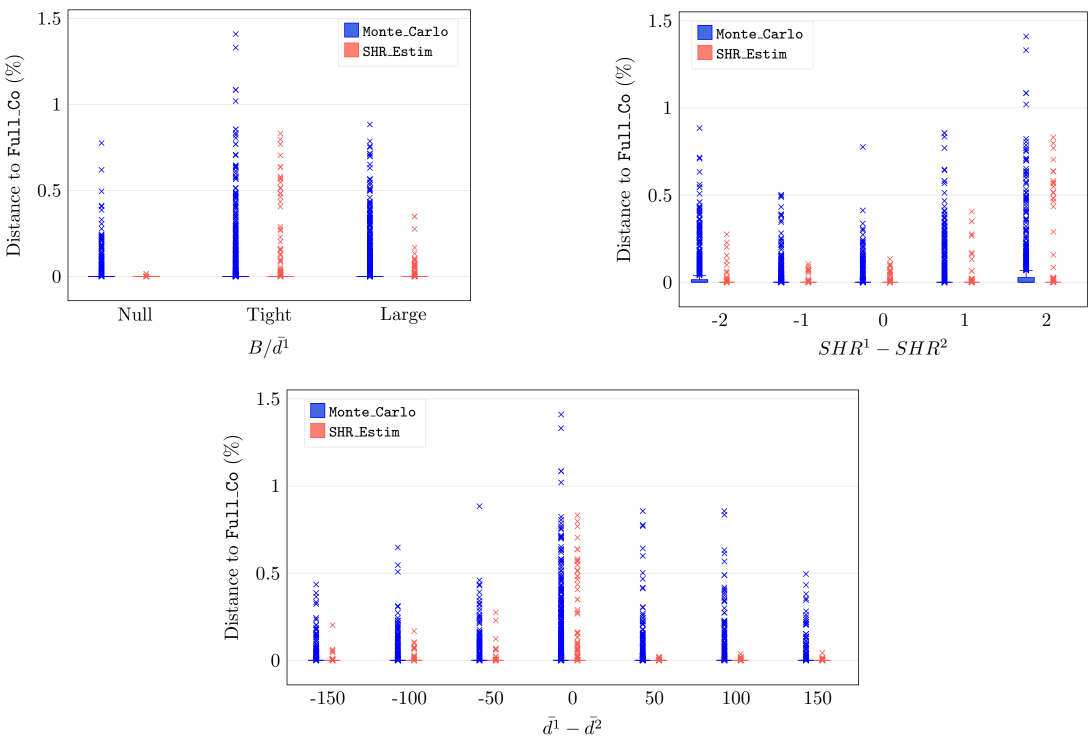


Figure 6.2: Average gap to `Full_Co` (in %) depending on $B/\bar{d}^1$, $SHR^1 - SHR^2$ and $\bar{d}^1 - \bar{d}^2$ for 200 contracts

evaluate these contracts based on the scores introduced in 6.5.4. In this section, we analyze the impact of choosing a contract based on a given aggregation of these scores with respect to the contract obtained with `Full_Co`. To perform this comparison, first we compute the scores of `Full_Co` for each PU (PU1 and PU2) and the score based on the by-product exchange. They are calculated using formulas: (6.37) and (6.38). We denote these scores: $score^1_{Full\_Co}$, $score^2_{Full\_Co}$ for respectively PU1 and PU2, and $score^{exc}_{Full\_Co}$ for the by-product exchange.

The final contract chosen by the mediator depends on the importance given to each criterion. Let `u_v` be the final contract chosen by using criterion $\texttt{u} \in \{\texttt{Sat}, \texttt{Env}, \texttt{Eco}\}$ at first (i.e. $\mu_u = 1$ in Equation (6.40)) and then criterion $\texttt{v} \in \{\texttt{Sat}, \texttt{Env}, \texttt{Eco}\}$ with $\texttt{v} \neq \texttt{u}$. Denote by `All` the special case where all the criteria have the same importance when choosing the final contract, i.e. $\mu_{Sat} = \mu_{Env} = \mu_{Eco} = 1/3$ in Equation (6.40). For each combination of criteria `u_v` used to select the final contract and for each type of score $i \in \{1, 2, exc\}$, the gap to `Full_Co` is computed as follows:

$$Gap = 100 \times \frac{score^i_{Full\_Co} - score^i_{\texttt{u\_v}}}{score^i_{Full\_Co}}$$

Note that the gap is negative when the score obtained with the negotiation procedure is better than the one obtained with `Full_Co`.

Figure 6.3 provides the average gaps to `Full_Co` for seven combinations of criteria. It shows that favoring the economic criteria instead of the satisfaction of PUs and the environment (`Eco_Sat`, `Eco_Env`) leads to high gaps, that are positive for all scores. Even when the economic criterion is applied second in order to discriminate equivalent contracts on other criteria (`Sat_Eco`, `Env_Eco`), the economic criterion is not competitive: `Sat_Eco` provides gaps equivalent to `Sat_Env` in average and `Env_Eco` highly decreases the quality of the contract for PU2, i.e. $score^2$ becomes high, for a very small gains related to PU1, compared to `Env_Sat`. These bad results related to the economic criterion can be explained by the fact that a bad economic score for a contract does not mean that this contract is bad, but it is just less good than the other ones (see Example 6.1). The scale can be low or high depending on the set of contracts. Moreover, the true global cost can be different from the global cost estimated by the mediator. Indeed,

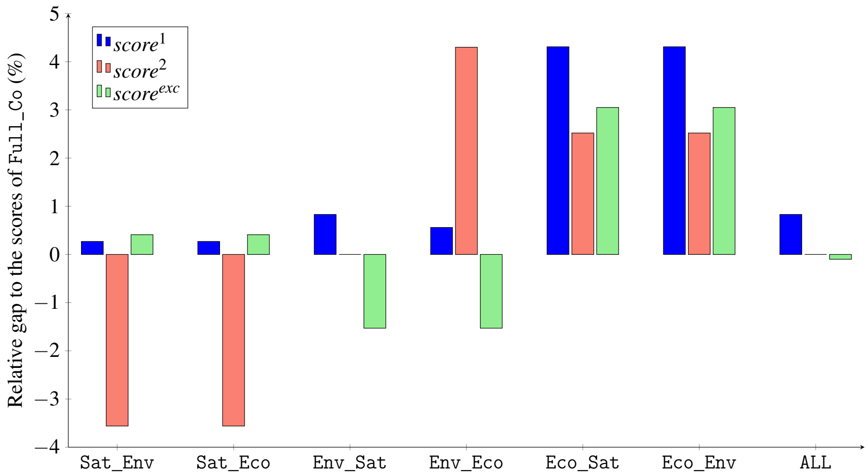


Figure 6.3: `Full_Co` against `u_v`, $\mathtt{u},\mathtt{v} \in \{\mathtt{Sat},\mathtt{Env},\mathtt{Eco}\}$: Average gaps

the mediator proposes contracts only composed of exchange plans, but PUs are free to make their own production plans and the true production plan of each PU is not necessarily the same as the one computed by the mediator. Consequently, choosing the contract with the best economic score is not always relevant, as illustrated in Figure 6.3.

The contract selected by formula (6.40) with $\mu_{sat} = \mu_{env} = \mu_{eco}$ does not allow to keep the environmental gain and the satisfaction of PU2 as interesting as when $\mu_{eco} = 0$, because it is pulled by the economic criterion. In the following, combination `ALL` is changed by setting $\mu_{Eco} = 0$, and $\mu_{Sat} = \mu_{Env} = 1/2$, in Equation (6.40). It is denoted `ALL(Sat,Env)`.

Example 6.1

Consider an instance where $SHR^1 = 5$, $SHR^2 = 3$, $\bar{d}^1 = 200$, $\Delta = 1$, $\bar{d}^2 = 50$ and $B = 0$. Table 6.5 summarizes the non-dominated contracts over $score^1$, $score^2$, $score^{exc}$ and the economic criterion.

The contract selected with `Sat` as first criterion is Contract 2. It is very close to `Full_Co` for the scores provided by the PUs. Regarding the environmental score, Contract 2 leads to a loss of more than 21%. The environmental criterion allows us to select a contract environmentally as good as `Full_Co`, while satisfying PU1 on the account of PU2. The contract chosen with the economic criterion (Contract 4) is globally the worst one as all scores are deteriorated. Consequently, for this instance, the high economic score is not totally due to the low environmental score, otherwise the scores of PU1 and PU2 would be higher thanks to the high flexibility. This might be a direct consequence of the scale and the way of computing the economic score. A contract with a score at 10 reduces the local costs of only 2% with respect to a contract with a score at 0.

Table 6.5: Non-dominated contracts in Example 6.1

Contract	Scores			Criterion value		Gaps to `Full_Co` (%)		
r	$score^1$	$score^2$	$score^{exc}$	Eco_r	Sat_r	$score^1$	$score^2$	$score^{exc}$
1	2.33	4.55	9.08	8.16	3.44	-25.4	29.4	9.2
2	1.86	6.43	7.85	8.90	4.15	0	0	21.5
3	2.68	1.07	10	0	1.87	-44.0	83.4	0
4	1.06	3.93	3.97	10	2.50	42.9	38.9	60.7
`Full_Co`	1.86	6.43	10					

Table 6.6 summarizes the average economic and environmental gains calculated for the different final contracts with respect to the nominal cost. These gains are computed in the same way as for the baseline policies provided in Table 5.4. In the following, obtained results are compared the collaboration policy `Full_Co`. As previously discussed, the main difference between these two collaboration policies is the level of information sharing.

Globally, gains obtained with `Sat_Env`, `Env_Sat` and `ALL(Sat,Env)` are very close to those obtained by `Full_Co`, both economically and environmentally (+/-0.2% for the economic gains and -0.9% in the worst case for the environmental benefits). It shows that the lack of information sharing is not prohibitive. It can be explained by the context and market knowledge of the mediator of private parameters of cost and its thin estimation of the SHR, which is the parameter whose impact is the highest.

Table 6.6: `Nego` against `No_Co`: *Economic and environmental gains (in %)*

Gains	$B/\bar{d}^1$	`Sat_Env`			`Env_Sat`			`All(Sat,Env)`		
		min	avg	max	min	avg	max	min	avg	max
PU1	Null	0.1	7.0	21.6	-0.3	7.0	21.6	0.1	7.0	21.6
	Tight	-0.5	7.8	22.0	-0.5	7.8	22.0	-0.5	7.8	22.0
	Large	-0.5	7.8	21.9	-1.4	7.7	22.0	-0.5	7.7	22.0
PU2	Null	0.3	5.9	23.0	-3.0	5.7	23.0	-0.5	5.8	23.0
	Tight	0.3	6.4	21.8	-2.2	6.2	21.8	0.3	6.4	21.8
	Large	0.3	6.6	21.9	-0.3	6.6	22.0	0.4	6.6	21.8
Env.	Null	51.7	96.6	100.0	78.5	99.5	100.0	78.5	98.9	100.0
	Tight	70.4	98.5	100.0	89.2	100.0	100.0	82.3	99.5	100.0
	Large	67.5	98.7	100.0	99.9	100.0	100.0	76.0	99.8	100.0

avg: Average, Env.: Environmental

Let us now focus on the choice of the final contract. First, note that whatever the criterion used for the choice of the final contract is, the environmental gain is high, 96.6% of the total reusable quantity of by-product is reused on average in the worst case. `Sat_Env` is economically efficient especially when the by-product is unstorable because it costs nothing to both production units. The economic gain is equal to the one obtained with collaboration policy `Full_Co` (7.0% for PU1 and 5.9% for PU2) but the distribution of gains is better as the lowest gain is positive (0.1% for PU1 and 0.3% for PU2 instead of -0.9% and -0.7% respectively for PU1 and PU2 with collaboration policy `Full_Co`). When the by-product is storable with a limited capacity, `Sat_Env` still provides good economic benefits (7.8% for PU1 and 6.5% in average for PU2 against 7.9% for PU1 and 6.4% in average for PU2 for collaboration policy `Full_Co`), even if they can be punctually negative. These economic gains imply that the environmental gains are penalized. On the contrary, `Env_Sat` increases the percentage of reuse of the by-product by at least 1.2% compared to collaboration policy `Full_Co` and it implies an economic loss of only 0.2% for PU1 and PU2 in the worst case. This combination of criteria can be very interesting when the by-product is forbidden in landfills.

Generally speaking, the best choice is the trade-off between economic and environmental gains, namely, option `All(Sat,Env)`. It leads to a percentage of reuse of the by-product close to 100% on average and the economic gains are similar to `Sat_Env` and slightly below `Full_Co` (0.2% below).

6.7 Managerial implications

In this section, let us discuss the economic and environmental opportunities induced by the exchange of by-products between two production units, by examining the collaboration policies described in Section 6.6.

Focus on a specific instance. To illustrate the previously discussed findings, let us take focus on a specific instance. This instance is characterized by unbalanced demands and SHRs that make the setup costs of PU2 much higher than the setup costs of PU1. The contracts obtained under different policies are represented in a 3D score-based system of coordinate in Figures 6.4 and 6.5.

- **Collaboration policy** `Nego`**:** Figure 6.4 represents the contracts obtained with the negotiation procedure (that we denoted `Nego`) with respect to other collaboration policies for a specific instance, depending on three score-based coordinates: from PU1, from PU2 and environmental. The contracts are mostly located in the center of the figure. We notice that the deviation between the scores of production units is relatively low. The contract obtained with `Full_Co` is found by `Nego`, but it is never selected by the proposed scores. The chosen contract of `Nego` is indicated by an arrow in Figure 6.4. It is the best regarding the environment and the satisfaction of PUs. This contract dominates collaboration policies `PU1_First` and `Opp_Co` and looks better than `Full_Co` as it is situated in the quarter $[5,10] \times [0,5]$ formed by coordinates representing the contract rating by PU1 and PU2. However, `Nego` cannot be better than `Full_Co` because `Full_Co` provides the minimal global cost. It leads that a high gain for PU1 that does not compensate a low loss for PU2. As illustrated via this particular instance, the scale of 3D coordinate system based on scores can be misleading but the mediator cannot know the true costs since it is blinded.

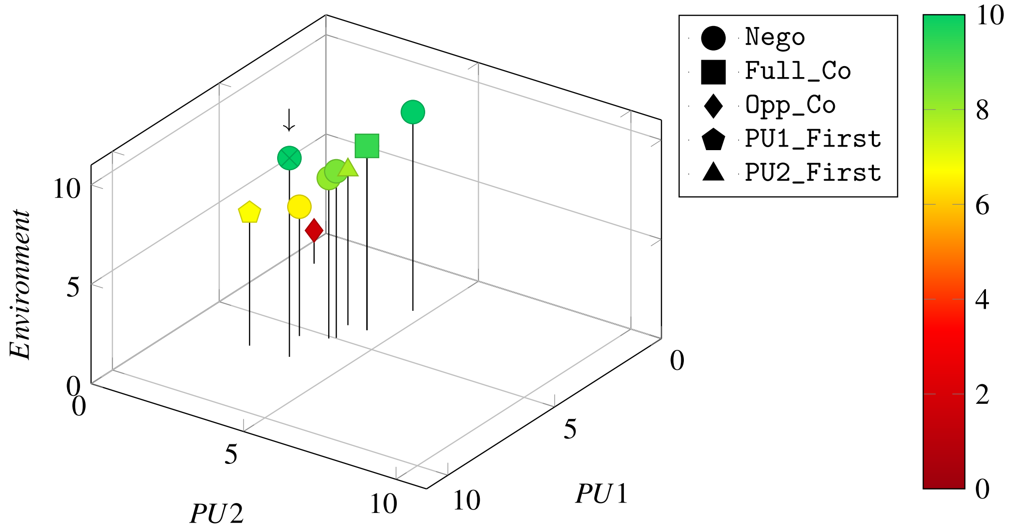


Figure 6.4: Positioning of contracts for the instance $SHR^1 = 3$, $SHR^2 = 5$, $\bar{d}^1 = 50$, $\bar{d}^2 = 100$, $\delta = 1$ and $B = 0$, depending on three scores: from PU1, from PU2 and environmental

- **Collaboration policies** `PU1_Leader` **and** `PU2_Leader`**:** In the following, we position contracts obtained in the case pf one-sided asymmetric information sharing using game-theoretic collaboration policy (`PU1_Leader` and `PU2_Leader`). Figure 6.5 represents the contracts obtained with `PU1_Leader` and `PU2_Leader` for different Θ with respect to other collaboration policies for a specific instance, depending on three scores: from PU1, from PU2 and environmental. The contracts corresponding to different unknown parameters and Θ form a Pareto front with contracts related to `PU1_Leader` on one side and contracts related to `PU2_Leader` on the other side. Note that, there is no contract in the middle due to a bad distribution of costs despite the side payments. Figure 6.5 highlights that the more the score of the follower is low the more the environmental score is good. Generally speaking, the collaboration policies based on the game theory are environmentally better than the sequential collaboration policies.

  Regarding the economic gains, the score of the follower is between 0 and the score obtained within the sequential collaboration policy when it starts first. On the contrary, the score of the leader is between 10 and its score obtained with the sequential collaboration policy. It can be explained by the fact that the leader allows itself to move production periods and increases the storage of the main product to reduce in return the quantities of disposed by-products and purchased raw materials. The gains related to the reuse of the by-product can allow the follower to compensate losses due to the storage of the main product but it does not allow to make higher benefits.

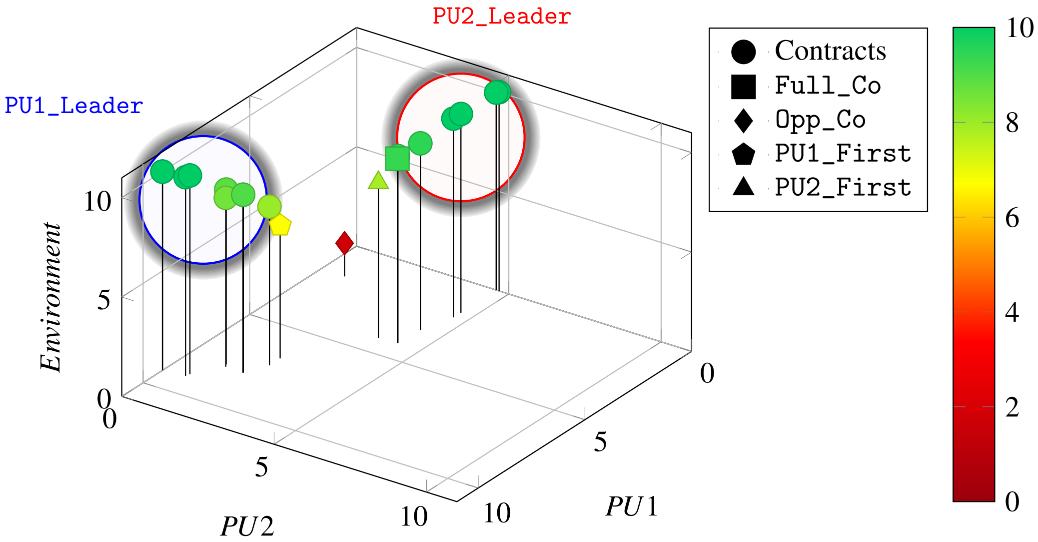


Figure 6.5: Positioning of contracts for the instance with $SHR^1 = 3$, $SHR^2 = 5$, $\bar{d}^1 = 50$, $\bar{d}^2 = 100$, $\Delta = 1$ and $B = 0$, depending on three scores: from PU1, from PU2 and environmental

Discussion on the incentives. Incentives can be: intra-network and coming from outside the network. Inside a network, incentive, generally, economic, aims at compensating a loss of money of one actor by another actor in the form of side payments. Table 6.6 shows that the gains are not always positive for both PUs. Even if an actor may accept a loss for the global interest, this situation is very scarce. In general, without a side payment, an actor will simply refuse the collaboration if its production plan leads to a loss of money. To summarize, the side payment is a way of changing the distribution of gains, i.e. moving the contract from the quarters $[0,5] \times [5,10]$ and $[5,10] \times [0,5]$ to quarter $[5,10] \times [5,10]$ defined by the 2D plan (from PU1, from PU2) in Figure 6.3.

Side payments are allowed in collaboration policies `PU1_Leader` and `PU2_Leader`. However, in the majority of the cases, no side payment is given (see Tables 6.3 and 6.4). It is mainly due to the definition of the problem that requires the leader to propose to the follower contracts at least as good as collaboration policy `No_Co`. Reusing the by-product is sufficient to reduce the costs of the follower. Then, a side payment is not required to meet constraint (6.4) contrary to problems studied in the literature. Moreover, the leader wants to minimize its own costs so it will not give money to the follower if it is not necessary for collaboration. Figure 6.5 shows that the gain of the leader is high while the gain of the follower is very low and even negligible. A side payment seems necessary in this configuration. The way of computing side payments deserves to be improved by using: the exchanged quantity of by-product, the costs or the scores.

Even if an intra-network incentive is well-calculated, it is not always sufficient for a successful implementation of the industrial symbiosis. When the gain of reusing the by-product is low for all actors, the risks related to an industrial symbiosis (e.g. production disruption), time and human energy lost in the implementation of a new collaboration can dissuade the actors to collaborate, despite the environmental benefits. In this case, governments and/or institutions, promoting the industrial symbiosis, have to propose convincing incentives of different types, for instance: **(i)** *economic*, i.e. a kind of environmental bonus that allows production units to move the contracts from the quarter $[0,5] \times [0,5]$ to quarter $[5,10] \times [5,10]$ in Figure 6.3 and, **(ii)** *environmental and legislative* i.e. a directive like Directive 1999/31/EC of the Council of 26 April 1999 on the landfill of waste[30], by which some by-products are not allowed in landfills anymore.

Limitations of the industrial symbiosis-based collaboration policies. In the current chapter, industrial symbiosis-based collaboration policies are given for each level of information sharing. As it has been identified in Chapter 5, there is a strong relationship between the level of information sharing and the results obtained by the different collaboration policies. In fact, collaboration policies will not provide the same behaviour for all levels of information sharing. However, other limitations may appear, they can be global or specific to a collaboration policy. In the following we will discuss some of them:

- **Automating of the negotiation procedure:** Due to the high number of proposed contracts, the negotiation procedure requires an automated generation of contracts and their evaluations. It supposes that the mediator has an IT platform and that the supplier and the receiver both have their own platform.
- **Honesty of each actor:** The success of collaboration policy `Nego` lies in the fact that both the supplier and the receiver are honest in the evaluation of the contracts. If one of them decides to underestimate its true score for each contract, i.e. push the negotiation in its sense, the other actor and the environmental indicator may suffer.
- **Economy-based collaboration policies:** Although the global economic gain is directly related to the quantity of exchanged by-product, the collaboration policies are commonly based on the minimization of the global cost and do not consider the maximization of the by-product exchanges directly in the objective function. Except the negotiation scheme, the collaboration policies investigated in this chapter only propose one final contract that does not allow to favor the environmental aspect.
- **Multiple objectives:** The collaboration policy `Nego` has the advantage of allowing to choose the final contract depending on several criteria. The chosen combination of criteria is only applied on the final contract proposed to PU1 and PU2. This way of choosing the final contract does not take into account the potential difference of objectives between the two PUs.

6.8 Conclusion and perspectives

This chapter investigates collaboration policies for partial information sharing within an industrial symbiosis network, composed by two actors: a supplier and a receiver. For one-sided asymmetric information sharing, two collaboration policies based on the game theory are studied and, for symmetric information sharing, a negotiation scheme managed by a blinded impartial mediator is proposed. The economic and environmental impacts associated with these collaboration policies are discussed with respect to several baseline industrial symbiosis-based collaboration policies introduced in Chapter 5.

We show that the negotiation scheme is a good alternative to the centralized collaboration policy. The obtained gains are very close and the private information may remain private. Regarding the game-theoretic-based collaboration policies, their gains are clearly better than sequential collaboration policies. Another information more significant is required, namely the local optimal cost for each scenario. It is one of the limits identified for the proposed industrial symbiosis-based collaboration policies. The main limits of the negotiation procedure are the possibility for the actors to lie and the need to automatize the process at each level.

In our approaches, the collaboration policies are based on economic benefits and the environmental aspect is only considered at the end of the process. Future works should be dedicated to integrating the environmental aspect in all the collaboration policies as part of the negotiation process. Moreover, the different actors in the network may have different systems of objective prioritization, which should be explicitly considered. Another perspective of this chapter is to integrate incentives, both between the actors of the industrial symbiosis and from outside the network.

Chapter 7

Conclusions and outlooks

7.1 Conclusions

Given the circular economy concerns imposing increasingly stringent constraints to the industrial world, this thesis focuses on the production planning in the framework of the industrial symbiosis. Recall that an industrial symbiosis is a sustainable way of converting production residues into high value-added products, by their reuse by another production unit. Recovered production residues are called by-products. An industrial symbiosis network involves at least two actors, a *supplier* of by-products and its *receiver* that uses it as raw materials.

To circumscribe the scope of the thesis, a taxonomy of the different processes related to the circular economy is proposed. The definitions of notions revolving around the circular economy are clarified and discussed in accordance with the proposed taxonomy. A literature review focused on mid-term production planning is performed to feed into the reflection on how recovery options can make sustainable the traditional production environments. Despite the extensive efforts dedicated to supporting the transition from a linear towards a circular economy, the production planning problems posed in the framework of industrial symbiosis networks are little studied. This thesis intends to contribute to the production planning literature, by studying original lot-sizing problems raised within an industrial symbiosis.

Before addressing the joint production planning at a network level, the problem encountered by the by-product supplier is addressed as a single-item lot-sizing problem including the management of a by-product and inventory capacities (ULS-B problem). Its complexity is studied for time-dependent and constant capacity. We show that the problem is weakly *NP*-Hard when the capacity is time-dependent. For the case with a constant capacity, an $\mathscr{O}(T^6 \log T)$ dynamic programming algorithm is proposed. The proposed algorithms are based on structural properties of the optimal solution and dynamic programming.

The problem encountered by the supplier in Chapter 4 is extended by integrating the production planning of a receiver of by-products. The resulting problem becomes a two-level single-item lot-sizing problem (ULS-IS problem) with a supplier of by-products at the first level and its receiver at the second level. As presupposed in an industrial symbiosis network, intermediate incoming and outgoing flows of production residues and raw materials are allowed. The ULS-IS problem is studied for storable with a limited capacity and unstorable by-products. We have shown that the ULS-IS problem is *NP*-Hard regardless the storability of the by-product. Due to the structure of the ULS-IS problem, a Lagrangian decomposition is applied to separate it into two subproblems easier to solve. Given the complexity of the subproblem of the supplier, the Lagrangian decomposition is coupled with a Lagrangian relaxation. Heuristics, enhanced by a local search procedure, are proposed to derive feasible solutions of good quality. To evaluate the performance of the proposed approach, several heterogeneous classes of instances have been generated for different values of critical parameters. Numerical experiments show that our solution method is more competitive than a state-of-the art leading solver.

To frame the symbiotic relationship between the actors of an industrial symbiosis, a number of collaboration policies is proposed and analyzed for different levels of information sharing. The full information sharing is considered within a centralized collaboration policy, expressed by the ULS-IS problem. When no information are shared between actors, sequential decentralized and opportunistic collaboration policies are examined. Under partial information sharing settings, two collaboration policies based on the offer of multiple contracts are investigated: **(i)** a game-theoretic based policy for one-sided asymmetric information sharing, and **(ii)** a negotiation scheme managed by a mediator for symmetric information sharing.

An in-depth analysis is conducted with respect to the collaboration policies for full and no information sharing modeled by mixed-integer programming. The strengths, shortcomings and implications of the studied collaboration policies are quantified and discussed. Decentralized policies for no information sharing lead to significant economic and environmental gains, but not always well-distributed. We also found that the collaboration policy based on the game theory allows the partners to converge towards results close to that obtained by the centralized collaboration policy, with the advantage of keeping sensitive information private.

7.2 Perspectives

This section consolidates and extends the perspectives discussed throughout this manuscript, while transposing the problems and solution approaches proposed in this manuscript into real-life environments. On the basis of industrial needs, let us provide, in Section 7.2.1, some extensions of the single-item lot-sizing problem with a by-product and inventory capacities (ULS-B problem) studied in Chapter 4. On a wider scale, Section 7.2.2 includes explanations about several possible extensions of the industrial symbiosis network studied in Chapters 5-6. On top of all of these research directions, the sustainability impact is discussed in Section 7.2.3.

7.2.1 Dealing with by-products in production planning

In Chapter 4, we deal with a generic lot-sizing problem with a by-product and inventory capacities. More specifically, the problems studied in this thesis deal with a single final product and a single by-product. In practice, the supply chains are much more complex, due to the large number of products produced by a company and the intricate interactions between companies. One perspective consists in extending the problem, by considering several co-products produced at the same time as a main product and/or several by-products with different characteristics. Several by-products, like steam or waste food, can be stored before their reuse during a limited time. Such a constraint could be considered. In the same way, different by-products can be used to produce the same product with different qualities and after different treatment processes. Moreover, multiple transportation modes can be used depending on the quantity of by-product and the distance between production units (see Figure 7.1).

The integration of multiple co-products or by-products in Model (4.1)-(4.9) should inherit the complexity of the ULS-IS problem: **(i)** *NP*-Hard: when there is at least one time-dependent inventory capacity, and **(ii)** polynomial, otherwise. The complexity of the dynamic programming (DP) based algorithms, which are already high, would increase in the same time as the number of states increases, until becomes intractable in decomposition approaches, where the DP-based algorithms has to be executed a large number of times.

7.2.2 Industrial symbiosis network

The thesis focuses on an industrial symbiosis network composed of one supplier and one receiver of by-products. This kind of network can be often found in eco-industrial parks, in which companies are supposed to work, while fostering the sustainable development . Consequently, as illustrated by the existing industrial symbiosis, such a network often includes more than two actors (see for instance the Kwinana symbiosis[29], whose by-product synergies are schematically illustrated in Figure 7.2). In terms of the morphology of relationships between actors, several extensions of the industrial symbiosis network studied in this thesis could be considered:

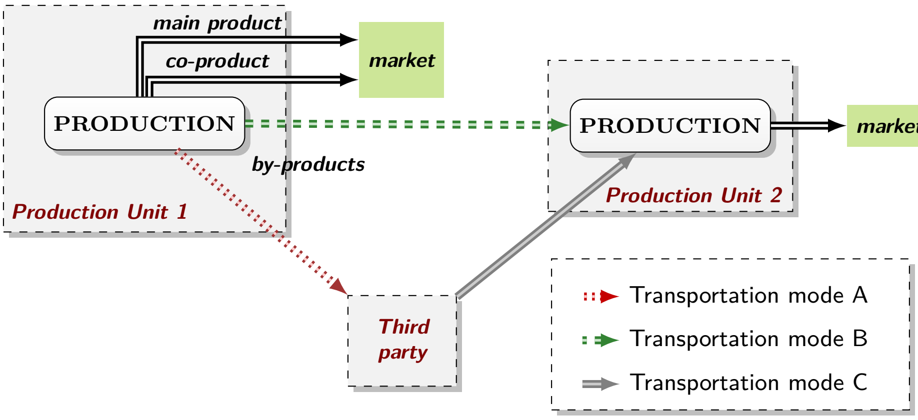


Figure 7.1: Industrial symbiosis network with multiple co-products, multiple by-products and a third party

With a third party. In this thesis, we study a lot-sizing problem modeling an industrial symbiosis network, where the supplier and the receiver directly interact. There exist cases where the by-product requires a treatment before its reuse, that neither the supplier, nor the receiver can perform, a third party can be needed. The by-product can thus be transported twice and the transportation modes are sometimes different, combining fixed and unitary costs. For instance, a solid element can be reused after extraction from a liquid or gaseous by-product. The solid element will be transported by trucks, whereas the liquid or gaseous by-product can be transported by a pipeline as illustrated in Figure 7.1.

The intervention of a third party can also involve a lead time between the time period when the by-product is generated and sent to the third party and the time period when it can be reused. Another consequence of the integration of a third party is the growing complexity and sophistication in the collaboration mechanisms between all the actors, especially if the third party has a limited flexibility due to its own local constraints (e.g. limited capacity in the process, the storage and/or the transportation).

One-to-many relationship. We identify two types of one-to-many relationships between the supplier and the receiver depending on the direction of the by-products flows:

- **One supplier-multiple receivers:** In this configuration, there are multiple receivers for a given by-product. For instance, in the Kwinana symbiosis[29] the demineralised water produced by the coal-fired power station is reused by two different production units, namely a gas fired power plant and an industrial chemical & fertilizer producer (illustrated by dashed green arrows in Figure 7.2).

- **Multiple suppliers-one receiver:** This industrial symbiosis network is composed of multiple suppliers and one receiver of by-products. The by-products are not necessarily the same for all suppliers. A same substance can be generated by the production of many products. For instance, in the Kwinana symbiosis[29], the titanium dioxide producer receives several types of wastewater from several different companies (illustrated by dotted red arrows in Figure 7.2).

The problems to solve are special cases of supplier/receiver selection problem [Lamba and Singh, 2019, Zouadi et al., 2018] with additional constraints. Several different criteria can be examined to find the best supplier (receiver) at each period: the distance between the suppliers and the receiver (e.g. in Figure 7.2, the industrial chemical and fertilizer producer is far from the titanium dioxide producer compared to other companies), the environmental impact, the quality of the by-product, with or without the pre-treatment of the by-product, etc.

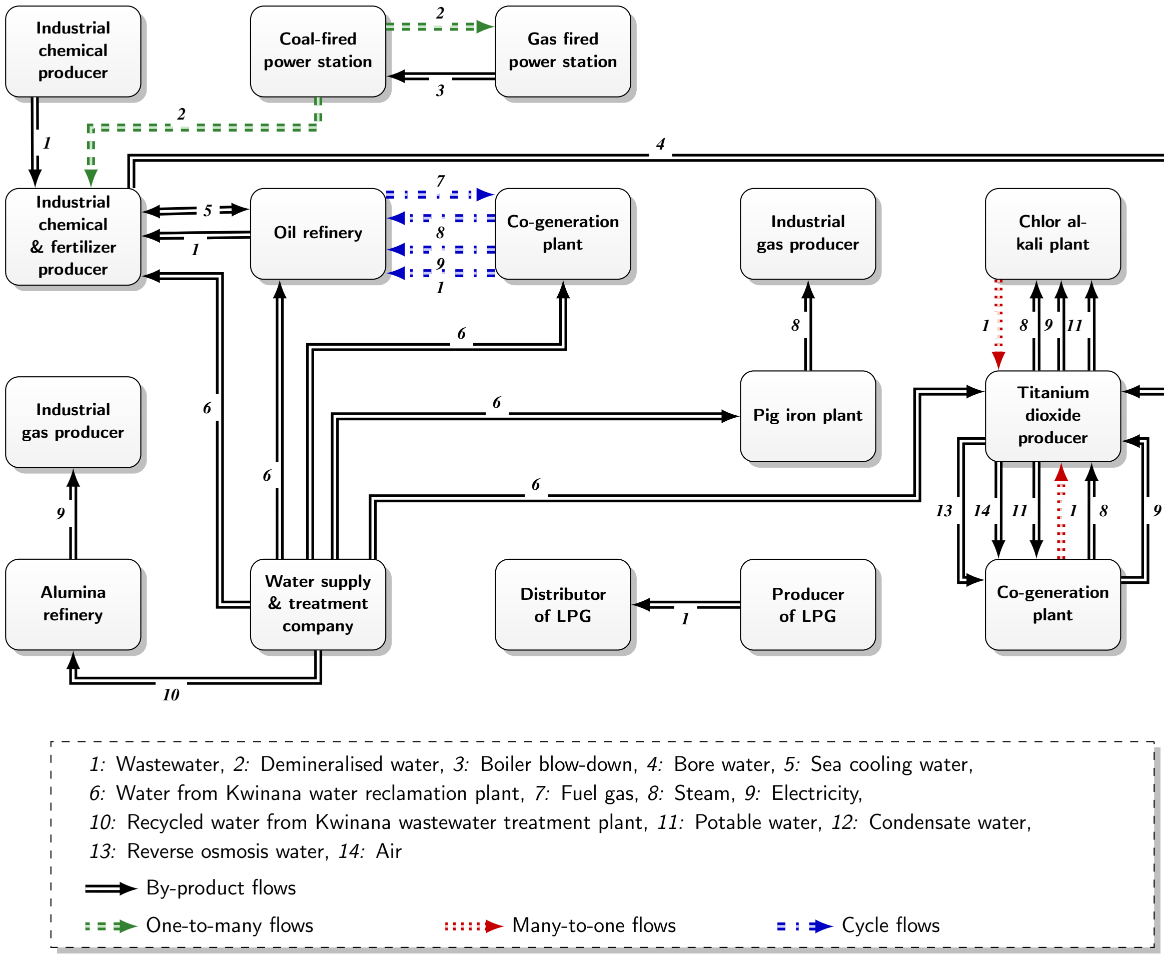


Figure 7.2: By-product synergies network flows of the Kwinana symbiosis in 2008 [van Beers, 2008]

One-to-many relationships lead to new issues, in particular, when not all the actors can be satisfied. This situation is opening up to competition between multiple suppliers or receivers. In our industrial symbiosis network, we do not consider a storage option for the by-products/raw materials for the receiver because, due to the absence of competition, we made the assumption that the receiver can buy the by-product/raw materials just in time. In a competition context, a storage of the raw materials by the receivers is needed to be considered in order to prevent the disruption to supplying the production with raw materials.

The algorithm based on the Lagrangian decomposition proposed in Chapter 5 to solve the ULS-IS problem can be easily tailored to cope with networks involving multiple suppliers and/or multiple receivers within a centralized collaboration policies or an opportunistic exchange of by-products. To do this, it is sufficient to consider an additional sub-problem per new actor that can be solved in parallel. Regarding the proposed collaboration policies, let us discuss the possible extensions of the proposed solution methods from one supplier-one receiver to multiple suppliers-multiple-receivers of by-products:

- *Sequential collaboration policies:* These policies do not seem realistic, as they require to sort (i.e. to create discrimination between) the receivers of by-products. The complexity of the particular case, where one (or multiple) leader(s) proposes (propose) a menu of contracts to its (their) followers, can become heavily tractable for sophisticated relationships between actors.

- *Negotiation-based policy:* The main difficulty to make the negotiation scheme able to handle more than three

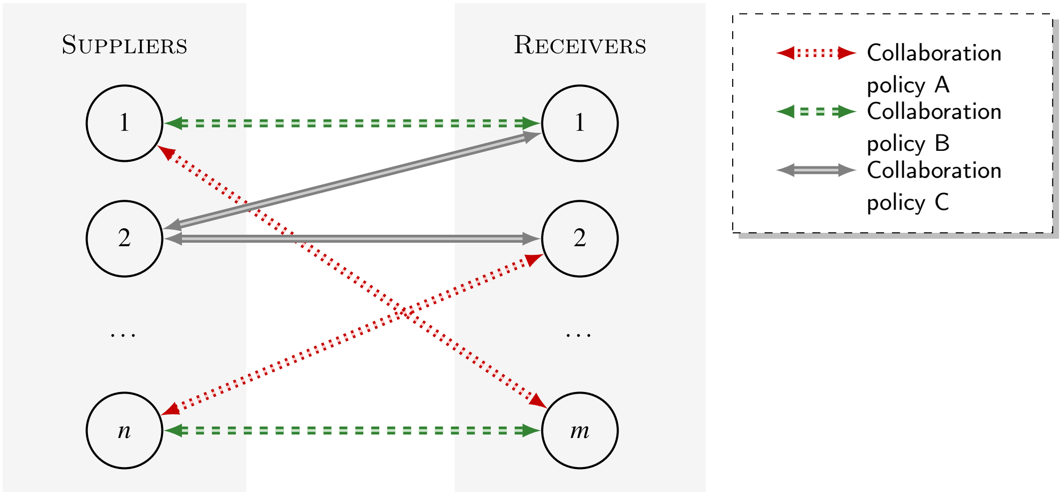


Figure 7.3: Relationships between n suppliers of by-products and m receivers in an industrial symbiosis framework

actors (namely, the third party, the supplier and the receiver of by-products) is related to the choice of a single final contract. It is challenging to find a contract, which satisfies all the actors.

- *Game-theoretic based collaboration policy:* This policy takes all its sense as there are both collaboration and competition to deal with at the same time.

Multiple suppliers-multiple receivers. The multiple suppliers-multiple receivers configuration can be also encountered in real-life eco-industrial parks (see e.g. the Kwinana symbiosis represented in Figure 7.2). The issues discussed in the previous paragraph apply. As long as more than two actors are linked by a symbiotic relationship, the framing of collaboration policies can become particularly difficult. The level of information sharing can be different for each couple of actors, and a combination of several collaboration policies could be more suitable. Even more complicated, the choice of best partner(s) of a given actor can be time-dependent (see Figure 7.3).

Moreover, the suppliers and receivers of by-products can face, at the same time, both: **(i)** a competition between them, as previously described, and **(ii)** a collaboration to share transportation or other costs, by involving a third party. Recall that in this thesis, the term *industrial symbiosis* is used for a by-product synergy. Utility sharing and joint provision of services are also parts of the industrial symbiosis, as highlighted in Chapter 2.4, and it could be interesting to consider these elements of an industrial symbiosis.

Cycle of industrial symbiosis. Another extension of interest identified in this thesis is the particular network where each actor is a supplier and a receiver at the same time. Let us denote this configuration a cycle of industrial symbiosis (see for instance, the cycle composed by the oil refinery and the co-generation plant highlighted by dashed-dotted blue arrows in Figure 7.2). In a cycle consisting of two actors, each actor can choose to produce in a period which is not the most advantageous, in return for receiving, in a next period, the by-product generated by another actor. These possible clauses make collaboration mechanisms more complicated to draw. In a cycle composed by at least three actors, other issues appear from the twofold role capability of a production unit to be at the same time a supplier and a receiver of by-products, e.g.: the limited flexibility regarding the production periods due to the demands of main products, the demands of by-products by the receiver and the availability of by-product/raw materials at the supplier. Besides the twofold technical role, production units may play different role in terms of leadership, whose coordination poses new interesting problems.

7.2.3 Sustainability impact

As previously mentioned, numerous extensions are worth to be pursued. In the context of international trade and climate change, far more attention should be paid to the consideration of environmental implications into production planning, while including recovery operations (see Chapter 3). If the economic and environmental dimensions of the sustainability are more or less studied in the production planning literature, the only found social-aware studies operate in the framework of traditional (continuous-time) economic order quantity (EOQ) models (see e.g. Battini et al. [2014], Andriolo et al. [2016]).

Besides the economic and environmental impacts, lot-sizing decisions also affect the workers health and security in terms of such human implications as the number of working hours, human fatigue and recovery, learning/forgetting effects, metabolic energy consumption and rest allowance [Andriolo et al., 2016]. The integration of human factors and ergonomics into production planning is a particularly topical subject stressed by the increasing concern for a humanly-friendly production planning. The human well-being and ergonomics in production systems remain to be explored.

To suitably respond to the growing request of the sustainability in the actual society, lot-sizing decisions are expected to adhere simultaneously to all three economic, environmental and social goals. Significant work is still needed to balance economic, environmental and social performances in production planning.

Industrial symbiosis is built on the three pillars of the sustainable development. In this thesis, the economic benefits of implementing an industrial symbiosis are largely discussed and the environmental benefits are quantified. However, some aspects remain unexplored:

- **Economic objectives:** It is important to notice that in this thesis we deal with economic objectives for each actor, but in reality, the actors do not necessarily have the same local objective. For instance, some by-products being no longer admitted in landfills, the need to reuse them is strong for the supplier. Thus, it can have an environmental objective to reach. On the contrary, the receiver can be encouraged by the economic benefits. In this configuration, it is difficult to consider a centralized collaboration policy like the one corresponding to Model (5.1)-(5.13) or a negotiation process proposed in Algorithm 6.1.

- **Environmental benefits:** The environmental benefits related to the industrial symbiosis are implicitly considered in this thesis, being expressed in terms of the reduction of waste and raw materials extraction (see Chapter 6). The environmental concerns deserve to be directly integrated in the models either by new constraints, for instance on a maximum quantity of by-products to be disposed of, or by a multi-objective optimization that maximizes the exchanges or the greenhouse gas emission savings due to the geographic proximity between the actors in eco-industrial parks (see Section 3.6).

- **Societal considerations:** The societal impact of an industrial symbiosis lies among other things in the creation of local jobs that increase the local economy. This aspect is not considered in this thesis, and poorly studied in the literature, as underlined in Chapter 3. Moreover, the societal benefits related to an industrial symbiosis are directly related to the by-products flows and then increase in the same time as the environmental benefits. However, quantitative tools and metrics have to be implemented and used while dealing with of an industrial symbiosis.

To summarize, the environmental and societal aspects have the same importance as the economic benefits in an industrial symbiosis, but they do not appear explicitly in the objective function of the problems studied in this thesis. They are only discussed to discriminate the proposed contracts in Chapter 6. A multi-objective optimization could be applied to integrate economic, environmental and societal criteria. Solution approaches proposed in this thesis, i.e. dynamic programming-based algorithms and algorithms based on the Lagrangian decomposition, as well as collaboration policies based on a negotiation process and the game theory, can be extend to deal with multiple objectives. We believe that the explicit consideration of sustainable criteria and the development of the framing of elaborated collaboration policies can provide significant value added to the existing IT platforms, which are currently focusing only on the opportunistic linkage, i.e. on the research of partnership for only short-term (mainly one-time) collaborations.

Appendix

A Computational results: Algorithm 5.1 and its variants

Table 1: Algorithm 5.1 and its variants: Gap distribution (in %) between UB and the solution obtained with `AGG`

B	Variant	Mean	Standard deviation	Max	Median
Null	`LD`	0.20	0.35	2.09	0.00
	`LD-MS`	0.17	0.32	2.09	0.00
	`LD-LS`	0.16	0.30	2.09	0.00
	`LD-MS-LS`	0.14	0.27	2.09	0.00
	`LD-LP`	0.67	0.88	5.13	0.21
	`LD-LP-MS-LS`	0.31	0.50	3.12	0.02
Non-null	`LD`	0.29	0.49	3.76	0.02
	`LD-MS`	0.27	0.47	3.76	0.01
	`LD-LS`	0.15	0.27	1.66	0.00
	`LD-MS-LS`	0.14	0.26	1.66	0.00
	`LD-LP`	0.50	0.72	4.92	0.13
	`LD-LP-MS-LS`	0.25	0.41	2.99	0.02

Table 2: Gaps (in %) of LD-MS-LS for small size instances

*SHR*1	*SHR*2	**Null** ***B***			**Tight** ***B***			**Large** ***B***		
		UB-LB	UB-OPT	OPT-LB	UB-LB	UB-OPT	OPT-LB	UB-LB	UB-OPT	OPT-LB
3	3	0.11	0.00	0.11	0.16	0.01	0.15	0.18	0.01	0.17
3	4	0.57	0.09	0.48	0.64	0.13	0.51	0.65	0.15	0.51
3	5	1.34	0.27	1.07	1.32	0.26	1.06	1.28	0.32	0.97
4	3	0.68	0.31	0.38	0.65	0.23	0.42	0.58	0.15	0.43
4	4	0.07	0.00	0.07	0.14	0.00	0.14	0.16	0.01	0.16
4	5	0.26	0.07	0.18	0.38	0.08	0.30	0.38	0.13	0.24
5	3	1.14	0.44	0.70	1.02	0.43	0.59	0.83	0.35	0.48
5	4	0.19	0.04	0.15	0.33	0.11	0.22	0.34	0.12	0.21
5	5	0.09	0.01	0.08	0.13	0.01	0.12	0.17	0.02	0.15
d^1	d^2	UB-LB	UB-OPT	OPT-LB	UB-LB	UB-OPT	OPT-LB	UB-LB	UB-OPT	OPT-LB
L	L	0.73	0.18	0.55	0.81	0.17	0.64	0.79	0.19	0.60
L	M	0.46	0.15	0.32	0.49	0.16	0.33	0.44	0.16	0.28
L	H	0.31	0.08	0.23	0.33	0.10	0.23	0.28	0.08	0.20
M	L	0.39	0.12	0.28	0.40	0.12	0.28	0.37	0.09	0.28
M	M	0.73	0.19	0.54	0.83	0.20	0.63	0.83	0.21	0.62
M	H	0.46	0.14	0.31	0.49	0.15	0.34	0.48	0.16	0.32
H	L	0.27	0.08	0.19	0.20	0.05	0.15	0.21	0.07	0.14
H	M	0.39	0.11	0.28	0.39	0.11	0.28	0.36	0.09	0.27
H	H	0.70	0.19	0.52	0.82	0.19	0.63	0.82	0.21	0.61
Average gaps		0.49	0.14	0.36	0.53	0.14	0.39	0.51	0.14	0.37

L: Low, M: Medium, H: High, UB: Upper bound, LB: Lower bound, OPT: Optimality.

Table 3: Average CPU time needed to reach the optimal solution for small size problems using formulations `AGG` and `FAL`

Param values		**Null B**						**Tight B**						**Large B**					
	$SHR1$	3		4		5		3		4		5		3		4		5	
$SHR2$	Δ	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`
3	L	0.29	0.28	0.65	0.84	0.76	0.98	0.32	2.65	0.54	6.27	0.68	8.02	0.25	2.09	0.49	7.72	0.57	5.22
3	M	0.30	0.27	0.63	0.79	0.75	1.04	0.33	2.41	0.53	6.27	0.69	9.18	0.25	2.11	0.47	6.64	0.62	5.60
3	H	0.33	0.26	0.62	0.78	0.76	1.05	0.34	2.47	0.52	5.49	0.67	9.60	0.25	2.13	0.47	6.46	0.60	5.28
4	L	0.38	0.88	0.50	0.23	0.50	0.48	0.49	7.13	0.50	2.70	0.68	3.66	0.39	5.86	0.50	2.53	0.64	4.20
4	M	0.39	0.88	0.47	0.22	0.51	0.50	0.48	7.37	0.52	2.62	0.68	4.09	0.42	5.81	0.48	2.51	0.64	4.27
4	H	0.40	0.97	0.49	0.20	0.51	0.59	0.50	7.25	0.54	2.56	0.66	4.43	0.42	6.13	0.47	2.44	0.61	4.47
5	L	0.47	1.14	0.50	0.59	0.55	0.27	0.55	16.18	0.62	7.17	0.64	2.67	0.48	9.71	0.54	5.83	0.59	2.61
5	M	0.48	1.03	0.46	0.57	0.55	0.27	0.59	13.87	0.60	6.75	0.63	2.70	0.51	8.62	0.57	5.55	0.59	2.49
5	H	0.45	1.12	0.53	0.41	0.52	0.28	0.58	12.41	0.63	6.55	0.63	2.71	0.49	7.33	0.53	5.21	0.63	2.55
	d^1	L		M		H		L		M		H		L		M		H	
d^2	Δ	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`	`AGG`	`FAL`
L	L	0.59	0.84	0.47	0.45	0.45	0.37	0.64	8.34	0.56	4.94	0.44	2.35	0.57	7.94	0.42	2.23	0.42	2.00
L	M	0.58	0.79	0.48	0.47	0.46	0.38	0.65	8.23	0.53	5.62	0.41	2.42	0.60	7.59	0.45	2.48	0.39	2.05
L	H	0.59	0.73	0.53	0.57	0.44	0.43	0.63	7.92	0.53	5.93	0.43	2.45	0.58	7.08	0.45	2.56	0.41	2.06
M	L	0.51	0.68	0.59	0.83	0.50	0.41	0.57	6.20	0.63	8.77	0.54	4.67	0.53	4.59	0.57	9.09	0.41	2.30
M	M	0.47	0.65	0.56	0.81	0.48	0.48	0.56	5.70	0.66	8.48	0.51	5.30	0.53	4.42	0.62	8.21	0.41	2.50
M	H	0.52	0.65	0.55	0.75	0.51	0.58	0.58	5.21	0.68	8.28	0.55	5.66	0.49	4.16	0.60	8.11	0.48	2.57
H	L	0.45	0.60	0.47	0.72	0.59	0.80	0.51	5.63	0.54	7.01	0.62	8.53	0.45	4.25	0.52	4.81	0.57	8.58
H	M	0.45	0.55	0.51	0.63	0.56	0.79	0.51	5.30	0.56	6.31	0.65	7.90	0.48	4.01	0.49	4.36	0.59	7.96
H	H	0.45	0.53	0.50	0.67	0.51	0.72	0.48	4.85	0.52	5.51	0.65	7.66	0.43	3.87	0.48	4.04	0.55	7.54
Avg CPU time (s)		0.51	0.63					0.56	6.12					0.50	4.87				

L: Low, M: Medium, H: High.

Table 4: Gaps (in %) and CPU times: AGG *and* LD-MS-LS

Parameters		**Null** *B*						**Tight** *B*						**Large** *B*					
*SHR*1	*SHR*2	Gaps				CPU time		Gaps				CPU time		Gaps				CPU time (s)	
		$\mathtt{AGG_u}$	$\mathtt{LD_u}$	$\mathtt{AGG_l}$	$\mathtt{LD_l}$	AGG	LD	$\mathtt{AGG_u}$	$\mathtt{LD_u}$	$\mathtt{AGG_l}$	$\mathtt{LD_l}$	AGG	LD	$\mathtt{AGG_u}$	$\mathtt{LD_u}$	$\mathtt{AGG_l}$	$\mathtt{LD_l}$	AGG	LD
3	3	0.00	0.03	0.00	0.18	1.95	1.61	0.00	0.04	0.00	0.22	0.65	8.49	0.00	0.03	0.00	0.22	0.68	8.91
3	4	0.00	0.31	0.05	0.10	8.94	1.57	0.00	0.26	0.00	0.41	8.32	8.03	0.00	0.18	0.00	0.3	6.12	8.50
3	5	0.00	0.73	0.18	0.07	9.07	1.50	0.00	0.56	0.02	0.34	9.82	8.11	0.00	0.36	0.02	0.26	7.95	8.36
4	3	0.00	0.38	0.09	0.06	9.09	1.60	0.00	0.32	0.00	0.57	6.68	7.83	0.00	0.33	0.00	0.43	5.53	8.50
4	4	0.01	0.06	0.31	0.03	9.88	1.44	0.00	0.09	0.00	0.33	4.09	7.58	0.00	0.13	0.00	0.23	5.30	8.45
4	5	0.00	0.30	0.65	0.01	10.02	1.40	0.00	0.23	0.04	0.16	9.75	7.77	0.00	0.22	0.04	0.10	8.28	7.95
5	3	0.00	0.61	0.22	0.05	10.01	1.49	0.00	0.47	0.01	0.60	8.05	7.56	0.00	0.36	0.01	0.47	7.16	7.67
5	4	0.00	0.37	0.70	0.00	10.02	1.42	0.00	0.28	0.06	0.27	8.91	7.34	0.00	0.28	0.05	0.21	8.63	7.66
5	5	0.00	0.05	0.64	0.00	10.01	1.29	0.00	0.04	0.02	0.25	8.09	7.07	0.00	0.08	0.02	0.19	7.59	7.86
d^1	d^2	$\mathtt{AGG_u}$	$\mathtt{LD_u}$	$\mathtt{AGG_l}$	$\mathtt{LD_l}$	AGG	LD	$\mathtt{AGG_u}$	$\mathtt{LD_u}$	$\mathtt{AGG_l}$	$\mathtt{LD_l}$	AGG	LD	$\mathtt{AGG_u}$	$\mathtt{LD_u}$	$\mathtt{AGG_l}$	$\mathtt{LD_l}$	AGG	LD
L	L	0.00	0.42	0.34	0.11	9.06	1.47	0.00	0.35	0.01	0.67	7.91	7.70	0.00	0.31	0.01	0.51	8.17	8.44
L	M	0.00	0.20	0.32	0.03	8.73	1.49	0.00	0.18	0.04	0.17	7.68	7.30	0.00	0.16	0.03	0.15	7.49	7.41
L	H	0.00	0.11	0.21	0.06	8.41	1.49	0.00	0.10	0.04	0.14	7.05	7.27	0.00	0.08	0.03	0.12	6.73	7.23
M	L	0.00	0.38	0.36	0.03	8.72	1.48	0.00	0.28	0.01	0.25	6.71	8.55	0.00	0.16	0.01	0.19	3.73	9.10
M	M	0.00	0.40	0.33	0.10	8.95	1.46	0.00	0.36	0.00	0.69	7.95	7.58	0.00	0.35	0.01	0.50	8.56	8.37
M	H	0.00	0.21	0.33	0.03	8.73	1.50	0.00	0.21	0.04	0.17	7.87	7.36	0.00	0.17	0.03	0.15	7.47	7.46
H	L	0.00	0.27	0.22	0.04	8.54	1.49	0.00	0.15	0.00	0.12	4.31	7.99	0.00	0.16	0.00	0.10	3.11	8.30
H	M	0.00	0.41	0.38	0.02	8.78	1.48	0.00	0.29	0.01	0.26	6.91	8.47	0.00	0.18	0.01	0.21	3.63	9.11
H	H	0.00	0.44	0.33	0.10	9.06	1.46	0.00	0.38	0.01	0.69	7.97	7.56	0.00	0.38	0.01	0.50	8.32	8.45

L: Low, M: Medium, H: High.

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The work previously described leads to several communications.

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NNT :

Élodie SUZANNE

Production planning for industrial symbiosis

Specialty: Industrial Engineering

Keywords: Circular economy, Production planning, By-product, Lot-sizing, Industrial symbiosis

Abstract:

Under the pressure of global warming and depletion of Earth natural resources, the increasing willingness to produce sustainable goods determines us to stop thinking linearly and to shift towards a circular approach by closing material loops. Consistent with these global trends, the industrial symbiosis seeks to generate sustainable advantages by binding traditionally separate industrial processes into a joint production approach, involving physical exchange of by-products or other collateral resources. This thesis introduces new production planning problems raised within industrial symbiosis networks, namely: (i) *at the production unit level*: a single-item lot-sizing problem with inventory capacities, which integrates the management of by-products, and (ii) *network-wide:* a two-level lot-sizing problem, which aims to align lot-sizing decisions of two actors: one supplier of by-products and its receiver. The complexity of the studied problems is analyzed and efficient algorithms based on dynamic programming and Lagrangian decomposition are proposed. To support the framing of symbiotic partnerships within an industrial symbiosis network, we investigate several collaboration policies for different levels of information sharing: (i) *centralized and decentralized collaboration policies* based on mixed-integer linear programming, for full and none information sharing, (ii) a *game-theoretic collaboration policy* for asymmetric information sharing, and (iii) a *contractual-based collaboration policy* obtained via a negotiation-based scheme managed by a blinded mediator, for symmetric feedback sharing. Through extensive numerical experiments, the proposed collaboration policies are discussed according to three dimensions: the satisfaction of actors, environmental implications and economic benefits.

École Nationale Supérieure des Mines
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NNT : *Communiqué le jour de la soutenance*

Élodie SUZANNE

Planification de la production pour la symbiose industrielle

Spécialité: Génie Industriel

Mots clefs : Économie circulaire, Planification de la production, Lot-sizing, By-product, Symbiose industrielle

Résumé :

Motivés par l'accumulation des déchets et la raréfaction des ressources naturelles, les modèles de production industrielle évoluent pour favoriser le développement d'une économie circulaire. Dans ce sens, la symbiose industrielle, vise à valoriser les déchets via un procédé industriel impliquant les échanges physiques de by-products ou d'autres ressources collatérales liants plusieurs processus de production. Cette thèse introduit des nouveaux problèmes de planification de la production posés par les symbioses industrielles: (i) *au niveau d'une unité de production :* un problème de lot-sizing avec des capacités de stockage, et (ii) *à l'échelle d'un réseau :* un problème de lot-sizing à deux niveaux, qui vise à coordonner les décisions de lot-sizing d'un fournisseur et d'un receveur de by-products. La complexité de ces problèmes est analysée et des algorithmes efficaces basés sur la programmation dynamique et la décomposition Lagrangienne sont proposés. Pour cadrer les partenariats symbiotiques, nous étudions plusieurs politiques de collaboration (PC) pour différents niveaux de partage d'information : (i) *des PCs centralisées et décentralisées* basées sur la programmation linéaire mixte, pour un partage d'information complet et nul, (ii) *une PC basée sur la théorie des jeux* pour un partage d'information asymétrique, et (iii) *une PC basée sur des contrats* obtenue par un schéma de négociation piloté par un médiateur impartial, pour un retour d'information symétrique. Au travers d'expérimentations numériques, les politiques de collaboration proposées sont discutées selon trois dimensions : la satisfaction des acteurs, les implications environnementales et les gains économiques.